THE

PRINCIPLES AND PRACTICE

OF

COMMERCIAL ARITHMETIC

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BY

P. W. NORRIS, M.A. (Lond.)
B.Sc. (Hons.)
HEAD OF MATHEMATICAL DEPARTMENT,
ALLEYN'S SCHOOL, DULWICH

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PREFACE

ARITHMETIC is one of the most ancient branches of knowledge, and in course of time several hundreds of books have been written on this subject. It thus falls to my lot to make a justification for the production of the present volume.

Text-books in Arithmetic a hundred or so years ago were almost purely academic in character, and some of the problems they contained are found very amusing by students of the present day. For example, in an old Italian text-book, a snake was stated to have climbed up the inside of a well a certain distance by day and to have slid down a certain distance each night, and the question was to calculate the time taken to reach the top. The answer worked out to be a number of hundreds of days, but the economic question as to how the poor creature obtained its daily supply of food was left to the imagination. As time went on methods were improved, and examples have become more of a practical nature. It has been with the hope of continuing this process of evolution that the present book has been written.

In many of the text-books published in recent years the methods have been excellent, and it would be presumptuous for an author to claim that his methods were preferable to these. In Section I of this book, all the fundamental laws of Arithmetic which are necessary for the solution of the commercial problems in the later portions have been set forth as briefly as possible, certain processes have been analysed, and in the worked examples have been adopted those methods which have reduced the amount of mechanical work to a minimum. Sections II–V give information concerning Business Undertakings, Trade, Banking, Finance, and Insurance. It is here that the great varieties of problems calling for Arithmetical solution arise, and the writer finds his justification in seeking to answer these calls.

This book is primarily meant to be of use to advanced students who wish to obtain a thorough knowledge of Commercial subjects and are preparing for the advanced stages of the examinations in Commercial Arithmetic held by the Royal Society of Arts, The Institute of Bankers, The London Chamber of Commerce, and other examining bodies. It is also hoped it will be found useful by teachers, for it contains sufficient matter for a course lasting four or five years.

My warmest thanks are due to those examining bodies who have so kindly permitted me to use a number of examples taken from recent examination papers; and to Messrs. C. E. Kerridge, B.Sc., and B. E. Lawrence, for valuable assistance in the work of compiling answers. I should also like to express my sincere appreciation of the helpful advice given to me by Dr. J. Stephenson, M.A., M.Com., D.Sc., and my gratitude to Mr. S. Sherring and other gentlemen for supplying information otherwise difficult to obtain.

The Appendix contains a set of tables which are of considerable practical utility.

P. W. N.

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THE PRINCIPLES AND PRACTICE OF COMMERCIAL ARITHMETIC

SECTION I

PRINCIPLES OF ARITHMETIC

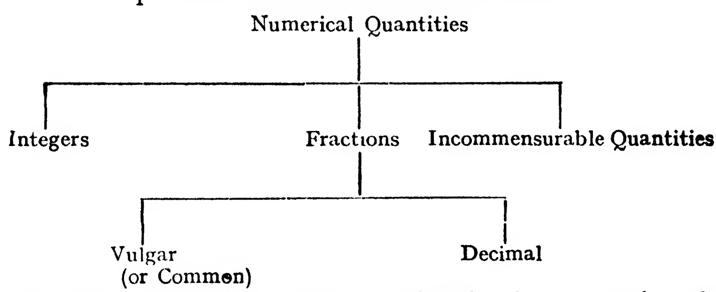
INTRODUCTION

A NUMBER considered by itself is an **abstract quantity,** but when associated with a unit the number and unit together form a **concrete quantity.**

In problems connected with business, the measurements of concrete quantities play an important part; and, in order to deal with these measurements, certain laws established by consideration of number in the abstract are employed: hence a thorough knowledge of these laws is essential.

Arithmetic is a **science** as regards the establishment of these laws and an **art** in the applications of these laws to solve problems of everyday life.

Numerical quantities are subdivided as follows—



An integer is a whole number. A fraction is a part of a whole one, and can be expressed in either of the two ways, although

to express certain fractions in the decimal notation, recurring decimals are involved. Any vulgar fraction having a denominator which is a factor of a power of 10 can be expressed in the decimal notation without recurring figures, but those whose denominators are not factors of a power of 10 can only be expressed exactly by employing recurring decimal figures. The number of figures that recur cannot exceed the number expressing the denominator. A recurring decimal fraction can always be expressed exactly as a vulgar fraction, *e.g.*, $\cdot 8\dot{2}8571\dot{4} = \frac{29}{35}$; but in calculations, as vulgar fractions are much more easy to manipulate than recurring decimal fractions, it is obvious that, although the latter may have theoretical interest, they have no practical value, and so have no claim for consideration in this book.

Most quantities met with in practical life are incommensurable quantities, which are such as to be incapable of exact expression. If a nugget of gold be discovered, the chances are thousands to one against its weight being such that it can be exactly expressed. It possesses a definite weight, but this probably cannot be expressed, and thus its weight is probably an incommensurable quantity. Another example of an incommensurable quantity is $\sqrt{2}$, which can be expressed exactly neither as a vulgar nor as a decimal fraction. The length of a metre, if measured in inches, also results in an incommensurable quantity. In practice, whenever a quantity is incommensurable, its value is considered as a commensurable amount which, however, differs from the actual amount by something sufficiently small as to be negligible. Thus 1 metre is, in some problems, considered as being 39·37 inches, which is a commensurable quantity differing from 1 metre by an amount less than $\frac{1}{200}$ inch. Denoting an incommensurable quantity by a commensurable quantity whose value differs from the former by a small amount is known as approximating. Many mistakes arise in the working out of problems through the approximations made being such as finally result in an error which is not negligible. The chapter dealing with Approximations is the most difficult part of the book, but it should be mastered by the student who aims for thoroughness, although by employing logarithms the necessity for the application of certain of the rules given is avoided.

A knowledge of Algebra, however elementary, is a mental

equipment of great value in the study of more advanced Arithmetic. This Section aims at giving the student a comprehensive knowledge of Arithmetic, and therefore just sufficient of the notation of Algebra is dealt with as is necessary to render this possible. The time spent in mastering this fringe of Algebra will repay the student a hundredfold, in that he will be able to grasp the generalizations and will be enabled to obtain solutions to questions with the minimum of calculations—a great asset, especially for examination purposes.

It is assumed that the student reading this book already possesses knowledge of the elements of Arithmetic, so that the subjects dealt with are grouped in accordance with the ideas involved, but not necessarily in the order that they should be taught to the young when first learning these subjects. For example, it is almost generally accepted that the Metric System should immediately follow the work on the British System of Weights and Measures, and that the decimal notation and the simple rules of decimals should be based on the Metric System and taught before the rules involved in Vulgar Fractions are dealt with. Here, however, the Metric System is introduced after the work on contracted methods, in order to afford illustration of the ideas involved in approximations.

Although this Section attempts to establish the principles of Arithmetic, yet direct applications are given with the purpose of ensuring their being understood. The exercises, therefore, although being as practical as possible, are such that they do not require a knowledge of commercial terms or practice other than those met with in ordinary affairs of everyday life. This Section, then, as far as Arithmetic is concerned, is complete in itself, and contains exercises having a commercial bias.

Those paragraphs and exercises which might with advantage be left until a second reading are marked with an asterisk.

CHAPTER I.

INTEGERS—DEFINITIONS OF TERMS AND SYMBOLS—APPLICATIONS.

1. It is assumed that the student is perfectly familiar with the processes of addition, subtraction, multiplication, and division as regards whole numbers.

The result of adding two or more numbers together is called the **Sum** of the numbers.

The result of subtracting one number from another is called the **Difference** of the numbers.

The result obtained by multiplying two or more numbers together is called the **Product** of the numbers. It should be noticed that multiplication is a condensed form of addition when the numbers to be added together are equal. Thus, when twelve nines are added together, the result is one hundred and eight.

When one number is divided by another, the result is called the **Quotient,** while the former number is called the **Dividend** and the latter the **Divisor.** While division can be regarded as the art of finding the result when a quantity is divided into a number of equal parts, it can also be looked upon as a condensed form of repeated subtraction when the numbers subtracted are all equal. Thus, 3 could be taken from 45 fifteen times.

2. A **Factor** of a given number is a number that is contained in the given number a whole number of times. Thus, 7 is a factor of 21 or 35, but not a factor of 18 or 40.

In the case of division, when the divisor can be factorized, each factor being not greater than 12, it is sometimes advisable from the point of view of speed to divide, short division, by each factor in turn, the remainder being obtained as shown by consideration of the following example.

Example (i)—

An exporter has 374,850 tins of fruit, which are to be packed in cases each capable of holding 196 tins. How many cases will be required? How many tins will the last case contain?

The number of cases will be obtained by dividing 374,850 by 196, the factors of which are 4, 7, and 7.

$$\begin{array}{r|l}
4 & 374{,}850 \\ \hline
7 & 93{,}712 \text{ rem. } 2 \\ \hline
7 & 13{,}387 \text{ rem. } 3 \\ \hline
 & 1{,}912 \text{ rem. } 3
\end{array}\left.\vphantom{\begin{array}{l}a\\b\\c\end{array}}\right\}\text{total rem. } 98$$

Ansr—1,913 cases: 98 tins

NOTE 1.—374,850 is a number of tins, and on dividing by 4 the result is 93,712 groups of 4 tins and a remainder of 2 tins; on dividing by 7 the result is 13,387 groups of 28 tins and a remainder of 3 groups of 4 tins (*i.e.*, 12 tins); on again dividing by 7 the result is 1,912 groups of 196 tins and a remainder of 3 groups of 28 tins (*i.e.*, 84 tins). Thus, the total remainder is obtained by adding 2, 12, and 84 tins (*i.e.*, 98 tins).

NOTE 2.—If it were required to divide a smaller number, such as 1,450, by 196, the long division method would entail less work than division by factors.

3. THE "SHORTHAND" OF ARITHMETIC.

As the solution of most problems consists of a number of statements following in logical order one from another, in order to save time a number of shorthand signs are introduced, which are as follows—

$+$	(or *plus*)	signifies	**Addition**
$-$	(or *minus*)	,,	**Subtraction**
$\times$		,,	**Multiplication**
$\div$		,,	**Division**

$=$ means "is equal to" and signifies **Equality**

$\therefore$ signifies **Therefore**

EXAMPLES SHOWING USE OF SIGNS—

(ii) On Monday, at noon, at a certain place the temperature was 63° F. On the following five days the rise or fall was as follows: Rise of 5° F., fall of 3° F.; fall of 6° F.; fall of 11° F.; rise of 5° F. What was the temperature on the last day at noon?

Temperature on last day at noon $= 63 + 5 - 3 - 6 - 11 + 5$ degrees F.
$= 73 - 20$ degrees F.
$= 53°$F.

(iii) What is the total cost of 5 articles at 3s. each, 3 articles at 2s. each, and 1 article at 4s.?

Total cost $= 5 \times 3 + 3 \times 2 + 4$ shillings
$= 15 + 6 + 4$,,
$= 25/-$

NOTE 3.—A sign affects only the quantity immediately after it. When $+$, $-$, $\times$, and $\div$ signs occur together, the convention is that the value of the expression is that obtained by dealing with the $\times$ and $\div$ signs first and then the $+$ and $-$ signs afterwards. The $\div$ sign, although sometimes used, is not really necessary, for it is much more convenient to denote division by placing the divisor under the dividend; *e.g.*, $\frac{9}{3} = 3$.

The following simple examples of simplification illustrate these conventions—

$5 - 12 \div 3 + 3 \times 2 \times 7 = 5 - 4 + 42$
$= 47 - 4$
$= 43$

$$\frac{7 \times 5 - 3}{5 + 3} = \frac{35 - 3}{8} = \frac{32}{8} = 4$$

4. CANCELLING

Division, being the reverse process to multiplication, it is obvious that if a given number be multiplied by a number and the product divided by the same number, the quotient will be the original number.

Thus, $\frac{365 \times 117}{117} = 365.$

Suppose it be required to know the number of times 12 lb of tea is contained in 96 lb. of tea. The number is, of course, $\frac{96}{12}$. Now, if the tea be wrapped in packets each containing 2 lb. of tea, the question now becomes: How many times are 6 packets contained in 48 packets? The result is $\frac{48}{6}$. Again, if the tea be wrapped in packets each containing 3 lb. of tea, the question now becomes: How many times are 4 packets contained in 32 packets? The result is $\frac{32}{4}$.

Thus $\frac{96}{12} = \frac{48}{6} = \frac{32}{4}$.

From considerations such as the above, the following rule is established—

If the dividend and the divisor are each multiplied or each divided by the same number, the quotient is unaltered.

NOTE 4.—The 5 in the dividend cannot be cancelled with the 5 in the divisor in the expression $\frac{5 + 7}{5 \times 6}$, for by so doing 5 is **subtracted** from the dividend, whereas the dividend is **divided** by 5. Cancelling can only take place when the dividend and the divisor are capable of being divided by the same factor.

EXAMPLE (iv)—

If the cost of paper in the production of 75,000 magazines is £305, for how many similar magazines would £732 worth of paper be sufficient?

For £1, paper could be bought sufficient for $\frac{75,000}{305}$ magazines

$\therefore$ „ £732, „ „ „ $\dfrac{\overset{15,000}{\cancel{75,000}} \times \overset{12}{\cancel{732}}}{\underset{\cancel{61}}{\cancel{305}}}$ magazines

$= 180,000$ magazines.

NOTE 5.—In cancelling, dividend and divisor are first divided by 5 and then by 61.

5. USE OF BRACKETS.

Consider the following example: A merchant bought 17 bags each containing 40 lb. of rice, 17 bags each containing 56 lb. of

rice, and 34 bags each containing 75 lb. of rice. What was the total number of lbs. of rice bought ?

The answer is obviously $17 \times 40 + 17 \times 56 + 34 \times 75$ lb. Now suppose the merchant divided the rice into 17 equal groups each containing 1 bag of 40 lb., 1 bag of 56 lb., and 2 bags of 75 lb., then each group contains $40 + 56 + 2 \times 75$ lb. In order to retain this latter as a single quantity, it is enclosed in a bracket. Thus each group contains $(40 + 56 + 2 \times 75)$ lb. $\therefore$ 17 groups contain $17 \times (40 + 56 + 2 \times 75)$ lb.

Thus $17 \times 40 + 17 \times 56 + 34 \times 75 = 17 \times (40 + 56 + 2 \times 75)$. The right-hand side expression entails one multiplication by 17, whereas the left-hand side requires three multiplications by 17. In any expression the quantities separated by plus or minus signs are known as terms, and by considerations similar to the above the following important rule is deduced—

An expression consisting of the sum or difference of a number of terms each having a common factor is equal to the product of this common factor and a quantity formed by dividing the expression by the common factor term by term.

EXAMPLE (v)—

Evaluate $719 + 719 \times 193 - 719 \times 134$.

$$\begin{aligned} 719 + 719 \times 193 - 719 \times 134 &= 719 \times (1 + 193 - 134) \\ &= 719 \times 60 \\ &= 43{,}140 \end{aligned}$$

NOTE 6.—$10 \times 753 = 10 \times (700 + 50 + 3) = 7{,}000 + 500 + 30 = 7{,}530$. Similarly, $1{,}000 \times 753 = 753{,}000$, etc. Also by applying the reverse process $\frac{7{,}534}{100} = \frac{7{,}500 + 34}{100} = 75$ with a remainder of 34. If it be required to divide 17,951 by 900, proceed as follows—

$$\begin{array}{l|l} 9 & 179(51 \\ \hline & 19 \text{ rem. } 851 \end{array}$$

Mark off the last two figures (*i.e.*, divide the number by 100). The result is 179 groups of 100 and rem. 51 units. On dividing by 9, 19 groups of 900 are obtained, and 8 groups of 100 is the rem. Thus, the complete rem. is $8 \times 100 + 51 = 851$.

NOTE 7.—The principle established in paragraph 5 is the basis of the method of long multiplication—

e.g., $4{,}573 \times 829 = 4{,}573 \times (800 + 20 + 9)$

$$\begin{array}{r l} 4{,}573 & \\ 829 & \\ \hline 36584 & = 4{,}573 \times 800 \\ 9146 & = 4{,}573 \times 20 \\ 41157 & = 4{,}573 \times 9 \\ \hline 3{,}791{,}017 & = 4{,}573 \times 829. \\ \hline \end{array}$$

It is understood that "0"s should be present at the places where gaps occur in the units and tens columns.

EXAMPLE (vi)—

A tea merchant blended together 31 cases of tea each containing 56 lb, 36 cases each containing 84 lb. and 28 cases each containing 112 lb. All the tea was then put into cases each containing 84 lb. How many of the latter were required?

$$\text{Total amount of tea} = 31 \times 56 + 36 \times 84 + 28 \times 112 \text{ lb.}$$
$$= 28 \times (31 \times 2 + 36 \times 3 + 112) \text{ lb.}$$
$$= 28 \times (62 + 108 + 112) \text{ lb.}$$
$$= 28 \times 282 \text{ lb.}$$

$$\text{No. of cases required} = \frac{\cancel{28} \times \overset{94}{\cancel{282}}}{\underset{3}{\cancel{84}}} = \underline{\underline{94 \text{ cases}}}$$

6. AVERAGE.

The average of a set of quantities is their sum divided by the number of quantities.

EXAMPLE (vii)—

The following data refers to the tramways of a provincial town. Find to the nearest whole number the average number of passengers carried by each tram per day throughout the whole period, the trams running on Sundays.

	1925.	1926.	1927.	1928.	1929.	1930.
Average number of passengers per day . .	18,740	20,450	23,100	26,420	28,360	27,430
Average number of trams per day	40	44	51	55	56	52

Total number of passengers—

$= 365 \times (18,740 + 20,450 + 23,100 + 26,420 + 28360 + 27,430) + 26,420$
$= 365 \times 144,500 + 26,420 = 52,768,920$

```
    144,500
     36 5
  ---------
  43 35
   8 670
     722 5
  ---------
 52,742,500
```

Total number of daily runs—

$= 365 \times (40 + 44 + 51 + 55 + 56 + 52) + 55$
$= \quad 365 \times 298 \quad + 55$
$= \quad 365 \times (300 - 2) + 55$
$= 109,500 - 730 + 55$
$= 108,825$

∴ Average number of passengers per tram per day—

3,517,928	484
10,553,784	7,255)3,517,928
52,768,920	29,020
108,825	61,592
21,765	58,040
7,255	35,528
	29,020
= 485 passengers	6,508

NOTE 8.—All the working required by this example is shown. The additional term 26,420 outside the bracket occurs as 1928 is a Leap Year; the product 366 × 26,420 = 365 × 26,420 + 26,420. As the remainder 6,508 is more than half the divisor, 7,255, the quotient to the nearest whole number is 485.

NOTE 9.—The Italian method of long division combines the steps of multiplying and subtracting, and thus the working of the above long division is as follows

```
        484
7,255)3,517,928
        61,592
         35,528
          6,508
```

7. GENERALIZATIONS.

It very often happens that a number of problems of a similar type are required to be solved, and the methods by which the solutions are obtained are the same in each. If the methods are at all involved, it entails a great amount of wording in stating the rule by which the answer can be obtained. In order to present any rule in a clear and concise form, a form of shorthand can be introduced as the following example shows—

A quantity of nuts is to be equally divided among a number of boys and girls. State the rule giving the number of nuts each will receive. The rule, of course, is: Divide the number of nuts by the sum of the numbers of boys and girls. This rule can be expressed in "shorthand" as follows—

Let $N \equiv$ no. of nuts to be divided*
" $b \equiv$ " boys
" $g \equiv$ " girls
" $n \equiv$ " nuts each boy and girl receives

then $n \equiv \dfrac{N}{b+g}$

This result $n = \dfrac{N}{b+g}$ is called a **formula**, so that a formula is really a statement in "shorthand" giving the rule as to how a

* The symbol $\equiv$ signifies "stand for."

particular type of problem can be solved. In higher work in Arithmetic, the formation, interpretation, and manipulation of formulae play so important a part, that the student should as soon as possible take an intelligent interest in them.

When letters are used denoting certain numbers, the notation as regards the symbols $+$, $-$, $\times$, and $\div$ is the same as with the numbers themselves, except that the multiplication sign is generally left out. As with numbers, whenever a bracket occurs, its contents must be regarded as a single numerical quantity.

The sum of	a and b is	$a + b$
The difference of	a and b is	$a - b$ or $b - a$
The product of	a and b is	ab
The ,,	a and a is	a^2
The ,,	a, b, c is	abc
The ,,	a^2, ab, c is	a^3bc, etc.
The quotient	$a \div b$ is	$\frac{a}{b}$
The product of a and $(x + y)$ is		$a(x + y)$
The quotient obtained by dividing the sum of a and b by their product is		$\frac{a + b}{ab}$

EXAMPLE (viii)—

A tailor has m rolls of cloth each containing x yds, and n rolls each containing y yds. How many coats could he make with this cloth if each coat requires c yds.?

Let $n \equiv$ no. of coats that can be made

$$\text{then } n = \frac{mx + ny}{c}$$

EXAMPLE (ix)—

A boy earns s shillings per week. He gives a shillings per week to his parents, spends b shillings a week, and saves the remainder. How long will he take to save 50 shillings?

Let $w \equiv$ no. of weeks taken to save 50/-

$$\text{then } w = \frac{50}{s - a - b}$$

The important rule stated in paragraph 5 can be expressed in the following way: If a, b, c, and x represent any four integers, then

$$\mathbf{ax + bx - cx = x(a + b - c)}*.$$

EXAMPLE (x)—

The following has been stated to be one of the "wonders" of number: Write down any number consisting of three digits and form the number by repeating these digits; then the latter is always divisible (*i.e.*, capable of being divided without a remainder) by 7, 11, and 13. Explain the reason of this.

* This identity is shown to be true when a, b, c, and x are fractions.

Suppose a start is made with the number 713.

Then it is required to show that 713,713 is divisible by 7, 11, and 13.

Now, $713{,}713 = 713{,}000 + 713$
$= 713 \times (1{,}000 + 1) = 713 \times 1{,}001$

As $1{,}001 = 7 \times 11 \times 13$, 713,713 must be divisible by 7, 11, and 13

The generalization of this is as follows—

Suppose a, b, and c are the digits;
then the original number is $100a + 10b + c$
(for comparing with 713, the latter is $700 + 10 + 3$).

Then it is required to show that
$100{,}000a + 10{,}000b + 1{,}000c + 100a + 10b + c$
is divisible by 7, 11, and 13.

This expression can be put in the form
$$1{,}000 \times (100a + 10b + c) + (100a + 10b + c)$$
$$= (100a + 10b + c)(1{,}000 + 1)$$
$$= 1{,}001 \times (100a + 10b + c),$$ so that 1,001 is a factor.

Therefore, 7, 11, and 13 are factors.

EXAMPLE (xi)—

A dealer bought n electric torches at S shillings each. He sold m of them for T shillings each and the remainder for R shillings each. What was his profit?

Let x sh. $\equiv$ his profit
$$\text{then } x = mT + R(n - m) - nS$$
$$= mT + Rn - Rm - nS$$
$$= m(T - R) + n(R - S)$$

EXAMPLE (xii)—

A number consists of four digits. Show that if their sum is divisible by 3 and 9, the number is divisible by 3 and 9 respectively.

Let a, b, c, and $d \equiv$ the digits;

then $1{,}000a + 100b + 10c + d \equiv$ the number.

This can be written in the form
$$999a + 99b + 9c + (a + b + c + d),$$
the first three terms of which are divisible by 3 and 9, so that if $(a + b + c + d)$ is divisible by 3 or 9, the whole quantity is divisible by 3 and 9 respectively.

TEST EXERCISES I, 1.

1. Add the following in rows and columns, and verify your answers by finding the grand total in each case.

							TOTAL.
714	4,691	3,026	2,198	4,638	629	817	
3,469	1,238	907	3,687	794	2,954	2,468	
85	497	4,698	1,438	8,329	368	947	
392	563	324	7,105	146	7,191	1,393	
1,468	2,486	762	96	1,799	785	746	
5,049	719	3,496	564	483	6,543	2,375	
Total							

(2) Write down columns of figures as in Question (1), and add in rows and columns in order to gain practice in speed and accuracy. Test your answers by comparing the grand total in each case.

(3) The areas of Kent, Surrey, Middlesex, Essex, and Yorkshire are 995,392, 485,129, 181,317, 987,032, and 3,882,851 acres respectively. How much greater is the area of Yorkshire than the total area of Kent, Surrey, Middlesex, and Essex?

(4) A track is 1,056 yds. round. How many times would a cyclist have to go round in order to travel 30 miles (1 mile = 1,760 yds.)?

(5) 2,485 bricks are used per yard to build a tunnel 1,896 yds. long, and 2,268 bricks per yard to build another tunnel 2,513 yds. long. How many more are used for one tunnel than for the other?

(6) A fruit grower fills 748 boxes each containing 18 doz. oranges, and has 247 oranges left over. How many more oranges will he require to fill exactly 1,000 boxes?

(7) It was found that the yearly cost of supporting a certain army of 1,000,000 soldiers, of whom 26,500 were officers, was £235,648,000. Assuming that the average cost per officer was twice that per man, find the average yearly cost to the nearest £ to support each man.

(8) A reservoir supplied 345,000 000 gall. in a year to a town of 23,850 inhabitants. Find to the nearest gallon the average amount of water used by each person per day. (1 year = 365 days.)

(9) A coal mine used in the years 1927, 1928, 1929, 1930, and 1931 an average of 1,456, 1,649, 1,826, 1,720, and 1,945 pit props respectively per working day. If the number of working days were 307, 309, 306, 308, and 307 respectively, what was the average number of pit props used per working day for the whole period?

(10) The area of Great Britain and Northern Ireland is 95,030 square miles and that of France is 213,000 square miles. The population of the former is 44,500,000 and that of the latter is 41,000,000. Calculate the population per square mile in each case.

(11) In 1930, the quantity of wheat imported into Great Britain and Ireland was 105,006,857 cwt. Reckoning that 100 acres of land will produce 1,880 cwt. of wheat, find to the nearest 1,000 acres the area of land that must be put under wheat cultivation to render the importation of wheat unnecessary.

(12) AUSTRALIA: AREA AND POPULATION.

States and Capitals.	Population.			
	Area (English sq miles).	Census of 1911.	Census of 1921.	Estimated March 31, 1931.
New South Wales (Sydney)	309,432	1,646,734	2,100,371	2,504,536
Victoria (Melbourne)	87,884	1,315,551	1,531,280	1,795,522
Queensland (Brisbane)	670,500	605,813	755,972	952,483
South Australia (Adelaide)	380,070	408,558	495,160	582,928
Western Australia (Perth)	975,920	282,114	332,732	420,124
Tasmania (Hobart)	26,215	191,211	213,780	219,694
Territories.				
North Australia (Darwin)	287,227	3,310	3,867	4,613
Central Australia (Alice Springs)	236,393			
Federal Capital Territory (Canberra)	940	1,714	2,572	8,807

(*a*) Calculate the increases in population, 1911–21 and 1921–31.

(*b*) Find the average population per 100 square miles for the whole of Australia in 1931.

(13) If a boat travelled a distance of 1,848 miles in 231 hours, how long ought it to take to travel 3,500 miles at the same average rate? Also how far would it travel in 35 hours?

(14) A garrison of 550 men have provisions sufficient to last a fortnight; they have a reinforcement of 130 men. For how many days will the provisions last?

(15) If the rent of 325 ac. of ground be £1,430, what should be the rent for 115 acres at the same rate per acre? Also how many acres could be rented for £572?

(16) A bankrupt pays to a man to whom he owes £910 the sum of £462. What sum should be paid to another man to whom he owes £195?

(17) The weight of a case containing n similar packets of salt is W lb. and the weight of the empty case is w lb. What is the weight of each packet of salt?

(18) A merchant bought n bags of rice each containing x lb., and three times the number of bags each containing y lb. of rice. How many packets each containing z lb. could he make from the total amount?

(19) An oblong lawn is a ft. long and b ft. broad. How many times would one have to walk round the lawn in order to walk 1,000 ft.?

(20) Telegraph posts are u yds. apart. What is the distance between the 1st post and the n^{th} post? Also show that if a man takes s min. to walk from the 1st to the m^{th} post he would take $\frac{s(n-1)}{m-1}$ min. to walk the former distance at the same rate.

(21) A dealer bought n articles at p pence each. How many articles could be bought for the same money if the price of each be increased by one penny?

(22) Verify by evaluating each side independently that—

(i) $245 \times 713 + 246 \times 593 = 245 \times (713 + 593) + 593.$

(ii) $315 \times 298 \times 592 + 315 \times 296 \times 613 - 315 \times 297 \times 1{,}149$
$= 315 \times [297 \times (592 + 613 - 1{,}149) + 592 - 613].$

CHAPTER II

BRITISH WEIGHTS AND MEASURES.

8. UNIFORMITY.

FROM earliest times, difficulties arose through the lack of uniformity as regards the units used in weighing and measuring. Many attempts have been made by law to enforce the use of the same units throughout the land: for example, King Edgar decreed that there should be but one Standard Measure; namely, that kept at Winchester, and by the 27th section of Magna Charta there was to be one standard weight for all England. In spite of these efforts, however, numerous customary weights and measures continued in use. In 1824 and 1878 Acts were passed rendering illegal all old local and customary weights and measures, other than Imperial ones; and enacted severe penalties on those found to be employing false and unverified weights and measures.

The **Imperial Standard Yard** and the **Imperial Standard Pound** are in the custody of the Board of Trade; and copies are also deposited at the Royal Mint, the Royal Observatory at Greenwich, the New Palace at Westminster, and with the Royal Society of London. Sections of the standards of length can be seen on the outer walls of Greenwich Observatory, and a length of 100 ft. and another of 1 chain marked in brass are let into the granite step at the back of Trafalgar Square. Besides the Imperial Standards, there are Board of Trade Standards, copies of which are supplied to local authorities for use by their inspectors of weights and measures.

It does not follow that the names given to quantities occurring in various branches of trade are necessarily to be the same: *e.g.*, a *sack* of potatoes is known by the trade as being 168 lb., whereas a *sack* of flour is 280 lb., but in each case the unit-lb. is the same. Even when the article of produce is the same, it is sometimes found that different values are attached to units at different markets. Thus, in some parts of England a stone of wool weighs 16 lb., whereas in Scotland it is taken to be 24 lb.; but although the unit "stone" is different, the unit "lb." is the same in each

case. The dealer knows the values of the different customary units at the place where he is buying his goods, and, as he regulates his orders accordingly, and as the fundamental units *pound sterling*, *yard*, *pound* (*Avoirdupois*), and *gallon* are the same everywhere in the United Kingdom, little ambiguity arises.

9. The tables of the **Imperial Weights and Measures** are given below, and should be committed to memory. Certain customary trade units are tabulated in Section III, Chapter XIV, para. 128.

*Length—Imperial Standard . . . the Yard.

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod, pole, or perch
22 yards or 4 poles	= 1 chain (ch.)
10 chains or 40 poles	= 1 furlong (fur.)
1,760 yards or 8 furlongs	= 1 mile (ml.)
3 miles	= 1 league

3 barleycorns	= 1 inch
$2\frac{1}{4}$ inches	= 1 nail
3 ,,	= 1 palm
4 ,,	= 1 hand
9 ,,	= 1 span
18 ,,	= 1 cubit
$2\frac{1}{2}$ feet	= 1 pace (military)

Land Measure.

100 links	= 22 yards or 1 chain
25 ,,	= 1 pole
10 chains	= 1 furlong
80 ,,	= 1 mile

Nautical Measure.

6 feet	= 1 fathom
120 fathoms	= 1 cable's length
6080 feet	= 1 nautical mile
69·121 miles	= 1 degree
*60 nautical miles	= 1 ,,
360 degrees	= circumference of Earth

1 knot is a speed of 1 nautical mile per hour.

NOTE 1.—The Imperial Standard yard is the distance, at 62° F. and 30 in. bar. pressure, between two gold pins in the bronze bar cast in 1845 as a result of the work of a Commission, which sat from 1843 to 1854, and which was formed for the purpose of standardizing British weights and measures.

Weight—Imperial Standard . . . the Pound.

Avoirdupois Weight (Av.).

16 drams (dr.)	= 1 ounce (oz.)
16 ounces	= 1 pound (lb.) = 7,000 grains

* The tables of Area, as they are derived from the table of Length, are given in Chapter V.

Avoirdupois Weight (Av.)—(*contd.*)

	14 pounds	=	1 stone (st.)
	28 pounds	=	1 quarter (qr.)
112 pounds =	4 quarters	=	1 hundredweight (cwt.)
	20 hundredweights	=	1 ton
	100 pounds	=	1 cental
	8 ,,	=	1 stone of butcher's meat

Troy Weight.

24 grains (gr.)	=	1 pennyweight (dwt.)
20 pennyweights	=	1 ounce (oz. Tr.)

Apothecaries' Weight.

20 grains (gr.)	=	1 scruple (℈)
3 scruples	=	1 drachm (ʒ)
8 drachm	=	1 ounce (℥)

Note 2.—By the Weights and Measures Act, 1878, all articles sold by weight must be sold by Avoirdupois weight, except precious metals and stones, which are sold by Troy weight, and drugs which, in dispensing, are weighed by Apothecaries' weight. Avoirdupois weight is used in the British Pharmacopoeia.

The Imperial Standard for determining the weight of an imperial standard pound is made of platinum, is of cylindrical shape, and was made in 1844. Its weight in vacuo determines the Imperial Standard pound.

A grain is the same in all units, so that 1 oz. is different from 1 oz. Tr. or 1℥; but the latter two, however, are the same, each being 480 gr. The unit "grain" was derived from the weight of a dried grain of wheat.

The fineness of pure gold is said to be 24 carat: 9-carat gold means that out of 24 parts of the metal, 9 parts are pure gold. "Carat" is also a unit of weight, thus: 1 gold carat = 240 grains, 1 diamond carat = $3\frac{1}{8}$ grains.

Capacity—Imperial Standard . . . the Gallon

Dry Measure and Fluid Measure

4 gills	=	1 pint (pt.)
2 pints	=	1 quart (qt.)
4 quarts	=	1 gallon (gall.)

Dry Measure

2 gallons	=	1 peck (pk.)
4 pecks	=	1 bushel (bush.)
8 bushels	=	1 quarter (qr.)
3 ,,	=	1 bag
4 ,,	=	1 comb
5 ,,	=	1 sack of flour
12 bags (36 bush.)	=	1 chaldron
5 quarters (40 bush.)	=	1 wey *or* horse load
2 weys (10 qr.)	=	1 last

Apothecaries' Fluid Measure.

60 minims (min.) or drops	=	1 fluid drachm
8 fluid drachms	=	1 ,, ounce
20 ,, ounces	=	1 Imperial pint
1 tea-spoonful	=	1 fluid drachm
1 dessert-spoonful	=	2 ,, ,,
1 table-spoonful	=	4 ,, ,,

NOTE 3.—The **Imperial Standard gallon** contains 10 lb **weight** of distilled water weighed in air, the temperature of water and air being 62° F. and the barometric pressure 30 in.

1 gallon = 277 274 cubic inches.

Paper Measure.

24 sheets	=	1 quire
20 quires	=	1 ream
516 sheets	=	1 printer's ream
2 reams	=	1 bundle
10 ,,	=	1 bale

NOTE 4.—Paper is made in sheets of recognized sizes, and according to its size it is given a name: *e.g.*, the sizes of foolscap writing paper and Double Demy printing paper are 17″ × 13½″ and 35½″ × 22½″ respectively.

Time

60 seconds (sec.)	=	1 minute (min.)
60 minutes	=	1 hour (hr.)
24 hours	=	1 day

Quarter Days (England)

Lady Day	. 25th March
Midsummer	. 24th June
Michaelmas	. 29th Sept.
Christmas	. 25th Dec.

NOTE 5.—The units of time are the same for all countries.

The numbers of days in the months are January, 31; February, 28 (29, Leap Years); March, 31; April, 30; May, 31; June, 30; July, 31; August, 31; September, 30; October, 31; November, 30; and December, 31.

The Earth takes 365 days 48 minutes 50 seconds to make one complete revolution round the Sun. If we considered each year as being 365 days, we should have no permanent relation between the date of any day and the season of the year: for example, Christmas Day would not necessarily occur in the Winter. In order to overcome this difficulty, the Gregorian Calendar was introduced. By this, years whose dates are divisible by 4, excluding those years ending the centuries and which are not divisible by 400, were to consist of 366 days (Leap Year) instead of 365 days. Thus 1684, 1712, 1908, and 1600 were Leap years, but 1900 and 1906 were not Leap Years. The extra day is 29th February.

Places on the Earth's surface having the same longitude have the same local time. The longitude is measured in degrees east and west from the line of longitude 0° passing through Greenwich. The Earth rotates 360° in 24 hours, thus there is a difference of 1 hour in the local times of places 15° of longitude apart (*i.e.*, 4 minutes per degree difference in longitude.) Degrees of Latitude or Longitude, as well as degrees of angular measurement, are subdivided into minutes and seconds: 60 seconds = 1 minute and 60 minutes = 1 degree. A longitude of 87 degrees 43 minutes 27 seconds East is written: long. 87° 43′ 27″ E.

Money.

4 farthings	=	1 penny (*d.*)
12 pence	=	1 shilling (*s.*)
20 shillings	=	1 pound (£)

NOTE 6.—The guinea was a gold coin in circulation from 1664 to 1817, and it derived its name from the fact that it was first coined out of gold from the Guinea Coast. The term "guinea" is often used by professional men, and simply denotes £1 1s.

The letters £ *s. d.* are the initials of the words *Libra, Solidus,* and *Denarius,* which are the names of Roman coins corresponding respectively to pound, shilling, and penny.

It has been proposed that a decimal system should be introduced in the United Kingdom. One suggestion is that 1,000 mils should be equivalent to £1, so that 1 florin would be equivalent to 100 mils, 1 shilling to 50 mils and 10 bronze coins each of value 5 mils, would be equivalent to 1 shilling.

10. REDUCTION.

In order to denote concrete quantities, a number and a unit are necessary. In adding and subtracting concrete quantities, no meaning of value is obtained by adding or subtracting the numbers, unless the units to which they refer are the same in each case. For example, suppose 7 shillings is to be added to 14 pence: by adding 7 and 14, the result 21 is obtained, which might give some information regarding the number of coins, but has no significance respecting the value of the combined sum of money. Thus, in order to add these quantities, it is necessary to know how to express a quantity given as a number of a certain unit, as a number of a different unit. Thus, 7s. 0d. + 1s. 2d. = 8s. 2d. or 84d. + 14d. = 98d. = 8s. 2d.

The process of altering the number and unit of a quantity, yet keeping the value of the quantity the same, is known as **Reduction.** It is called descending or ascending reduction, according to whether the new unit is smaller or larger than the original unit.

The principles involved in the following examples are those of Chapter I combined with Reduction.

EXAMPLE (i)—

How many books at 4s. 1½d. each can be bought for £11 13s. 9d. ?

```
         20  12
      £  s.  d.
      11 13  9
      233 shillings
    2,805 pence
 9 \ 5,610 half-pence
11 \  623 rem. 3 \
                  } 66
       56 rem. 7 /
```

Ansr.—56 books; 2/9 not spent.

NOTE 7.—All the necessary working is shown. The first line is obtained by multiplying the numbers of pounds by 20 and adding the number of shillings, and the next line by multiplying the number of shillings by 12 and adding the number of pence.

EXAMPLE (ii)—

What money is obtained by selling 1 ton 14 cwt. 3 qr. 16 lb. of potatoes at $1\frac{3}{4}$d. per lb.?

	20	4	28
tons	*cwt.*	*qr.*	*lb.*
1	14	3	16

34 cwt.
139 qr.
* 556
3,908 lb.
27,356 farthings
6,839 pence
569s 11d. *Ansr.*—£28 9s. 11d.

NOTE 8.—As it ought not to be necessary to put down a number like 4 or 7 when multiplying and dividing, all the working required is shown. The line marked * is obtained by multiplying 139 by 4, but it would be wrong to add in the 16 here, for 16 is a number of lb., whereas 556 is the number of groups of 7 lb. On multiplying by 7, the number of lb.. is obtained, so that the 16 lb. is now added.

EXAMPLE (iii)—

A manufacturer of cocoa wishes, for advertisement, to distribute 250,000 sample packets each containing ½ oz. of cocoa. How much cocoa will be required?

4 | 125,000 oz.
4 | 31,250
4 | 7,812 rem. 2 } 8 oz.
7 | 1,953
4 | 279 } 0 lb.
20 | 69 rem. 3 qr.
3 rem. 9 cwt. *Ansr.*—3 ton 9 cwt. 3 qr. 8 oz.

NOTE 9.—Although it is not necessary to put down the divisors when dividing, yet it is advisable in this case, in order to avoid confusion as regards the complete remainders.

11. Although it is assumed that the student is familiar with the processes of Addition, Subtraction, Multiplication, and Division of quantities occurring in the tables of Weights and Measures, it would be well to analyse briefly these processes and to see on what principles they depend.

12. ADDITION.

The addition of quantities consists of the repeated applications of the following processes: (1) Adding the numbers in a column to the number (if any) obtained by the previous reduction; and (2) reducing the result to the next higher unit, putting down the remainder in its assigned column.

EXAMPLE (iv)—

A, *B*, *C*, *D*, and *E* are five places in a road. The distances from *A* to *B*, *B* to *C*, *C* to *D*, and *D* to *E* are 1 fur. 1 ch. 17 yd. 1 ft. 7 ins, 3 fur. 2 ch. 9 yd. 2 ft. 10 in., 4 ch. 13 yd. 1 ft. 9 in., and 1 fur. 6 ch. 15 yd. 2 ft. 5 in. respectively. What is the distance from *A* to *E* ?

	10	22	3	12
fur.	*chain*	*yd.*	*ft.*	*in.*
1	1	17	1	7
3	2	9	2	10
	4	13	1	9
1	6	15	2	5
6	5	12	2	7

13. SUBTRACTION.

The principles underlying this are either (*a*) re-arrangement of the larger quantity, according to the figures occurring in the smaller quantity; or (*b*) the logical conclusion that if the same quantity be added to each of two given quantities, the difference between the latter will be unaltered.

	20	4	28	16
Tons	*cwt.*	*qr.*	*lb.*	*oz.*
5	13	1	7	11
1	17	0	11	14
3	16	0	23	13

(*a*) Keeping the value the same, re-arrange the upper quantity as follows:

Tons	*cwt.*	*qr.*	*lb.*	*oz.*
4	13+20	0	6+28	11+16

Then it is possible to subtract the numbers in each of the columns independently

(*b*) (1) As 14 is more than 11, add 16 oz. to the upper quantity and 1 lb. to the lower quantity, so that the difference is unaltered; then, as 16 – 14 + 11 = 13, the latter is put down in the oz. column.

(2) As 12 is more than 7, add 28 lb. to the upper quantity and 1 qr. to the lower; then, as 28 – 12 + 7 = 23, put the latter in the lb. column.

(3) 1 qr. from 1 qr. leaves 0 qr.

(4) As 17 is more than 13, add 20 cwt. to the upper quantity and 1 ton to the lower; then, as 20 – 17 + 13 = 16, put the latter in the cwt. column.

(5) 2 tons from 5 tons leaves 3 tons.

14. MULTIPLICATION.

The multiplication of a quantity by a given number not greater than 12 involves the repeated application of the following processes: (1) Multiplying the number in a column by the given number; (2) adding the number (if any) to be carried as the result of the previous reduction; and (3) reducing the result to the next higher unit, putting down the remainder in its assigned column.

348 × £3 13s. 8½d. is the same as

(*a*) (300 + 40 + 8) × £3 13s. 8½d. or (*b*) 348 × (£3 + 10/– + 3/4 + 4d. + ½d.).

(*a*)

£	s. (20)	d. (12)			£	s.	d.
3	13	$8\frac{1}{2}$					
36	17	1	=	10 times	3	13	$8\frac{1}{2}$
368	10	10	=	100 ,,	.	.	.
1,105	12	6	=	300 ,,	3	13	$8\frac{1}{2}$
147	8	4	=	40 ,,	.	.	.
29	9	8	=	8 ,,	.	.	.
1,282	10	6	=	348 times	3	13	$8\frac{1}{2}$

(*b*) The question arises as to how the sum of money shall be split up.

An *aliquot part* of a given quantity is a quantity which is contained in the given quantity a whole number of times (*e.g.*, 10s., 6s. 8d., 5s., 4s., 3s. 4d., 2s. 6d., 2s., 1s. 8d., etc.) are aliquot parts of £1.

The following method is known as *Simple Practice.*

	£	s.	d.				£	s.	d.
Cost of 348 articles at	1	0	0	each		=	348	0	0
,, ,,	3	0	0	,,		=	1,044	0	0
,, ,,		10	0	,,	$10/- = \frac{£1}{2}$	=	174	0	0
,, ,,		3	4	,,	$3/4 = \frac{10/-}{3}$	=	58	0	0
,, ,,			4	,,	$4d. = \frac{3/4}{10}$	=	5	16	0
,, ,,			$\frac{1}{2}$	,,	$\frac{1}{2}d. = \frac{4d.}{8}$	=		14	6

348 articles at £3 13s. $8\frac{1}{2}$d. each cost £1,282 10s. 6d.

NOTE 10.—In general, when the number of articles is fairly large, the Practice method entails less mechanical work.

NOTE 11.—The method of Simple Practice is not confined to sums of money, as the following example will show—

What is the total weight of 3,715 cases of tinned salmon, if the average weight of each case is 2 cwt. 3 qr. 16 lb. ?

	ton	cwt.	qr.	lb.			ton	cwt. (20)	qr. (4)	lb. (28)
Wt. of 3,715 cases, each weighing	1	0	0	0		=	3,715	0	0	0
,, ,, ,,		2	0	0	$2\text{ cwt.} = \frac{1\text{ ton}}{10}$	=	371	10	0	0
,, ,, ,,			2	0	$2\text{ qr.} = \frac{2\text{ cwt.}}{4}$	=	92	17	2	0
,, ,, ,,			1	0	$1\text{ qr.} = \frac{2\text{ qr.}}{2}$	=	46	8	3	0
,, ,, ,,				14	$14\text{ lb.} = \frac{1\text{ qr.}}{2}$	=	23	4	1	14
,, ,, ,,				2	$2\text{ lb.} = \frac{14\text{ lb.}}{7}$	=	3	6	1	10
Wt. of 3,715 cases, each weighing		2	3	16		=	537	6	3	24

15. DIVISION.

Both short and long division of a quantity by a given number involve the repeated application of the following processes: (1) Adding the number (if any) obtained by the previous reduction to the number in a column; (2) dividing the result by the given number, putting the quotient down in its assigned place; and (3) reducing the remainder to the next lower unit.

When the divisor is capable of being factorized, each factor being not greater than 12, it is in almost every case preferable to divide, short division, by each factor in its turn rather than to apply long division.

It often happens that a problem is such that it does not concern itself with the remainder obtained by division, but requires to know the quotient expressed to the nearest whole number of units. If the remainder expressed as a number of a certain unit is less than half the divisor, it should be ignored; but if greater than one-half, the quotient should be increased by 1 unit, the latter being the same as that in which the remainder is expressed. When this is done, the error is not greater than one-half of the unit in question.

E.g., if £5 be equally divided among 7 men, how much should each receive (1) to the nearest penny, (2) to the nearest farthing.

		20	12	
	£	*s.*	*d.*	
(1)	5	0	0	
		14	3	rem. 3d.

(1) Rem. is 3 pence and, as 3 is less than half of 7, the answer is 14/3.

		20	12	
(2)	£	*s.*	*d.*	
	5	0	0	
		14	3¼	rem. $\frac{5}{4}$d.

(2) Rem. is 5 farthings and, as 5 is greater than half of 7, the answer is 14/3½

The error in the first case is less than half a penny and, in the second case, less than half a farthing.

Further considerations of the subject of approximation are given in Chapter VI.

EXAMPLE (v)—

Assuming that 5,000,000 persons in Greater London are meat-eaters, how many tons of meat are consumed weekly if each person be allowed 2½ lb. of meat per week ?

Amount eaten per week = 12,500,000 lb.

$$= \frac{\overset{156,250}{\cancel{12,500,000}}}{\underset{28}{\cancel{2,240}}} \text{ tons} \qquad \begin{array}{r|l} 4 & 156,250 \\ \hline 7 & 39,062 \text{ rem. } 2 \\ \hline & 5,580 \text{ rem. } 2 \end{array} \Big\} \; 10$$

= 5,580 tons (to nearest ton).

EXAMPLE (vi)—

A dealer wishes to buy 3 cwt. of coffee at an average price of, as near as possible, 1s. 3d. per lb. He has bought 95 lb. at 1s. 1½d. per lb., 80 lb. at 1s. 2d. per lb., and 72 lb. at 1s. 4½d. per lb. At what price per lb. should he buy the remaining coffee?

Total money to be spent = 336 + 84 shillings
= 420 „

Amount already spent = 1/1½ × 95 + 1/2 × 80 + 1/4½ × 72
= 1/1½ × (95 + 80 + 72) + 3/4 + 18/-
= 1/1½ × 247 + 3/4 + 18/-
= 247/- + 30/10½ + 21/4
= 299/2½

∴ Average price per lb. of remainder = 120/9½ ÷ 89
= 1/4¼ (to nearest ¼d.)

$$\begin{array}{r|rr} & 1 & 4\tfrac{1}{4} \\ \hline & s. & d. \\ 89) & 120 & 9\tfrac{1}{2} \\ & 89 & \\ \hline & 31 & \\ & 381 & \\ & 356 & \\ \hline & 25 & \\ & 102 & \\ \hline \end{array}$$

EXAMPLE (vii)—

At a factory, n_1 men, n_2 women, and n_3 boys are employed. They receive p_1, p_2, and p_3 pence per hour respectively, and work 48, 44, and 40 hours per week respectively. What is the weekly wage bill in pounds?

$$\text{Total weekly wage} = £\frac{48n_1p_1 + 44n_2p_2 + 40n_3p_3}{240}$$

$$= £\frac{\cancel{4}(12n_1p_1 + 11n_2p_2 + 10n_3p_3}{\underset{60}{\cancel{240}}}$$

$$= £\frac{12n_1p_1 + 11n_2p_2 + 10n_3p_3}{60}$$

EXAMPLE (viii)—

The cost of conveying a tons b cwt. of produce a certain distance was x pounds y shillings. What is the cost in pence per lb.?

Cost of conveying $112(20a + b)$ lb. $= 12(20x + y)$ pence

$$\therefore \quad „ \quad „ \quad „ \quad 1 \text{ lb.} = \frac{12(20x + y)}{112(20a + b)} \;\; „$$

$$= \frac{3(20x + y)}{28(20a + b)} \;\; „$$

Example (ix)—

3 articles were bought at s shillings p pence each and 4 similar articles were bought at s shillings P pence each. Obtain a formula giving the average cost of each article.

Let x pence $\equiv$ average cost of each article,

$$\text{then } x = \frac{3(12s + p) + 4(12s + P)}{7} = \frac{36s + 3p + 48s + 4P}{7} = \frac{84s + 3p + 4P}{7}$$

TEST EXERCISES I, 2.

(1) A man's salary is £285 per annum. What are his weekly earnings to the nearest penny?

(2) Goods are sold at £63 per ton. What is the price in pence per lb.?

(3) 19 men each earn £2 13s. 9d. per week. How many could be completely paid from a sum of £45, and how much is still required to pay them all completely?

(4) Divide £3 7s. among A, B, and C so that A shall receive 14s. 1d. more than B, and B 5s. 10d. more than C.

(5)	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	*Totals.* £	s.	d.	
	5	13	9	24	11	5	21	17	9	31	5	9				
	3	7	4	26	13	6	23	4	6	26	3	8				
	11	18	6	19	19	4	7	0	9	17	4	3				
	3	1	11	7	4	8	3	5	1	23	8	5				
	25	9	5	2	15	4	15	9	5	15	17	4				
	18	19	3	13	2	10	3	16	9	7	11	9				
	19	4	8	7	3	11	2	18	4	23	15	10				
	13	16	10	29	14	8	17	11	8	16	13	5				
	5	13	11	15	11	4	23	15	9	9	2	2				
	12	5	2	23	19	8	16	10	11	11	4	6				
	39	0	8	16	14	2	7	2	7	23	7	11				
	15	4	7	7	3	9	25	5	10	15	11	4				
	23	8	9	25	7	7	29	0	11	26	13	10				
Totals																Grand Totals

Add up the above sums of money in columns and rows. Verify your answers by finding the grand totals in each case.

(6) Write down columns as in Question 5, and add up in rows and columns in order to gain speed and accuracy. Test your answers by comparing the grand totals in each case.

(7) A bankrupt owes £840. His net assets available for payment to his creditors amount to £514 10s. What should a creditor receive to whom he owes £525?

(8) A mile of a certain kind of cable weighs 6 ton 7 cwt. 55 lb. What is the weight of 7 miles 987 yds. of cable to the nearest lb.?

(9) How many oz. Troy are equal to 1,260 oz. Avoirdupois?

(10) One train leaves a station at 8.45 a.m. and travels at 18 miles per hour. An express train travelling at 55 miles per hour in the same direction passes through the station at 9.13 a.m. At what time (to the nearest sec.) will the express overtake the slow train and how far is the place from the station?

(11)

Boroughs	Rateable Value	Rates: 1931–32	Total Sum collected
	£	s. d.	
Battersea	1,072,027	12 2	
Camberwell	1,595,319	11 0	
Chelsea	1,210,584	10 0	
Fulham	1,120,845	10 4	
Hammersmith	1,133,533	11 10	
Kensington	3,256,628	9 7	
Lewisham	1,617,862	10 8	
Southwark	1,323,410	11 5	
Woolwich	1,038,277	13 0	

(*a*) Fill up the fourth column, giving the amount to the nearest pound.

(*b*) Find the average rate for the nine boroughs, giving the answer to the nearest penny.

(12) Find to the nearest ton the amount of meat consumed by 32,000,000 people per week on the assumption that the average amount consumed by each person per week is 1 lb. 3 oz.

(13) If 35 tons of coal, bought at the pit's mouth for £38 10s., be carried 128 miles at a charge of $\frac{3}{4}$d. per ton per mile, what is the total cost per ton on delivery?

(14) Find, by both methods, the total weight of 368 cases of silk goods, if the average weight of each case be 2 cwt. 3 qr. 18 lb.

(15) The population of Birmingham in 1931 was 1,002,413. Assuming the average amount of water used by each person per day is 31 gall., what weight in tons must be stored in order to supply the city for six weeks?

(16)* A steamer travelling at 14 knots took 18 hr. 33 min. to go from one port to another. How much less time would it take if its speed were increased by 2 knots?

(17) Show that (1) a dozen articles at x pence each cost x shillings; and (2) a cwt. of goods at £a per ton cost a shillings.

(18) Find the cost in shillings of 1 cwt. of rice at n lb. for 1s.

(19) A dealer bought m sewing machines for x pounds each, and later on bought n more at y pounds each. What was the average amount paid for each machine?

(20) A traction engine travelled 100 yds. in x seconds. What was its speed in miles per hour?

(21) A retailer bought 2 chests of tea at £5 6s. per chest, and 1 cwt. of a better kind at £7 15s. per cwt. He mixed the two and sold the mixture at 1s. 11d. per lb. How much did he gain? (1 chest of tea = 84 lb.)

(22) An American cask of rice contains 6 cwt. How many casks must be imported during a year to enable a population of 43,000,000 of people to consume $1\frac{1}{2}$ oz. each per week?

(23) A man, on behalf of his son, saved 1d. per day from 4th June, 1921, to the end of August, 1930, inclusive. How much did he save?

(24) How many American barrels of flour, each of which contains 196 lb., are necessary to supply 7,350,000 persons with sufficient to allow an average of 1 lb. 6 oz. per week to each person?

(25) The longitudes of Calcutta and Chicago are approximately 88° E. and 85° 30′ W. respectively. What are the local times at these towns when it is (1) mid-day at Greenwich, (2) 5.20 a.m. at Greenwich? Also (3) find the time at Chicago when it is 3.25 p.m. at Calcutta, and the time at Calcutta when it is 3.25 p.m. at Chicago.

CHAPTER III.

FACTORS—HIGHEST COMMON FACTOR—LOWEST COMMON MULTIPLE.

16. ONE number is said to be divisible by another number if, when the former is divided by the latter, the quotient is an integer and the remainder is zero. When this is the case, the second number is a **factor** or a measure of the first number, while the first number is a **multiple** of the second number. Thus 6 is a factor of 24 and 24 is a multiple of 6.

17. A **Prime Number** is one which is not divisible by any integers except itself and unity. Thus 2, 3, 5, 7, 11, 13, 17, 19, 23 . . . are prime numbers. A **Composite Number** is one which is divisible by other integers as well as by itself and unity.

The following tests of the divisibility of integers by 2, 3, 4, 5, 8, 9, and 11 are sometimes of use.

An Integer is Divisible by	
2	If last digit is divisible by 2.
3	If the sum of the digits is divisible by 3.
4	If number formed by last two digits is divisible by 4.
5	If last digit is zero or 5.
8	If number formed by last three digits is divisible by 8.
9	If the sum of the digits is divisible by 9.
11	If the difference between the sum of the digits in the even places and the sum of the digits in the odd places is zero or divisible by 11.

NOTE 1.—Consider the number 635,476.

This can be put in the form $635 \times 1{,}000 + 476$.

As 8 is a factor of 1,000, 8 is a factor of $635 \times 1{,}000$; therefore the number is divisible by 8, if 476 is divisible by 8.

NOTE 2.—The divisibility of a number by 3 and 9 has been considered in Chapter I.

The test as regards the divisibility by 11 is proved as follows—

Suppose the number consists of five digits, a, b, c, d, and e.
Then the number is

$$10{,}000a + 1{,}000b + 100c + 10d + e,$$

which can be written in the form

$$9{,}999a + 1{,}001b + 99c + 11d + (a - b + c - d + e).$$

The first four terms are always divisible by 11, whatever the values of a, b, c, and d; therefore the number is divisible by 11, if $(a - b + c - d + e)$; i.e., $(a + c + e) - (b + d)$ is divisible by 11.

NOTE 3.—By combining these tests, it can at once be seen if the number be divisible by 6, 15, 24, 33, etc.

If a composite number be expressed as a product of two factors, then if either or both of the factors are composite numbers, one or both can be expressed as products of two factors. Again, it may be possible to express the latter as products of factors; and so, proceeding in this way, the original number can be expressed as a continued product of prime factors.

EXAMPLE (i)—

Express 1,008 as a continued product of prime factors, and thus find all the factors of 1,008.

$$\begin{aligned} 1{,}008 &= 2 \times 2 \times 2 \times 126 \\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \end{aligned}$$

The factors of 1,008 are—

Prime Factors.	Products of 2 Prime Factors.	Products of 3 Prime Factors.	Products of 4 Prime Factors.	Products of 5 Prime Factors.	Products of 6 Prime Factors.
2	2×2= 4	2×2×2= 8	2×2×2×2= 16	2×2×2×2×3= 48	2×2×2×2×3×3=144
	2×3= 6	2×2×3=12	2×2×2×3= 24	2×2×2×3×3= 72	2×2×2×2×3×7=336
3	2×7=14	2×2×7=28	2×2×2×7= 56	2×2×2×3×7=168	2×2×2×3×3×7=504
7	3×3= 9	2×3×3=18	2×2×3×3= 36	2×2×3×3×7=252	
	3×7=21	2×3×7=42	2×2×3×7= 84		
		3×3×7=63	2×3×3×7=126		

NOTE 4.—As 8 is a factor of 1,008, write down 2×2×2 and, dividing 1,008 by 8, write down the factor 126. As 6 is a factor of 126, write down 2 × 3, and dividing 126 by 6, put down the other factor 21 in the factorized form 3 × 7.

NOTE 5.—Applying the notation, namely, that $a \times a \times a \times a$ is written more shortly a^4, etc., the number 1,008 can be expressed in the form $2^4 \times 3^2 \times 7$.

18. The Highest Common Factor (or Greatest Common Measure) of two or more given numbers is the highest number that is a factor of every one of the given numbers.

EXAMPLE (ii)—

Find the Highest Common Factor (H.C.F.) of 210, 168, and 231.

$$\begin{aligned} 210 &= 2 \times 3 \times 5 \times 7 \qquad & \text{H.C.F.} &= 3 \times 7 \times \cancel{11} \\ 168 &= 2 \times 2 \times 2 \times 3 \times 7 & &= 21 \\ 231 &= 3 \times 7 \times 11 \end{aligned}$$

NOTE 6.—After expressing each number as a product of prime factors, write down one set (usually the one containing the least number of factors) and cross out the factor or factors which do not occur in every set. The factors not crossed out will each be common factors, as well as the products of them taken two, three, or more at a time. Thus the H.C.F. will be the product of *all* the prime factors not crossed out.

19. The Least Common Multiple of two or more given numbers is the lowest number that is a multiple of every one of the given numbers.

EXAMPLE (iii)—

Find the Least Common Multiple (L.C.M.) of 24, 42, 56, and 70.

$24 = 2 \times 2 \times 2 \times 3$ L.C.M. $= 2 \times 2 \times 2 \times 3 \times 7 \times 5$

$42 = 2 \times 3 \times 7$ $= 840$

$56 = 2 \times 2 \times 2 \times 7$

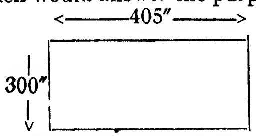
$70 = 2 \times 5 \times 7$

NOTE 7.—Write down the first factor in the first set and mark it off *once* in every set of factors where it occurs. Write down the second factor in the first set and mark it off once in every set where it occurs. Proceed in this way until all the factors in the first set are marked off, then write down the first factor in the second set which is not marked off and mark it off once in every set where it occurs unmarked. Proceed thus until every factor in every set is marked off. The L.C.M. is the product of the prime factors written down.

It is easily seen that the set of factors written down contains each of the given sets. Also by marking off the factors as they are written down, no more factors than are necessary for this to happen are put down. Thus, because the final product contains each of the given products, it is a common multiple of the latter; and because the minimum number of factors for this to happen occurs in the product, the latter is the least common multiple.

EXAMPLE (iv)—

A rectangular courtyard 33′ 9″ long and 25′ broad is to be paved by square tiles of the same size. What is the length of the side of the largest tile which would answer the purpose? Also how many tiles will be required?



The number of inches expressing the length of the side of a tile is contained in 405 a whole number of times.

∴ it is a factor of 405.

Similarly it must be a factor of 300.

As the largest size possible is required, the number of inches giving the length of side will be the H.C.F. of 405 and 300.

$405 = 3 \times 3 \times 3 \times 3 \times 5$ H.C.F. $= 3 \times \cancel{3} \times \cancel{3} \times \cancel{3} \times 5$

$300 = 2 \times 2 \times 3 \times 5 \times 5$ $= 15$

Ansr.—1′ 3″

15 is contained in 405, $3 \times 3 \times 3$ *i.e.*, 27 times
15 ,, 300, $2 \times 2 \times 5$ *i.e.*, 20 ,,

Thus each row will contain 27 tiles, and there will be 20 rows of tiles.

Ansr.—540 tiles.

EXAMPLE (v)—

What is the least whole number of pounds sterling which contains 2s. 4d. and also 1s. 4d whole numbers of times?

The sum required contains £1, 2s. 4d., and 1s. 4d. each a whole number of times. The number of pence in this sum is a common multiple of 240, 28, and 16. Also, as the least sum possible is required, the number of pence will be the L.C.M. of 240, 28, and 16.

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$28 = 2 \times 2 \times 7$

$16 = 2 \times 2 \times 2 \times 2$

L.C.M. $= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 7$

and, by dividing this product by $2 \times 2 \times 2 \times 2 \times 3 \times 5$, it is seen that it contains 240 exactly 7 times.

Ansr—£7.

Example (vi)—

What number multiplied by itself gives 81,225 as the result? In other words, what is the *square root* of 81,225? (Or, more briefly, evaluate $\sqrt{81,225}$.)

$$\begin{aligned} 81,225 &= 3 \times 3 \times 9,025 \\ &= 3 \times 3 \times 5 \times 1,805 \\ &= 3 \times 3 \times 5 \times 5 \times 361 \\ &= 3 \times 3 \times 5 \times 5 \times 19 \times 19 \\ &= \overline{3 \times 5 \times 19} \times \overline{3 \times 5 \times 19} \\ &= 285 \times 285 \end{aligned}$$

Thus $\sqrt{81,225} = 285$

Example (vii)—

Find the H.C.F. and L.C.M. of $10a^2bc$, $2abx - 2aby$, $6cx - 6cy$.

$$\begin{aligned} 10a^2bc &= 2 \times 5a^2bc \\ 2abx - 2aby &= 2ab(x-y) \\ 6cx - 6cy &= 6c(x-y) \\ &= 2 \times 3c(x-y) \end{aligned}$$

H.C.F. $= 2 \times \cancel{5} \times a \times a \times b \times c$
$= 2$

L.C.M. $= 2 \times 5 \times a^2 \times b \times c \times 3 \times (x-y)$
$= 30a^2bc(x-y)$

Note 8.—When different powers of the same quantity occur, the H.C.F. is the lowest power and the L.C.M. is the highest power. Thus, considering the powers x^3, x^7, x^5, and x^4, the H.C.F. is x^3 and the L.C.M. is x^7.

TEST EXERCISES I, 3

(1) What is the least number of articles at 4s $7\frac{1}{2}$d. each that is equivalent in value to a number of articles at 4s $1\frac{1}{2}$d. each?

(2) What is the least weight that contains an exact number of ounces Troy and of ounces Avoirdupois?

(3) A tailor has 3 rolls of cloth containing 31 yd. 1 ft. 6 in., 12 yd. 9 in. and 43 yd. 2 ft. 3 in. of cloth respectively. He wishes to cut the whole into portions of equal length in such a way that none is wasted. What is the greatest possible length of the portions?

(4) Two companies of 210 and 238 men respectively are to be divided into squads, each containing the same number of men, but the two companies are to remain distinct. What is the greatest number of men in each squad and what is the total number of squads?

(5) A hall is 59′ 6″ long and 45′ 6″ wide. Four brackets for electric lights are fixed, one at each of the four corners of the walls. Other brackets are to be fixed so that the distance between any one and the next is to be the same all round the hall. What is the smallest number of brackets necessary?

(6) What is the shortest distance that contains an exact number of miles and geographical miles, if the latter be taken as equivalent to 6,080 ft.

(7) Three vans are laden with packages, the weight of each package

being the same. The total weights of the packages in the vans are 2 ton 2 cwt. 18 lb., 1 ton 16 cwt., 6 lb. and 1 ton 10 cwt. 24 lb. What is the greatest weight each package could have been ?

(8) Two omnibus routes have the same starting-point. Each 'bus on the first route is expected to begin a journey 1 hr. 12 min. after the start of the preceding journey, and the time in the second case is 1 hr. 24 min. Two 'buses, one on each route, start off together at 6 40 a.m. What will be the time when next they commence a journey together ?

(9) A triangular field has its sides of length 111 yd. 2 ft., 119 yd. 4 in., and 134 yd. respectively. A farmer wishes to have a wire fence round the field, and supports are to be fixed at equal distances apart all round the field. What is the greatest distance apart of the supports, in order that one should be fixed at each of the three corners ? Also how many supports would there be and what would be the cost at 3s. 10d. each ?

(10) What is the least number of cases each weighing 3 cwt. 3 qr. 12 lb. that would together weigh an exact number of tons ?

(11) Find the H.C.F. and L.C M. of $6a^2b$, $10ab - 12ac$, and $15b^2 - 18bc$.

(12) Express in £, the least sum that contains a shillings 6 pence and x pounds y shillings exactly.

CHAPTER IV.

VULGAR FRACTIONS.

20. A **Fraction** of a quantity is a part of the quantity. It is the business of Arithmetic to devise a means whereby that part can be measured.

If a quantity be divided, say, into five equal parts, each part is commonly called one-fifth, and is written $\frac{1}{5}$ of the quantity. The portion which contains, say, exactly three of these parts is called three-fifths, and is written $\frac{3}{5}$ of the quantity. Thus to evaluate $\frac{3}{5}$ of a quantity it would be correct to divide the quantity by 5 and multiply the result by 3.

Now, from the notation of Chapter I, $\frac{3}{5}$ of a quantity denotes the result of dividing three times the quantity by 5. It is necessary, therefore, to see that these distinct conceptions as to the meaning of $\frac{3}{5}$ of a quantity are in accordance one with the other.

AB denotes a quantity. It is divided into 5 equal parts, and AX consists of 3 of these parts. Therefore from the first conception, AX $= \frac{3}{5}$ of AB.

CD is exactly three times AB. It is divided into 5 equal parts, and CY is one of these parts. Therefore from the second conception,

$$\text{CY} = \frac{3 \text{ times AB}}{5}$$

The diagram shows that AX = CY, and thus the two ideas are in accordance with one another.

Thus the fraction $\frac{x}{y}$ of a quantity is evaluated by either

(1) dividing the quantity by y and multiplying the result by x

or (2) **multiplying the quantity by x and dividing the result by y.**

It is advisable to use the second method for a reason which the following example shows.

Example (i)—

Evaluate $\frac{3}{7}$ of £4 7s. 9d. to the nearest penny.

		20	12	
	£	*s.*	*d.*	
(1)	4	7	9	
		12	6	$= \frac{1}{7}$ of £4 7s. 9d. to nearest penny.
	1	17	6	$= \frac{3}{7}$ of £4 7s. 9d. This result is incorrect.

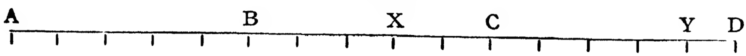
		20	12	
(2)	£	*s.*	*d.*	
	4	7	9	
	13	3	3	= 3 times £4 7s. 9d.
	1	17	7	$\doteq \frac{3}{7}$ of £4 7s. 9d. to nearest penny.

Note 1.—The reason that the result in the first case is wrong is as follows: $\frac{1}{7}$ of £4 7s. 9d. is exactly 12s. $6\frac{3}{7}$d., and so writing 12s. 6d. results in an error of $\frac{3}{7}$d. On multiplying by 3, this error is magnified 3 times.

Note 2.—In the second case, the last line neglects a remainder of 2d., which has not been divided by 7. If this remainder were divided also, the answer expressed exactly would be £1 17s. $7\frac{2}{7}$d.

21. In the previous diagram, if AB denotes unity, AX or CY denotes the magnitude $\frac{3}{5}$, and it is obvious that $\frac{5}{5}$ is the same as unity, that a number of fifths less than 5 is less than unity, and a number of fifths greater than 5 is greater than unity.

In general, the fraction $\frac{x}{y}$ denotes the magnitude obtained by either dividing unity by y, and multiplying the result by x or by dividing x units by y. If x be less than y, $\frac{x}{y}$ is less than unity, and is called a **Simple** Fraction: if x is greater than y, $\frac{x}{y}$ is greater than unity and is called an **Improper** Fraction.



AD is 3 units and each unit is divided into 5 equal parts, so that each part is $\frac{1}{5}$ of a unit.

AX consists of 8 of these parts $\therefore$ AX $= \frac{8}{5}$;
But AX = AB + BX $= 1 + \frac{3}{5}$, which is written $1\frac{3}{5}$.

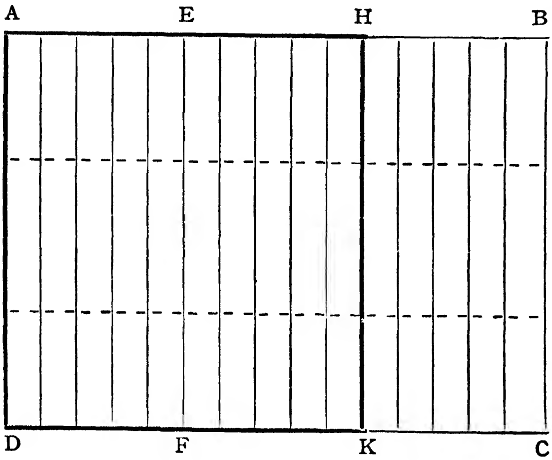
$$\therefore \tfrac{8}{5} = 1\tfrac{3}{5}.$$

Similarly, by considering AY, $2\frac{4}{5} = \frac{14}{5}$.

A **quantity consisting of a whole number together with a simple fraction is called a Mixed Number, which, it can be seen, can be readily expressed as an improper fraction;** and *vice versâ*.

A fraction expressed in the form $\frac{x}{y}$ is called a **Vulgar** (meaning Common) **Fraction**: x is called the **Numerator** and y is called the **Denominator.**

22.



ABCD is a rectangle which is divided into 3 equal parts by the lines EF and HK.

As the portion AHKD contains 2 of these parts,

AHKD is $\frac{2}{3}$ of ABCD.

Subdividing each part into 3 equal parts, ABCD contains 9 parts and AHKD contains 6 of these parts.

$$\text{Thus, AHKD is } \frac{6}{9}, \text{ i.e., } \frac{2 \times 3}{3 \times 3} \text{ of ABCD.}$$

Again, subdividing each part into 5 equal parts, ABCD contains 45 parts, and AHKD contains 30 of these parts.

$$\text{Thus, AHKD is } \frac{30}{45}, \text{ i.e., } \frac{2 \times 15}{3 \times 15} \text{ of ABCD}$$

$$\therefore \frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{2 \times 15}{3 \times 15}. \quad \text{Also } \frac{30}{45} = \frac{30 \div 5}{45 \div 5} = \frac{30 \div 15}{45 \div 15}$$

From this and similar examples, the following rule is established: **The value of a fraction is unchanged if the Numerator and Denominator be each multiplied or divided by the same number.**

NOTE 3.—This result is in accordance with the result obtained in Paragraph 4.

23. If a fraction be such that the numerator and the denominator have a common factor, the fraction is simplified by dividing each by that common factor. This can be repeated until the numerator and denominator have no common factor, in which case the fraction is expressed in its **Lowest Terms.**

24. 1 is $\frac{1}{7}$ of 7, so that 5 is $\frac{5}{7}$ of 7. Similarly, $13 = \frac{13}{15}$ of **15.** When two quantities of the same kind are considered, it is possible to express one quantity as a fraction of the other, for the quantities can be expressed in the same unit.

Thus as 16/8 = 200 pence and £1 12s. = 384 pence, 16/8 is

$$\frac{\overset{25}{\cancel{200}}}{\underset{48}{\cancel{384}}}, \; i.e., \; \frac{25}{48} \text{ of £1 12s.}$$

EXAMPLE (ii)—

Evaluate £$\frac{9}{14}$ (1) to the nearest penny (2) exactly.

	£	s.	d.	
2	9	0	0	
7	4	10	0	
	12	10 rem. 2.		} rem. 4d.

Ansr.—(1) 12/10, (2) 12/10$\frac{2}{7}$

NOTE 4.—£$\frac{9}{14}$ can be regarded either as 9 times the fourteenth part of £1, or as the fourteenth part of £9. If the fourteenth part of £1 be obtained first and expressed to the nearest penny, there will be an error not exceeding $\frac{1}{2}$d. On multiplying by 9, however, this error is magnified 9 times, and so the result will not be correct to the nearest penny. Thus, in evaluating a fraction of a quantity, multiply by the numerator before dividing by the denominator.

The remainder, 4d., divided by 14, equals $\frac{2}{7}$ penny.

EXAMPLE (iii)—

Evaluate $\frac{4}{15}$ of 7 cwt. 2 qr. 17 lb.

	cwt.	qr. (4)	lb. (28)	oz. (16)		
	7	2	17			
3	30	2	12			
5	10	0	22	10	rem. 2	} rem.
	2	0	4	8	rem. 2	} 8 oz.

Ansr.— 2 cwt. 0 qr. 4 lb. 8$\frac{8}{15}$ oz.

EXAMPLE (iv)—

Express 14s. 7$\frac{1}{2}$d. as a fraction of £1 in its lowest terms.

14/7$\frac{1}{2}$ = 175$\frac{1}{2}$ pence = 351 half-pence
£1 = 480 half-pence

$$14/7\tfrac{1}{2} = £\frac{\overset{117}{\cancel{351}}}{\underset{160}{\cancel{480}}} = £\frac{117}{160}.$$

25. MULTIPLICATION AND DIVISION OF A FRACTION BY AN INTEGER.

From paragraph 20 it is seen that $\frac{3}{5}$ of 4 is $\frac{12}{5}$. Now 4 times 3-fifths is 12-fifths, *i.e.*, $4 \times \frac{3}{5} = \frac{12}{5}$, from which it is clear that to multiply a fraction by an integer, the numerator is multiplied by the integer,

the denominator remaining unchanged. Also it is seen that $\frac{3}{5}$ of 4 is the same as $4 \times \frac{3}{5}$.

To obtain one-third of a quantity, the latter is, of course, divided by 3, so that one-third of 12 twenty-fifths is 4 twenty-fifths, *i.e.*, $\frac{1}{3}$ of $\frac{12}{25} = \frac{4}{25}$ or $\frac{12}{25} \div 3 = \frac{4}{25}$. **Thus to divide a fraction by an integer, the numerator is divided by the integer, the denominator remaining unchanged.**

From the above statement it follows that $\frac{13}{25} \div 3 = \frac{4\frac{1}{3}}{25}$, which is not a convenient way of expressing the result.

Now, $\frac{13}{25} = \frac{13 \times 3}{25 \times 3}$, so that, on dividing by 3 by the above rule, the result becomes $\frac{13}{25 \times 3}$; *i.e.*, $\frac{13}{25} \div 3 = \frac{13}{25 \times 3}$.

Thus, **to divide a fraction by an integer, the denominator is multiplied by the integer, the numerator remaining unchanged.**

26. MULTIPLICATION OF A FRACTION BY A FRACTION.

Suppose it be required to evaluate $\frac{3}{7}$ of $\frac{4}{5}$. From paragraph 20, the result is obtained by multiplying $\frac{4}{5}$ by 3 and then dividing the product by 7.

Now $\frac{4}{5} \times 3 = \frac{4 \times 3}{5}$ and $\frac{4 \times 3}{5} \div 7 = \frac{4 \times 3}{5 \times 7}$.

Thus $\frac{3}{7} \times \frac{4}{5} = \frac{3 \times 4}{7 \times 5} = \frac{12}{35}$.

By repeating this argument, it is seen that $1\frac{3}{5} \times 2\frac{5}{8} \times \frac{4}{7}$,

$$i.e., \ \frac{8}{5} \times \frac{21}{8} \times \frac{4}{7} = \frac{\cancel{8} \times \overset{3}{\cancel{21}} \times 4}{5 \times \cancel{8} \times \cancel{7}} = \frac{12}{5} = 2\frac{2}{5}.$$

Thus **the product of a number of fractions is a fraction whose numerator is the product of the numerators and whose denominator is the product of the denominators of the fractions.**

27. DIVISION OF A QUANTITY BY A FRACTION.

As unity contains 5-fifths, the number of fifths in a quantity is 5 times the number of units. To see how many groups of 3-fifths

there are in a number of fifths, the latter is divided by 3. Thus, to find the number of times $\frac{3}{5}$ is contained in a quantity, the quantity is multiplied by 5 and the result divided by 3.

Suppose the quantity be $1\frac{7}{8}$, then $1\frac{7}{8} \div \frac{3}{5} = \frac{\cancel{15}^{5}}{8} \times \frac{5}{\cancel{3}} = \frac{25}{8} = 3\frac{1}{8}$.

The reciprocal of a fraction is the fraction formed by interchanging numerator and denominator, *e.g.*, the reciprocals of $\frac{3}{7}$ and 4 are $\frac{7}{3}$ and $\frac{1}{4}$ respectively.

Thus the quotient obtained by dividing a quantity by a fraction is the same as the product of the quantity and the reciprocal of the fraction.

NOTE 5.—This result can be established in another way as follows—

$$1\tfrac{7}{8} \div \tfrac{3}{5} = \frac{1\frac{7}{8}}{\frac{3}{5}} = \frac{\frac{15}{8} \times 40}{\frac{3}{5} \times 40} = \frac{15 \times 5}{3 \times 8} = \frac{15}{8} \times \frac{5}{3}.$$

This principle of multiplying N. and D. by the L.C.M. of the denominators is very useful; *e.g.*,

$$\frac{1\frac{1}{4} + 2\frac{2}{3} - 2\frac{1}{6}}{3 - 1\frac{3}{8} - \frac{7}{12}} = \frac{(\frac{5}{4} + \frac{8}{3} - \frac{13}{6}) \times 24}{(3 - \frac{11}{8} - \frac{7}{12}) \times 24} = \frac{30 + 64 - 52}{72 - 33 - 14} = \frac{42}{25} = 1\tfrac{17}{25}$$

EXAMPLE (v)—

What is the cost of 4 cwt. 2 qr. 21 lb. of potatoes at 17s. 4d. per cwt.?

$$\text{Cost of 4 cwt.} = £3 \ 9\text{s.} \ 4\text{d.}$$

$$\text{,, } 2\tfrac{3}{4} \text{ qr.} = \frac{2\frac{3}{4}}{4} \times 17\tfrac{1}{3} \text{ shillings}$$

$$= \frac{11 \times \cancel{52}^{13}}{\cancel{4} \times 4 \times 3} \text{ ,,}$$

$$= \tfrac{143}{12}$$

$$= 11/11$$

$$\therefore \text{ Total cost} = £4 \ 1\text{s.} \ 3\text{d.}$$

EXAMPLE (vi)—

$$\text{Evaluate } \frac{3\frac{1}{3} \div 2\frac{1}{2} \times 1\frac{4}{5}}{3\frac{1}{3} \div (2\frac{1}{2} \times 1\frac{4}{5})}.$$

$$\frac{\frac{10}{3} \div \frac{5}{2} \times \frac{9}{5}}{\frac{10}{3} \div (\frac{5}{2} \times \frac{9}{5})} = \frac{\frac{10}{3} \times \frac{2}{5} \times \frac{9}{5}}{\frac{10}{3} \div \frac{9}{2}} = \frac{\frac{10}{3} \times \frac{2}{5} \times \frac{9}{5}}{\frac{10}{3} \times \frac{2}{9}} = \frac{\cancel{10} \times \cancel{2} \times 9 \times \cancel{3} \times 9}{\cancel{3} \times 5 \times 5 \times \cancel{10} \times \cancel{2}} = \tfrac{81}{25} = 3\tfrac{6}{25}$$

EXAMPLE (vii)—

A man set aside $\frac{1}{6}$ of his income for rates and taxes, $\frac{1}{4}$ of remainder for rent and insurance, $\frac{2}{9}$ of the remainder for clothes and education, $\frac{5}{8}$ of the

remainder for food and sundries, and saved the amount left, which was £30 per annum. What was his income?

$$\begin{aligned}
&\text{1st remainder} &&= \tfrac{5}{6} &&\text{of his income}\\
\therefore\ &\text{2nd} \quad ,, &&= \tfrac{5}{6} \times \tfrac{3}{4} && ,,\\
\therefore\ &\text{3rd} \quad ,, &&= \tfrac{5}{6} \times \tfrac{3}{4} \times \tfrac{3}{5} && ,,\\
\therefore\ &\text{Amount saved} &&= \tfrac{5}{6} \times \tfrac{3}{4} \times \tfrac{3}{5} \times \tfrac{3}{8} && ,,\\
\therefore\ &\text{Total income} &&= £30 \div \frac{5 \times 3 \times 3 \times 3}{6 \times 4 \times 5 \times 8}
\end{aligned}$$

$$= £\frac{\overset{10}{\cancel{30}} \times \overset{2}{\cancel{6}} \times 4 \times \cancel{5} \times 8}{\cancel{5} \times \cancel{3} \times \cancel{3} \times 3} = £\frac{640}{3}$$

$$= £213\ 6s.\ 8d.$$

EXAMPLE (viii)—

A merchant bought 5 cwt. 2 qr. 24 lb. of sugar for £12 7s. 6d. What should another merchant pay for 14 cwt. 1 qr. 4 lb. of the same kind of sugar?

$$2\text{ qr. } 24\text{ lb.} = 2\tfrac{6}{7}\text{ qr.} = \frac{20}{7 \times 4}\text{ cwt.} = \frac{5}{7}\text{ cwt.}$$

$$1\text{ qr. } 4\text{ lb.} = 1\tfrac{1}{7}\text{ qr.} = \frac{8}{7 \times 4}\text{ cwt.} = \frac{2}{7}\text{ cwt.}$$

$$\text{Cost of 1 cwt. of sugar} = £\frac{12\frac{3}{8}}{5\frac{5}{7}}$$

$$\therefore\ ,,\quad 14\tfrac{2}{7}\quad ,, \quad = £\frac{99}{8} \times \frac{\cancel{7}}{\underset{2}{\cancel{40}}} \times \frac{\overset{5}{\cancel{100}}}{\cancel{7}}$$

$$= £\frac{495}{16} = \overset{£}{30}\ \overset{s.}{18}\ \overset{d.}{9}$$

EXAMPLE (ix)—

At a trial, a motor cycle covered 330 yd. in $10\frac{2}{5}$ sec. What was its average speed in miles per hour?

$$\text{Distance covered per sec.} = \frac{330}{10\frac{2}{5}}\text{ yd.}$$

$$,,\quad ,,\quad ,,\ \text{hour} = \frac{\overset{\overset{15}{\cancel{30}}}{\cancel{330}} \times 5 \times \overset{15}{\cancel{60}} \times \overset{3}{\cancel{60}}}{52 \times \underset{\underset{\cancel{4}}{\cancel{16}}}{\cancel{1760}}}\text{ ml.}$$

$$= \frac{3375}{52}\text{ ml.}$$

$$\therefore\ \text{Average speed} = 64\tfrac{47}{52}\text{ ml. per hr.}$$

28. ADDITION AND SUBTRACTION OF FRACTIONS.

The sum of two numbers has, generally, no concrete meaning, unless the units associated with the numbers are the same.

Thus, if it be required to add 3-eighths of an inch to 7-tenths of an inch, the sum of the numbers 3 and 7 (*i.e.*, 10) has no meaning as regards expressing the value of the sum of the two quantities.

To add or subtract fractions, the rule of paragraph 22 is first applied, in order to express each fraction so that the denominators are the same in each case. Then the numerators, being expressed in the same units, can be added or subtracted, thus giving a total numerator; and the denominator which expresses the unit in which the numerators are measured will remain the same. Thus—

$$\tfrac{3}{5} + \tfrac{7}{8} - \tfrac{5}{12} = \tfrac{72}{120} + \tfrac{105}{120} - \tfrac{50}{120} = \tfrac{127}{120} = 1\tfrac{7}{120}.$$

In order that the amount of mechanical work shall be a minimum, the common denominator should be the L.C.M. of the denominators, and it need only be written down once. If mixed numbers occur, these could be expressed as improper fractions; but time is saved if the whole numbers and the fractions are added separately.

EXAMPLE (x)—

$$\begin{aligned} & 23\tfrac{3}{4} - \tfrac{7}{16} + 2\tfrac{3}{10} - 17\tfrac{5}{12} - 1\tfrac{77}{80} \\ &= 23 + 2 - 17 - 1 + \tfrac{3}{4} - \tfrac{7}{16} + \tfrac{3}{10} - \tfrac{5}{12} - \tfrac{77}{80} \\ &= 7 + \frac{180 - 105 + 72 - 100 - 231}{240} \\ &= 7 - \frac{\overset{23}{\cancel{184}}}{\underset{30}{\cancel{240}}} = \underline{\underline{6\tfrac{7}{30}}} \end{aligned}$$

NOTE 6.—A fraction can be subtracted from a whole number by inspection, *e.g.*, $\frac{23}{30}$ from 1 (*i.e.*, $\frac{30}{30}$) $= \frac{7}{30}$ and 1 from 7 $= 6$; so that $7 - \frac{23}{30} = 6\frac{7}{30}$.

NOTE 7.—The same convention as regards signs and brackets applies to fractions as to integers. Thus, $\frac{3}{5} + \frac{2}{3} \times \frac{6}{7} = \frac{3}{5} + \frac{4}{7} = \frac{41}{35} = 1\frac{6}{35}$, whereas $(\frac{3}{5} + \frac{2}{3}) \times \frac{6}{7} = \frac{19}{15} \times \frac{6}{7} = \frac{38}{35} = 1\frac{3}{35}$.

29. In the case of Algebraic expressions, it is often necessary for the sake of simplification to remove certain brackets, so that like terms may be combined together. Thus, $3a^2 + 2a(a - 3b) = 3a^2 + 2a^2 - 6ab = 5a^2 - 6ab = a(5a - 6b)$, which result, a and b being integers, follows from paragraph 7.

Suppose from a length 30″ of wire, a portion 7″ is cut off and from the remainder another 5″ is cut off. The final remainder will be 30 − 7 − 5 in. Now if 12″, *i.e.* (7 + 5) in. had been cut off in one piece, the remainder would obviously be the same. Thus, 12 − (7 + 5) = 12 − 7 − 5. Again, if 2 pieces of length 7″ were cut off and afterwards 2 pieces of length 5″ cut off, the remainder would be 30 − 2 × 7 − 2 × 5 in. Now, if 2 lengths each of 12″, *i.e.* (7 + 5) in., had been cut off in one piece, the remainder would be the same. Thus, 12 − 2(7 + 5) = 12 − 2 × 7 − 2 × 5.

Suppose from a bag containing 30 apples, 7 apples are taken out and 5 put back. The number left in the bag is $30 - 7 + 5$. Now, if 2 apples, *i.e.* $(7 - 5)$ apples, had been taken out instead, the number left would obviously be the same. Thus, $12 - (7 - 5) = 12 - 7 + 5$. Again, if 2 groups of 7 apples were taken out and then 2 groups of 5 apples put back, the number left in the bag would be $30 - 2 \times 7 + 2 \times 5$. Now, if 2 groups of 2 apples, *i.e.* $2(7 - 5)$ apples had been taken out instead, the number left would obviously be the same. Thus, $30 - 2(7 - 5) = 30 - 2 \times 7 + 2 \times 5$.

It is seen from the above and similar arguments that the minus sign before a bracket has the effect, when the bracket is removed, of changing the sign of each term in the bracket. This result can be generalized as follows: If a, b, c, and x denote any integers, then $-x(a + b - c) = -ax - bx + cx$.

EXAMPLE (xi)—

The head of a department of a business house requires 250 copies of a circular to be typed. Three typists are employed on the work, and it is found that the times taken by them to type 10 copies are 50 min., 45 min. and 42 min. respectively. How long would it take them, working simultaneously, to perform the whole work, and how many copies would each typist complete?

In 1 min., the number of copies typed $= \frac{10}{50} + \frac{10}{45} + \frac{10}{42}$

$$= \frac{1}{5} + \frac{2}{9} + \frac{5}{21}$$

$$= \frac{63 + 70 + 75}{315} = \frac{208}{315}$$

Approximate time taken to type 250 copies $= 250 \times \frac{315}{208}$ min.

$= 378$ „

In 378 min., 1st typist can do $\frac{378}{5}$ copies $= 75\frac{3}{5}$ copies

„ „ 2nd „ „ $\frac{378 \times 2}{9} = 84$ „

„ „ 3rd „ „ $\frac{378 \times 5}{21} = 90$ „

The 1st typist can complete her 76th copy in 2 min.

Ansr—6 hr. 20 min. $\begin{cases} \text{1st typist does 76 copies in 6 hr. 20 min.} \\ \text{2nd „ 84 „ 6 hr. 18 min.} \\ \text{3rd „ 90 „ 6 hr. 18 min.} \end{cases}$

EXAMPLE (xii)—

Find the total cost in pounds of x oranges at a for a shilling and y lemons at b for a shilling.

$$\text{Total cost} = \frac{x}{a} + \frac{y}{b} \text{ shillings}$$

$$= \frac{bx + ay}{ab} \quad \text{„}$$

$$= \frac{bx + ay}{20ab} \text{ pounds.}$$

*EXAMPLE (xiii)—

One steamer performs a journey of d miles at x miles per hour and returns at y miles per hour. A second steamer performs both outward and homeward journeys at constant speed, which is the average of the speeds of the first steamer. How much less time would the second steamer take over the double journey than the first? Also find this time to the nearest second if $d = 500$, $x = 22\frac{1}{2}$, $y = 21\frac{1}{2}$.

$$\text{Total time taken by 1st steamer} = \frac{d}{x} + \frac{d}{y} \text{ hr.}$$

$$\text{,, ,, 2nd ,,} = \frac{2d}{\frac{x+y}{2}} \text{ hr.}$$

$$= \frac{4d}{x+y} \text{ hr.}$$

$$\therefore \text{Time 2nd steamer takes less than 1st} = \frac{d}{x} + \frac{d}{y} - \frac{4d}{x+y} \text{ hr.}$$

$$= \frac{dy(x+y) + dx(x+y) - 4dxy}{xy(x+y)} \text{ hr.}$$

$$= \frac{dxy + dy^2 + dx^2 + dxy - 4dxy}{xy(x+y)} \text{ hr.}$$

$$= \frac{d(x^2 + y^2 - 2xy)}{xy(x+y)} \text{ hr.}$$

$$x^2 + y^2 - 2xy = \frac{2{,}025 + 1{,}849 - 3{,}870\dagger}{4} = 1$$

$$x + y = 44$$

$$\therefore \text{Time 2nd steamer takes less than 1st} = \frac{500 \times 1 \times 60 \times 60}{\frac{45}{2} \times \frac{43}{2} \times 44} \text{ sec.}$$

$$\frac{500 \times \overset{4}{\cancel{60}} \times 60 \times 2 \times 2}{\underset{3}{\cancel{45}} \times 43 \times \underset{11}{\cancel{44}}} \text{ sec.}$$

$$= \frac{120000}{1419} \text{ sec.}$$

$$= 1 \text{ min. } 25 \text{ sec. to nearest sec.}$$

EXAMPLE (XIV)—

A man earned £525 per annum. The taxable income was the remainder after one-fifth of his earnings and £175 had been deducted. On £175 of taxable income he paid at the rate of 2s. 6d in the £ and on the remainder at 5s. in the £. What fraction of his earnings did he pay?

† Knowing that $x^2 - 2xy + y^2 = (x - y)^2$, this working would not have been necessary.

$$\text{Total deduction} = £105 + £175$$
$$= £280$$
$$\text{Taxable income} = £245$$
$$\text{Amount paid} = £\frac{175}{8} + £\frac{70}{4}$$
$$= £\frac{315}{8}$$
$$\text{Fraction of income paid} = \frac{315}{8 \times 525}$$
$$= \frac{3}{40}$$

EXAMPLE (xv)—

Find the cost to nearest penny of 7 ton 2 cwt. 2 qr. 24 lb. of a commodity at £2 8s. 4d. per ton.

Cost of 7 ton = £16 18s. 4d.

$$2 \text{ cwt. } 2 \text{ qr. } 24 \text{ lb.} = \frac{224 + 56 + 24}{2240} \text{ ton} = \frac{\overset{19}{\cancel{\overset{38}{\cancel{304}}}}}{\underset{140}{\cancel{\underset{280}{\cancel{2240}}}}} \text{ ton}$$

$$\therefore \text{ Cost of } 2 \text{ cwt. } 2 \text{ qr. } 24 \text{ lb.} = \frac{19}{\underset{28}{\cancel{140}}} \times \frac{\overset{29}{\cancel{145}}}{3} \text{ shillings}$$
$$= \tfrac{551}{84} \text{ shillings} = 6\tfrac{47}{84} \text{ shillings}$$
$$= 6/6\tfrac{5}{7}$$

$\therefore$ Total cost = £17 4s. 11d., to nearest penny

EXAMPLE (xvi)—

Find a multiplier to convert £ per quarter to pence per pint.

If cost of 1 qr. = £1

$$\text{then cost of 1 pt.} = \frac{20 \times 12}{8 \times 8 \times 8} \text{ pence}$$
$$= \frac{15}{32} \text{ pence}$$

$\therefore$ The multiplier is $\frac{15}{32}$

TEST EXERCISES I, 4

(1) Make out a bill for 11 yd. of calico at $8\frac{3}{4}$d. per yd.; $4\frac{1}{2}$ yd. of silk at 5s. $1\frac{1}{2}$d. per yd.; 10 oz. of wool at $6\frac{1}{2}$d. per oz.; 5 reels of cotton at $3\frac{1}{2}$d. per reel; 7 packets of needles at $1\frac{3}{4}$d. per packet.

(2) Make out a bill for $\frac{1}{2}$ doz. lb. of sugar at $5\frac{1}{4}$d. per lb.; $3\frac{1}{2}$ lb. of cheese at 1s. $6\frac{1}{2}$d. per lb.; $2\frac{1}{2}$ lb. of butter at 2s. 1d. per lb.; $6\frac{1}{2}$ lb. of butter at 1s. $9\frac{1}{2}$d. per lb.

(3) Find the cost of 5 ton 17 cwt. 2 qr. of hay at £3 16s. 8d. per ton.

(4) Two ricks containing $20\frac{1}{4}$ tons of hay and $29\frac{1}{2}$ tons respectively were sold. If the first was sold for £107 7s. 6d., what should the second have been sold for ?

(5) What was the cost of carrying 3 ton 13 cwt. 3 qr. 14 lb. at £3 17s. 6d. per ton ?

(6) A partner in a business worth £5,400 sells $\frac{5}{16}$ of his share for £675. What fraction of the business does he still own ?

(7) In a workshop 35 men were employed; the working hours were 53 per week and the weekly wages amounted to £85 0s. 5d. If 7 extra men were employed and they all worked 10 hrs. per week overtime for which they were paid $1\frac{1}{4}$ times the usual rate, calculate the increase of the total amount paid in wages per week.

(8) In a van there were 5 parcels each weighing 7 lb. $2\frac{3}{4}$ oz.; 9 parcels each weighing 13 lb. $11\frac{3}{8}$ oz.; and 14 parcels each weighing 9 lb. $7\frac{5}{8}$ oz. Find the average weight of the parcels.

(9) A rectangular plate of metal is 1 ft. $7\frac{17}{32}$ in. long and $10\frac{13}{16}''$ broad. What is the distance round and by how much is the length greater than the breadth ?

(10) Rectangular tiles are $7\frac{4}{5}''$ long and $6\frac{3}{10}''$ wide. What is the least number that could be placed together so as to form a square ?

(11) On a farm of 420 ac., $\frac{3}{5}$ of the land is arable. The farmer plants $\frac{1}{2}$ of this with wheat, allowing $3\frac{1}{4}$ bush. to the acre, $\frac{1}{3}$ of it with barley, allowing $2\frac{1}{2}$ bush. to the acre, and the remainder with other crops. How many bushels of barley and wheat will he require for seed ? Also how many acres are available for the growing of potatoes if $\frac{3}{8}$ of the remaining land be used for the purpose ?

(12) Find the cost of the wire for making a wire fence to run along both sides of a path $\frac{1}{4}$ mile long and using 5 rows of wire. The wire costs 12s. $7\frac{1}{2}$d. per cwt., and 100 yd. of wire weigh $24\frac{1}{4}$ lb

(13) The amount of corn required to sow a field of $11\frac{1}{4}$ ac. was $30\frac{3}{4}$ bush., and the yield was $418\frac{3}{8}$ bush. Find the yield per acre. Also find how many times over the farmer gets his seed back.

(14) A tank when $\frac{1}{4}$ full of oil weighs 3 lb. 14 oz., and when $\frac{2}{3}$ full weighs 7 lb. 4 oz. Find the weight of the tank (*a*) when empty, (*b*) when $\frac{1}{2}$ full of oil.

(15) $13\frac{1}{2}$ doz. parcels have to be made up by four boys. The first takes $3\frac{1}{2}$ min., the second takes 4 min., and each of the other two $4\frac{1}{2}$ min. to make up a parcel. If all begin at 8.45 a.m., at what time should the work be finished ?

(16) Find a multiplier to convert pence per lb. to £ per ton. Use it to find the loss incurred by a dealer who bought $2\frac{1}{2}$ tons of potatoes at £11 18s. 9d. per ton and sold then at $1\frac{1}{4}$d. per lb.

(17) Find the cost, to the nearest penny, of 7 oz. Troy 13 dwt. 18 gr. of metal at 1s. $6\frac{1}{2}$d. per oz. Troy.

(18) A, B, C, D, and E are in partnership with a total capital of £2,340. A's share is $\frac{1}{5}$ of this amount, B's share is $\frac{3}{8}$ of the remainder; and of the amount left, C's and D's shares are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. What are the capitals of the partners ?

(19) Find, to the nearest penny, the cost of 3 cwt. 3 qr. 18 lb. bacon at 125s. 6d. per cwt. What is the gross profit if the bacon is sold at an average price of 1s. $7\frac{1}{2}$d. per lb. ?

(20) Find the value of 9,000 rupees at 1s. $6\frac{5}{32}$d. per rupee.

(21) Given that 1 kilogram = $2\frac{1}{5}$ lb., 90 francs = £1, and 1 kilometre = $\frac{5}{8}$ mile, find a multiplier to convert pence per ton per mile to francs per 1,000 kilograms per kilometre.

(22) Find the cost, to the nearest penny, of 851 shares at 4s. $8\frac{3}{16}$d. each, if the broker charges $1\frac{1}{2}$d. per share.

(23) If a bar of metal which weighs 593 oz. 19 dwt. is £64 14s. 6d., in value, find the cost of a bar 763 oz. 13 dwt.

(24) How many rings each weighing 4 dwt. 20 gr. can a goldsmith make from a mixture of 22 oz. Troy 1 dwt. 18 gr. of pure gold with 7 oz. Troy 7 dwt. 6 gr. of alloy?

(25) A sum of money is to be divided amongst 11 men and 18 boys, and 5 men are to receive as much as 9 boys. When 4 men and 5 boys have received their shares, what fraction of the whole sum will remain?

(26) Two stations, A and B, are 150 miles apart. A goods train leaves A at 8.20 a.m. and travels towards B at $17\frac{1}{2}$ miles per hour. An express train leaves B at 9.40 a.m. on the same day and travels towards A at $57\frac{3}{8}$ miles per hour. When and where do they meet? (*Note*—Find where the goods train is at 9.40 a.m.; then find the time after this when they meet, using the fact that they approach one another at $17\frac{1}{2} + 57\frac{3}{8}$ miles per hour.)

(27) Two pipes together fill a reservoir in 10 hr. 20 min. One of them alone fills it in 18 hr. In what time would the other fill it?

(28) Four junior clerks are employed in addressing 2,400 envelopes. To address 50 envelopes they take 4 min. 25 sec., 4 min. 50 sec., 5 min. 18 sec., and 5 min. 48 sec. respectively. How long should they take working together and keeping up the same average rates?

(29) A tobacconist buys 1,000 packets of tobacco: a packets at p pence each, and the remainder at an additional penny per packet. What is the total cost?

(30) A tailor spent four sums each of £241 10s. in buying cloth at 3s. 10d., 3s. 6d., 3s. 4d., and 3s. per yard respectively. If the price of each kind had risen 6d. per yard, find to the nearest inch the amount less of each kind of cloth he could buy for the money.

(31) A train performed a journey of j miles in n hours. Find how many minutes less it would take on the journey if its average speed were to be increased by 2 miles per hour.

(32) One pipe can fill a swimming bath in x hr. and another can fill it in y hr. z min. How long would it take to fill the bath if both pipes were running?

(33) A dishonest draper has n yards of lace, which he claimed to sell at p pence per yard. In reality, his "yard" measure was x inches short. If he sells all the lace, by how many shillings has he defrauded his customers?

(34) A merchant sold $\frac{a}{b}$ of his goods and later on he sold $\frac{x}{y}$ of the remainder. What fraction of the original quantity remained unsold?

CHAPTER V.

SIMPLE MENSURATION.

30. An **Area** is an amount of surface.

A Square Inch is the amount of surface contained by a square whose side is 1 inch. A square inch is an amount, and has no reference to shape.

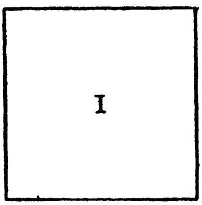


Fig. 1.

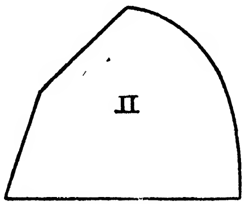


Fig. 2.

Fig. I is a square, side 1". Fig. II is an irregular figure, which has the same amount of surface as Fig. I. The area of Fig. II is, therefore, 1 sq. inch.

A square foot, square yard, square pole, and square mile are amounts of surface contained by squares whose sides are 1 foot, 1 yard, 1 pole, and 1 mile respectively. There is no such thing as a pole or perch of land. This term is sometimes used in dealing with land, but a square pole or square perch is meant, the word "square" being erroneously omitted.

31. AREA OF A RECTANGLE.

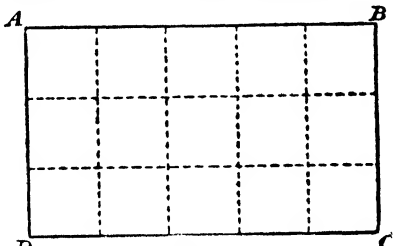


Fig. 3.

ABCD is a rectangle whose length is 5 units and breadth 3 units. The dotted lines divide it into a number of squares, the area of each being 1 sq. unit.

The number of squares in a row is equal to the number of units in the length, while the number of rows of squares is equal to the number of units in the breadth. Now, to obtain the number of squares, the number in one row is multiplied by the number of rows: so that the number of square units in the area is the product of the number of units in the length and breadth.

This result is true in the case when the length and breadth involve fractions of units, as the following diagram shows.

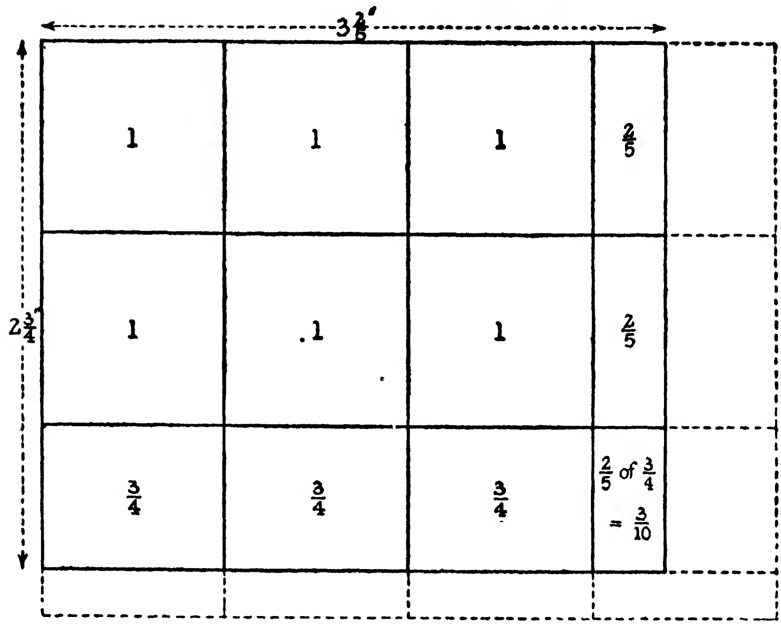


FIG. 4.

The rectangle is divided as far as possible into squares, whose sides are 1 inch.

The areas of the remaining portions are expressed as a fraction of a square inch.

$$\text{Area of rectangle} = 6 + 3 \times \tfrac{3}{4} + 2 \times \tfrac{2}{5} + \tfrac{3}{10} \text{ sq. in.}$$
$$= 8 + \tfrac{1}{4} + \tfrac{4}{5} + \tfrac{3}{10} \text{ sq. in.}$$
$$= 9\tfrac{7}{20} \text{ sq. in.}$$
$$\text{Now } 3\tfrac{2}{5} \times 2\tfrac{3}{4} = \tfrac{17}{5} \times \tfrac{11}{4} = \tfrac{187}{20} = 9\tfrac{7}{20}.$$

Thus the number giving the area of the rectangle in square inches is the product of the numbers of inches in the length and breadth respectively.

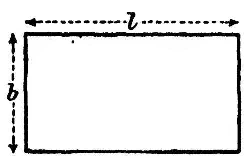


FIG. 5.

Let $l \equiv$ number of units in the length of a rectangle

" $b \equiv$ " *same* units in the breadth of the rectangle

" $A \equiv$ " of sq. units in the area of the rectangle

then $A = lb$.

If $b = l$, the figure is a square and the formula becomes $A = l^2$.

32. From the last result, the following table is obtained—

144 (*i.e.*, 12 × 12) sq. in.	=	1 sq. ft.
9 (*i.e.*, 3 × 3) sq. ft.	=	1 sq. yd.
$30\frac{1}{4}$ (*i.e.*, $5\frac{1}{2} \times 5\frac{1}{2}$) sq. yd.	=	1 sq. pole
1,760 × 1,760 sq. yd.	=	1 sq. mile

In dealing in land, the units *acres* and *roods* are employed, and these are defined as—

40 sq. poles	=	1 rood
4 roods	=	1 acre
Thus 1 acre	=	$4 \times 40 \times 30\frac{1}{4}$ sq. yd. = 4,840 sq. yd.

$$\text{and} \quad 1 \text{ sq. mile} = \frac{1760 \times 1760}{4840} \text{ acres} = 640 \text{ acres}$$

33. A **Parallelogram** is a four-sided rectilinear (*i.e.*, bounded by straight lines) figure, whose opposite sides are parallel.

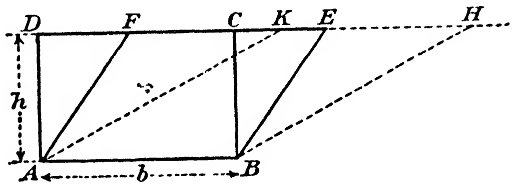


FIG. 6.

$ABCD$ is a rectangle, $ABEF$ and $ABHK$ are parallelograms. The common base is b units long, and the perpendicular heights of each are the same, namely, h units.

By cutting off the portion AFD and placing it in the position BEC, it is seen that the areas of $ABCD$ and $ABEF$ are equal.

By cutting off the portion AKF and placing it in the position BHE, it is seen that the areas $ABEF$ and $ABHK$ are equal.

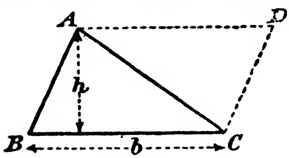


FIG. 7.

Let A sq. units $\equiv$ area of a parallelogram, base b units, perp. ht. h units

$$\text{then } A = bh.$$

34. The area of the triangle ABC is equal to that of ADC, and thus is half the area of the parallelograms $ABDC$.

Let A sq. units $\equiv$ area of a triangle, base b units, perp. ht. h units.

$$\text{Then } A = \frac{bh}{2}$$

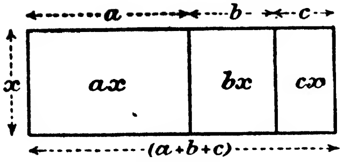


FIG. 8.

35. $x, a, b,$ and c denote integers or fractions, the unit of length in each case being the same. The total area is $ax + bx + cx$ sq. units. Now, considering the whole as one rectangle, the area is $x(a + b + c)$ sq. units.

$$\therefore\ x(a + b + c) = ax + bx + cx.$$

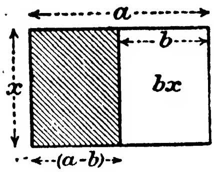


FIG. 9.

Considering the shaded area as being the difference of two rectangles, its area is $ax - bx$ sq. units. Regarding it as a single rectangle, its area is $x(a - b)$ sq. units.

$$\therefore\ x(a - b) = ax - bx.$$

Now, the argument of paragraph 29 applies equally well for fractions as for integers, so that if $x, a, b, c \ldots$ denote integers or fractions, the following results are true.

$$x(a + b - c \quad . \quad . \quad . \quad) = \quad ax + bx - cx \quad . \quad . \quad . \quad . \quad .$$
$$-x(a + b - c \quad . \quad . \quad . \quad) = -ax - bx + cx \quad . \quad . \quad . \quad . \quad .$$

36. Applications of the last result are now given, it being understood that in any one figure the letters denote numbers of the same units, and A denotes the area in square units in each case.

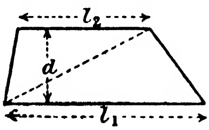


FIG. 10.

Trapezium.

$$A = \frac{l_1 d}{2} + \frac{l_2 l}{2} = \frac{d(l_1 + l_2)}{2}$$

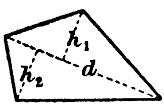


FIG. 11.

Quadrilateral.

$$A = \frac{dh_1}{2} + \frac{dh_2}{2} = \frac{d(h_1 + h_2)}{2}$$

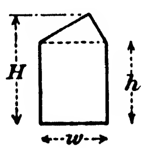


FIG. 12.

$$A = hw + \frac{(H - h)w}{2} = \frac{2hw + Hw - hw}{2}$$
$$= \frac{hw + Hw}{2} \qquad = \frac{w(h + H)}{2}$$

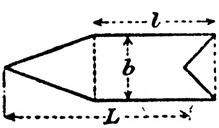


FIG. 13.

$$A = 2 \times \frac{b}{2} \times \frac{L + l}{2}$$
$$= \frac{b(L + l)}{2}$$

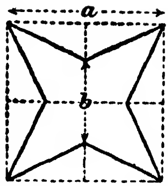


FIG. 14.

$$A = a^2 - 4 \times \frac{a}{2} \times \frac{a-b}{2}$$
$$= a^2 - a(a-b)$$
$$= a^2 - a^2 + ab$$
$$= ab$$

NOTE 1.—An angle is an amount of rotation. One quarter of a complete rotation is called a right angle. A right angle is subdivided into 90 equal parts, each part being called a degree. Also 60 min. = 1 degree and 60 sec. = 1 min. An angle of 35 deg. 13 min. 28 sec. is written 35° 13′ 28″.

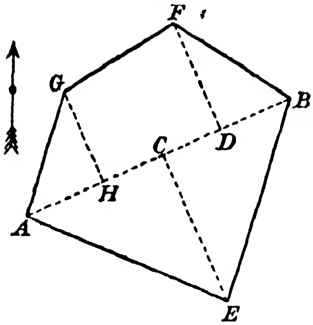


FIG. 15.

37. A surveyor who wishes to measure the area of a field as represented by the figure would proceed as follows—

A base line AB is chosen, in this case in a direction 64° 50′ E. of N.

G, F, and E are objects whose positions are determined by measuring AH and HG, AC and CE, AD and DF, H, C, and D being points on the base line, such that the angles AHG, ACE, and ADF are right angles.

Suppose A, B, E, G, and F are the corners of a field whose area is to be measured. The necessary measurements would be recorded in the Field-book as follows—

LINKS, 64° 50′ E. of N.

618 (F) 524 (G)	1,386 ⊙ B 987 (D) 705 (C) 362 (H)	746 (E)
	FROM ⊙ A	

Area of AEB $= \dfrac{1{,}386 \times 746}{2}$ sq. links

$= 516{,}978$ sq. links

Area of AGH $= \dfrac{362 \times 524}{2}$ sq. links

$= 94{,}844$ sq. links

Area of $HGFD$ $= \dfrac{625 \times (524 + 618)}{2}$ sq. links

356,875 sq. links

Area of DFB $= \dfrac{399 \times 618}{2}$ sq. links

$$
\begin{aligned}
& = 123{,}291 \text{ sq. links} \\
\therefore \text{ Total area} & = 1{,}091{,}988 \text{ sq. links} \\
& = 109 \text{ sq. chains } 1{,}988 \text{ sq. links} \\
& = \underline{\underline{10 \text{ acres } 9 \text{ sq. chains } 1{,}988 \text{ sq. links}}}
\end{aligned}
$$

NOTE 2.—

$$
\begin{aligned}
1 \text{ sq. chain} & = 100 \times 100 \text{ sq. links} \\
1 \text{ acre} & = \frac{4840}{22 \times 22} \text{ sq. chains} = 10 \text{ sq. chains} \\
1 \text{ sq. chain} & = \frac{22 \times 22}{30\frac{1}{4}} \text{ sq. poles} = 16 \text{ sq. poles}
\end{aligned}
$$

38. PLANS DRAWN TO SCALE.

In order to represent areas in their proper shapes, plans are drawn each to a scale which is convenient. For example, the plan of the site of a building might be $\frac{3}{8}''$ to a foot, which means that a line $\frac{3}{8}''$ long on the map represents actually a length of 1 foot. As $\frac{3}{8}'' = \frac{1}{32}'$, the scale of the plan could be stated as being $\frac{1}{32}$ full size, which means that the actual distance from any two points is 32 times longer than the corresponding distance on the map.

In dealing with areas, care must be taken; for $\frac{3}{8}$ sq. in. does not represent 1 sq. ft., nor does 1 sq. in. represent 32 sq. in. A square of side $\frac{3}{8}''$ on the map represents a square of side 1 ft.: thus, $\frac{3}{8} \times \frac{3}{8}$ sq. in., *i.e.*, $\frac{9}{64}$ sq. in., represents 1 sq. ft.

Similarly, a square of side 1 in. on the map represents a square of side 32 in. Thus,

1 sq. in. represents $\frac{32 \times 32}{12 \times 12}$ sq. ft. (*i.e.*, 1 sq. in. represents $7\frac{1}{9}$ sq. ft.)

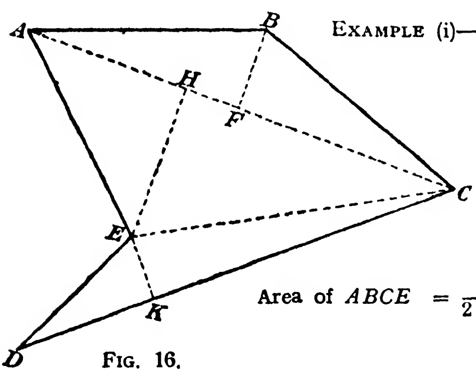


FIG. 16.

EXAMPLE (i)—

ABCDE is a plan of an estate drawn to a scale of 4″ to 1 mile. $AC = 2\frac{3}{8}''$, $BF = \frac{7}{16}''$, $EH = \frac{13}{16}''$, $DC = 2\frac{7}{16}''$, $EK = \frac{1}{3}''$.

Find the acreage of the estate and the cost at £125 per acre.

$$
\begin{aligned}
\text{Area of } ABCE & = \frac{19 \times \overset{5}{\cancel{20}}}{2 \times 8 \times \underset{4}{\cancel{16}}} \text{ sq. in.} = 1\tfrac{31}{64} \text{ sq. in.} \\
\text{,, } \quad DEC & = \frac{1}{2} \times \frac{\overset{13}{\cancel{39}}}{16} \times \frac{1}{\cancel{3}} = \frac{13}{32} \text{ ,,} \\
\text{,, } \quad ABCDE & = 1\tfrac{31}{64} + \tfrac{26}{64} \text{ sq. in.} = 1\tfrac{57}{64} \text{ ,,}
\end{aligned}
$$

$\therefore$ Now 4×4 sq. in. represents 640 acres

$\therefore$ 1 sq. in. ,, $\frac{640}{4 \times 4}$,,

$\therefore$ $1\frac{57}{64}$ sq. in. ,, $\dfrac{\overset{\overset{5}{\cancel{10}}}{\cancel{640}} \times 121}{4 \times \cancel{4} \times \underset{2}{\cancel{64}}}$ acres

$= 75\frac{5}{8}$ acres.

Cost of estate $= £75\frac{5}{8} \times 125$
$= £9,375 + £78\frac{1}{8}$
$= £9,453$ 2s 6d.

NOTE 3.—Usually in a plan of a particular estate, plot, or site, the actual distances are given, but on a map, as only the scale is given, the actual area of a portion of ground represented on the map can be obtained by measuring suitable distances and applying the method shown in the example.

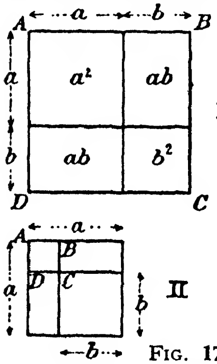


FIG. 17.

39. The following identities are sometimes useful—

I. $(a + b)^2 = (a + b)(a + b)$
$= a(a + b) + b(a + b)$
$= a^2 + ab + ab + b^2$

i.e., $(a + b)^2 = a^2 + 2ab + b^2$

II. Similarly, $(a - b)^2 = a^2 - 2ab + b^2$.

The areas of $ABCD$ in Figs. I and II are $(a + b)^2$ and $(a - b)^2$ respectively, and the above results can be verified by considering the figures.

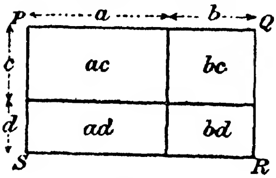


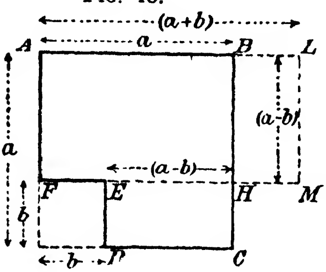
FIG. 18.

$(a + b)(c + d) = c(a + b) + d(a + b)$
$= ac + bc + ad + bd.$

The area of $PQRS$ is $(a + b)(c + d)$, and the figure verifies the above result.

Also $(a + b)(c - d) = c(a + b) - d(a + b)$
$= ac + bc - ad - bd$

and $(a - b)(c - d) = c(a - b) - d(a - b)$
$= ac - bc - ad + bd.$



The area of $ABCDEF$ is $a^2 - b^2$. Now, if the rectangle $EHCD$ is cut off and placed in the position $LMHB$, the rectangle $ALMF$ is formed. The area of $ALMF$ is $(a + b)(a - b)$, thus $\mathbf{a^2 - b^2 = (a + b)(a - b)}$

e.g., $1,934^2 - 1,924^2 = 3,858 \times 10 = 38,580$.

40. The formulae $A = lb$, $A = bh$, $A = \dfrac{bh}{2}$ are employed in finding the **areas** of a rectangle, parallelogram and triangle respectively, when the necessary linear measurements are given. It is necessary in some problems to find a linear measurement, the area being given.

Now for a rectangle $lb = A$. As the results obtained by dividing two equal quantities by the same quantity are equal, by dividing lb and A each by l, the result $\mathbf{b} = \dfrac{\mathbf{A}}{\mathbf{l}}$ is obtained.

Similarly, $\mathbf{l} = \dfrac{\mathbf{A}}{\mathbf{b}}$.

Similarly, in the case of the parallelogram, $\mathbf{b} = \dfrac{\mathbf{A}}{\mathbf{h}}$ and $\mathbf{h} = \dfrac{\mathbf{A}}{\mathbf{b}}$

By multiplying both sides of the formula $A = \dfrac{bh}{2}$ by 2, it is seen that $2A = bh$. By dividing each side by b, it is clear that $h = \dfrac{2A}{b}$. Thus in the case of the triangle,

$$\mathbf{h} = \frac{\mathbf{2A}}{\mathbf{b}} \text{ and } \mathbf{b} = \frac{\mathbf{2A}}{\mathbf{h}}$$

Example (ii)—

A lawn 14 yd. long and 12 yd. 9 in. wide is surrounded by a path 5′ 3″ wide. How many square tiles of side 10½″ will be required to cover the path?

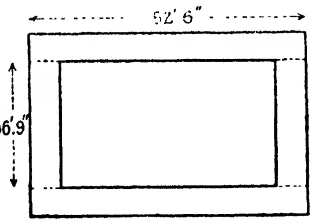


Fig. 20.

Length of path $= (52\frac{1}{2} + 36\frac{3}{4}) \times 2$ ft.

$= 2 \times 89\frac{1}{4}$ ft

$\therefore$ Area of path $= 2 \times 89\frac{1}{4} \times 5\frac{1}{4}$ sq. ft.

Area of 1 tile $= \frac{7}{8} \times \frac{7}{8}$ sq. ft.

$\therefore$ No. of tiles $= \dfrac{2 \times \overset{51}{\cancel{357}} \times \overset{3}{\cancel{21}} \times \overset{2}{\cancel{8}} \times \overset{2}{\cancel{8}}}{\cancel{4} \times \cancel{4} \times \cancel{7} \times \cancel{7}}$

$=$ 1,224 tiles.

Example (iii)—

540 pieces of metal of the shape indicated by the last diagram of paragraph 36 have to be cut out for the purpose of making medals. If $a = 1\frac{3}{4}''$, $b = \frac{13}{16}''$, and 1 sq. in. of the sheet of metal from which the medals are to be cut weighs $\frac{4}{5}$ oz. Troy, find the cost of the metal used at 2s. 10½d. per oz. Troy.

Area of 1 piece of metal $= \frac{7}{4} \times \frac{13}{16}$ sq. in.

Wt. of 540 pieces of metal $= \frac{7}{4} \times \frac{13}{16} \times \frac{4}{5} \times 540$ oz. Troy

Cost of 540 pieces of metal $= \frac{7}{4} \times \frac{13}{16} \times \frac{4}{5} \times 540 \times 2\frac{7}{8}$ shillings

$$= \frac{7 \times 13 \times \cancel{4} \times \overset{\overset{27}{\cancel{108}}}{\cancel{540}} \times 23}{\cancel{4} \times \underset{4}{\cancel{16}} \times \cancel{5} \times 8}$$

$$1{,}765\tfrac{31}{32} \text{ shillings}$$
$$= £88 \ 5s \ 11\tfrac{5}{8}d.$$

EXAMPLE (iv)—

A room is 18′ long, 13′ 6″ wide, and 8′ high. How many pieces of wall paper 12 yds. long and 21 in. wide would be required to cover the four walls? Find the cost of paper at 2s. 4d. per piece, assuming that the portions of the walls taken up by windows and door cause the wastage of the paper due to overlapping to be counteracted.

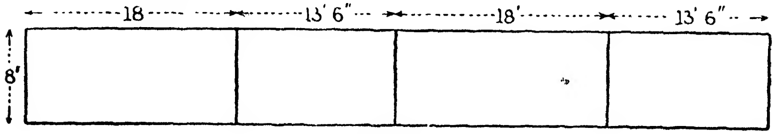


FIG 21.

$$\text{Perimeter of floor} = 2 \times 31\tfrac{1}{2} \text{ ft.}$$
$$\therefore \text{ Area of 4 walls} = 8 \times 63 \text{ sq. ft.}$$
$$\text{Also area of one piece of paper} = 36 \times \tfrac{21}{12} \text{ sq. ft.}$$

$$\therefore \text{ No. of pieces} = \frac{8 \times \overset{3}{\cancel{63}}}{\cancel{3} \times \cancel{21}} = 8$$

$$\therefore \text{ Cost of paper} = 8 \times 2/4$$
$$= 18/8.$$

NOTE 4.—The perimeter of a figure is the distance round the boundary of the figure.

EXAMPLE (v)—

A rectangular field measures 5 chains 75 links by 2 chains 16 links. It is required to divide it into allotments, each of area 10 sq. poles by lines parallel to the shorter sides of the field How many of these allotments will there be, what will be the width of each allotment, and what will be the area of the remaining portion?

$$\text{Number of allotments} = \frac{5\tfrac{3}{4} \times 2\tfrac{4}{25} \times 16}{10}$$

$$= \frac{23 \times 54 \times \overset{\overset{2}{\cancel{4}}}{\cancel{16}}}{\cancel{4} \times 25 \times \underset{5}{\cancel{10}}}$$

$$= 19\tfrac{109}{125}$$

Thus there will be 19 *allotments*, and

$$\text{Area of remaining ground} = \tfrac{109}{125} \times 10 \text{ sq. poles}$$
$$= 8\tfrac{18}{25} \text{ sq. poles}$$

$$\text{Width of each allotment} = \frac{10 \times 30\frac{1}{4}}{2\frac{4}{25} \times 22}\text{ yd.}$$

$$= \frac{\overset{5}{\cancel{10}} \times \overset{11}{\cancel{121}} \times 25}{4 \times 54 \times \underset{2}{\cancel{22}}}\text{ yd.}$$

$$= 6\text{ yd. } 1\text{ ft. } 1\tfrac{1}{8}\text{ in.}$$

EXAMPLE (vi)—

Turf to a width of 3′ 6″ is removed round a square lawn, whose side formerly was 25 yd. If a triangular lawn, base 110 ft., is to be formed with this turf, what will be the perpendicular distance from the corner of the lawn to its base?

$$\text{Area of turf removed} = 75^2 - 68^2\text{ sq. ft.}$$
$$= 143 \times 7\text{ sq. ft.}$$

$$\therefore \text{ Perp. dist. from corner to base} = \frac{2 \times 143 \times 7}{110}\text{ ft.}$$

$$= \frac{91}{5}\text{ ft.}$$

$$= 18\text{ ft. } 2\tfrac{2}{5}\text{ in.}$$

41. A Volume is an amount of space.

A **Cubic Inch** is the amount of space contained by a cube, each edge of which is 1 in. long. As in the case of a square inch, a cubic inch is an amount, and has no reference to shape.

A **Prism** is a solid object, such that the face opposite the base is parallel to and equal in all respects to the base, and the edges connecting the corners of the base to the corresponding corners of the opposite face are parallel. When these edges are at right angles to the plane of the base, the prism is called a **right** prism. If the base of the prism is a square, rectangle, triangle, hexagon (6-sided figure), or octagon (8-sided figure), the prism is called a square, rectangular, triangular, hexagonal, or octagonal prism respectively.

42. VOLUME OF A PRISM.

PQRS is a square side 1 unit of length, drawn on the base of the prism.

T, *U*, *V*, and *W* are vertically, 1 unit of length, above *P*, *Q*, *R*, and *S* respectively.

Then *PQRS* is the base of a cube; whole volume is 1 cubic unit.

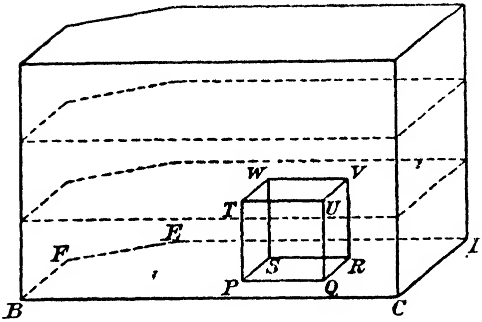


FIG. 22.

Let $A \equiv$ number of square units in area of base $BCDEF$.

Then the area of $BCDEF$ is A times the area of $PQRS$. Therefore the volume of the prism, base $BCDEF$ and height 1 unit, is A times the volume of the cube whose volume is 1 cubic unit. Thus the volume of the prism, base $BCDEF$ and height 1 unit, is A cubic units.

Let $h \equiv$ number of units in the height of the prism, then the volume of the prism will be h times the volume of prism whose base is $BCDEF$ and height 1 unit.

Thus if $V \equiv$ number of cubic units in the volume of the prism,

$$V = Ah$$

Also, dividing both sides in turn by A and h,

$$h = \frac{V}{A} \text{ and } A = \frac{V}{h}.$$

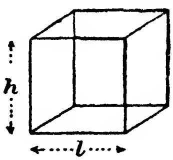


FIG. 23.

Square Prism. $A = l^2$

$\therefore \quad V = l^2h, \qquad h = \frac{V}{l^2}$

Total area of faces $= 2l^2 + 4lh = 2l(l + 2h)$

„ length of edges $= 8l + 4h = 4(2l + h)$

Rectangular Prism. $A = lb$

$\therefore \quad V = lbh, \; l = \frac{V}{bh}, \quad b = \frac{V}{lh}, \quad h = \frac{V}{lb}$

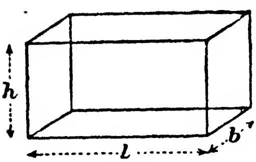


FIG. 24.

Total area of faces $= 2lb + 2lh + 2bh$
$= 2[lb + h(l + b)]$

„ length of edges $= 4l + 4b + 4h$
$= 4(l + b + h)$

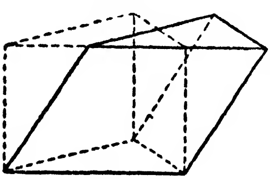


FIG. 25.

A prism can be regarded as made up of a number of layers (say, of cardboard), all of the same shape and size. If the layers be pushed to one side, as in the figure, the perpendicular height and the volume are unaltered. Therefore, prisms

having equal bases and equal perpendicular heights have equal volumes. Thus if $h \equiv$ perpendicular height of prism, the formula $V = Ah$ is true, whether the prisms are right prisms or not.

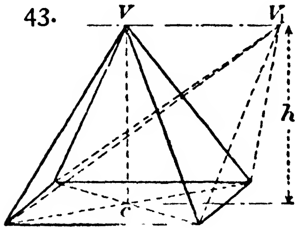
43. 

FIG 26

If the corners of any plane figure be joined to a point V, the solid figure formed is called a pyramid, V being the vertex. If the line joining V to the centre of the base of the pyramid be at right angles to the plane of the base, the pyramid is a right pyramid. It can be proved that the volume of a pyramid is one-third the volume of a prism having an equal base and an equal perpendicular height. Thus, if

$V \equiv$ vol. of pyramid, $A \equiv$ area of base, and $h \equiv$ perp. height,

$$V = \frac{Ah}{3}$$

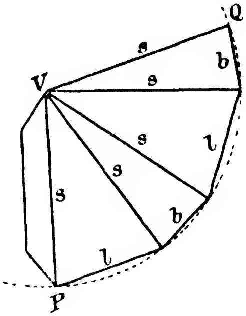


FIG. 27.

NOTE 5.—The proof of this is beyond the scope of this book, but the result can be verified by the student.

Cut out, in any suitable size, from a sheet of paper a figure as shown Fold the edges and use the flap to gum the edges VP and VQ together. A rectangular pyramid is obtained, and its perpendicular height (h) can be measured: s is the length of each sloping edge, and l and b are the length and breadth of the base.

Now cut out a figure as shown and thus make a rectangular prism open at one end, having an equal base and an equal perpendicular height as the pyramid.

Fill the pyramid with fine sand and empty into the prism. Repeat this twice and it will be found that the prism has three times the volume of the pyramid.

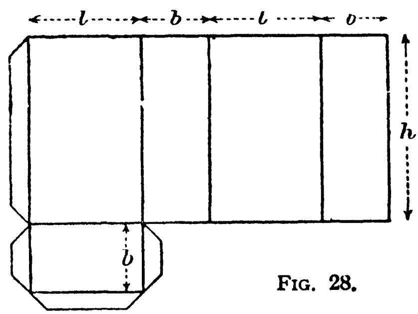


FIG. 28.

EXAMPLE (vii)—

Find the value of hay in a rick 11 yd 2 ft. long, its end view being as shown, if 14 cub yd of the rick weigh 1 ton of 40 trusses at 3s. 6d. per truss.

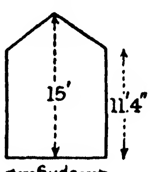


FIG. 29.

$$\text{Area of end section} = 6 \times 3\tfrac{7}{9} + \frac{6 \times 1\tfrac{2}{9}}{2} \text{ sq. yd.}$$

$$= 3 \times (7\tfrac{5}{9} + 1\tfrac{2}{9}) \text{ sq. yd.}$$

$$= 3 \times 8\tfrac{7}{9} \text{ sq yd.}$$

$\therefore$ Vol. of rick $= 3 \times 8\frac{7}{9} \times 11\frac{2}{3}$ cub. yd.

$$\therefore \text{ Wt. of hay} = \frac{\cancel{3} \times 79 \times 35}{9 \times \cancel{3} \times 14} \text{ tons}$$

$$\therefore \text{ Value of hay} = £\frac{79 \times \overset{5}{\cancel{35}} \times \cancel{40} \times 7}{9 \times \underset{2}{\cancel{14}} \times \cancel{40}}$$

$= £153\frac{11}{18} = £153 \ \ 12$s. $2\frac{2}{3}$d.

NOTE 6.—1 cub. yd. $= 3 \times 3 \times 3$ cub. ft. $= 27$ cub. ft.
1 cub. ft. $= 12 \times 12 \times 12$ cub. in. $= 1{,}728$ cub. in.

EXAMPLE (viii)—

The external dimensions of a closed rectangular wooden case are 5′ 6″ by 4′ 8″ by 3′ 4″. If the wood is $\frac{1}{2}$ in. thick, and the case and its contents weigh 12 cwt., find the weight of the contents to the nearest lb., given that 1 cub. ft. of the wood weighs 42 lb.

External vol. $= 66 \times 56 \times 40$ cub. in.
$= 147{,}840$ cub. in.

Internal vol. $= 65 \times 55 \times 39$ cub. in.
$= 139{,}425$ cub. in.

$$\therefore \text{ Wt. of wood} = \frac{\overset{935}{\cancel{8415}} \times \overset{7}{\cancel{42}}}{\underset{4}{\cancel{12}} \times \underset{4}{\cancel{12}} \times \underset{2}{\cancel{12}}} \text{ lb.}$$

$\therefore$ Wt. of contents $= 12$ cwt.—1 cwt. $92\frac{17}{32}$ lb.
$= 10$ cwt. 19 lb. to nearest lb.

EXAMPLE (ix)—

A swimming bath is 25 yds. long, 10 yds. wide, and its depth ranges from 3′ 6″ to 6′ 6″. Given that 1 cub. ft. $= 6\frac{1}{4}$ gall., find the number of gallons of water required to fill the bath.

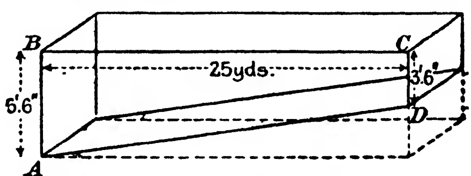


FIG. 30.

$$\text{Area of } ABCD = \frac{75 \times 10}{2} \text{ sq. ft.}$$

$\therefore$ Vol. of water $= 75 \times 5 \times 30$ cub. ft.

$$\therefore \text{ Quantity of water} = \frac{75 \times 5 \times 30 \times 25}{4} \text{ gall.}$$

$= 70312\frac{1}{2}$ gall.

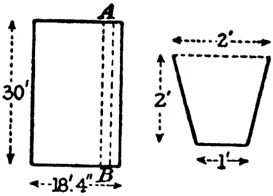


FIG. 31.

EXAMPLE (x)—

A garden, whose dimensions are shown, has a trench to be dug, the position being shown by AB and the dimensions of the cross-section by the second figure. If the mould excavated be spread evenly over the remaining part of the garden, how much will the level be raised?

Vol. of mould excavated $= 3 \times 30$ cub. ft.

$$\therefore \text{ Depth of mould spread over garden} = \frac{3 \times 30}{30 \times 16\frac{1}{3}} \text{ ft.}$$

$$= \frac{3 \times 3 \times 12}{49} \text{ in.}$$

$$= 2\tfrac{10}{49} \text{ in.}$$

EXAMPLE (xi)—

A stone monument has the shape of a square prism, on the top of which is a pyramid having an equal base. The total height is 18 ft., and the height of the prism is 13′ 6″. If the length of the side of the base be 2′ 8″ and 1 cub. ft. of the stone weighs 324 lb., find the total weight of the monument.

$$\text{Vol. of monument} = 2\tfrac{2}{3} \times 2\tfrac{2}{3} \times 13\tfrac{1}{2} + \frac{2\frac{2}{3} \times 2\frac{2}{3} \times 4\frac{1}{2}}{3} \text{ cub. ft.}$$

$$= 2\tfrac{2}{3} \times 2\tfrac{2}{3} \times (13\tfrac{1}{2} + 1\tfrac{1}{2}) \text{ cub. ft.}$$

$$\therefore \text{ Wt. of monument} = \frac{\overset{4}{\cancel{8}} \times \cancel{8} \times 15 \times \overset{36}{\cancel{324}}}{\cancel{3} \times \cancel{3} \times \underset{\underset{7}{\cancel{14}}}{\cancel{112}}} \text{ cwt.}$$

$$= 15 \text{ ton } 8 \text{ cwt. } 64 \text{ lb.}$$

EXAMPLE (xii)—

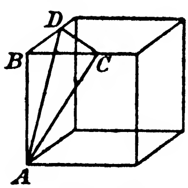


FIG. 32.

From a cube whose edges are a ft. long, a portion is cut off by the plane ACD. If $BC = x$ ft. and $BD = y$ ft., what is the volume of the remaining portion?

$$\text{Volume of remaining portion} = a^3 - \frac{ax}{2} \cdot \frac{y}{3} \text{ cub. ft.}$$

$$= a\left(a^2 - \frac{xy}{6}\right) \text{ cub. ft.}$$

• EXAMPLE (xiii)—

Figure 33 represents a reservoir. If $AB = 80$ ft., $AE = 56$ ft., $CD = 74$ ft., and $CF = 50$ ft., find the number of hundred gallons of water the reservoir can hold, given that 1 cub. ft. $= 6\frac{1}{4}$ gall., and that the depth of the reservoir is 12 ft.

The volume consists of—

(1) a rect. prism;

(2) two triangular prisms of length 74′ and two of length 50′, their cross-sections being the same, namely, base 3′ and height 12′;

(3) four square pyramids, the side of the square being 3′.

$$\therefore \text{ Vol.} = 74 \times 50 \times 12 + \frac{3 \times 12}{2} \times 248 + 4 \times \frac{3 \times 3 \times 12}{3} \text{ cub. ft.}$$

$$= 12 \times (3700 + 372 + 12) \text{ cub. ft.}$$

$$\therefore \text{Capacity} = \overset{3}{\cancel{12}} \times \overset{1021}{\cancel{4084}} \times \frac{\cancel{25}}{\cancel{4}} \times \frac{1}{\underset{4}{\cancel{100}}} \text{ hundred gallons}$$

$$= \underline{3{,}063 \text{ hundred gallons.}}$$

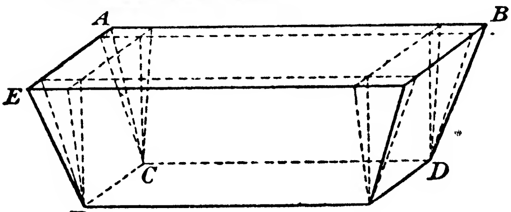


FIG 33.

TEST EXERCISES I, 5.

(1) A heap of gravel containing 12 cub. yds. is to be carted a distance of $1\frac{1}{4}$ miles. Find the cost of carting if 36 cub. ft. are taken in each load at 6d. per load, and 1d. extra for every furlong more than $\frac{1}{4}$ mile.

(2) A cow-house is $18\frac{3}{4}$ ft. long and 16 ft. wide. Two-thirds of this area are occupied by cow stalls. If 5 cows are kept in the building, what area is allowed to each stall?

(3) Find the number of cubic feet of timber in a log of oak 7 yd. 2 ft. 3 in. long, 1 ft. 8 in. wide, and 9 in. thick. What is it worth at 3s. 6d. per cubic ft.?

(4) Three fields, measuring respectively $7\frac{1}{2}$ ac., $5\frac{1}{3}$ ac., and $7\frac{7}{8}$ ac., were planted with potatoes. The total weight of potatoes produced was $123\frac{3}{4}$ tons. Find the average yield in tons per acre. How many lbs. of potatoes should be produced from an allotment of 6 sq. poles if the yield per acre be the same?

(5) If it cost £3 11s. $5\frac{1}{2}$d. to cover with linoleum the floor of a room 17′ 6″ long and 14′ wide, how much will it cost to cover the floor of a square room, the side of which is 15′, with linoleum which is $2\frac{1}{2}$d. per square yard dearer?

(6) The plan of a rectangular field is drawn to the scale of $\frac{3}{4}$ in. to 1 chain. The lengths of the sides on the plan are $5\frac{5}{8}$ in. and $9\frac{3}{4}$ in. The field is to be drained by four rows of pipes, running the length of the field and parallel to the longer side. Find the cost of draining at 6d. per yard. Find also the annual rental at £4 15s. per acre.

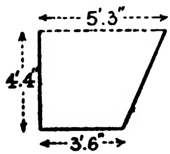


FIG. 34.

(7) A receptacle having its cross-section as shown, is 4 yd. long. Knowing that 1 bush. of corn occupies $1\frac{1}{4}$ cub. ft., how many bushels could be held by the receptacle?

(8) A piece of land has a side AB parallel to CD, and the side AC is perpendicular to CD. If AB measures 85 ft., CD 132 ft., and AC 33 ft., what is the area of the plot in square yards?

(9) A trough is 12′ 6″ long and the dimensions of its cross-section are shown in Figure 35. How many gallons of water can it hold, given that 1 cub. ft. = $6\frac{1}{4}$ gall. ?

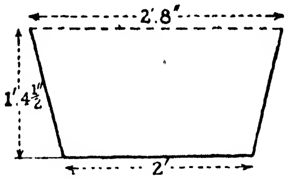


FIG. 35.

(10) A rectangular field 165 yd. long and 115 yd. 2 ft. broad was bought for £121 9s. and was afterwards sold at a profit of £5 17s. 6d. per acre. For what amount was the field sold ?

(11) A town of 450,000 inhabitants is supplied with water from a lake, the area of which is $5\frac{1}{2}$ sq. miles. If the level of the lake rises by 3 in., find how many days longer the water supply of the town will last, supposing that the average daily consumption per inhabitant is 32 gall. (1 cub. ft. of water weighs 1,000 oz.)

(12) Find the number of pieces of turf, each 9 in. wide and 2 ft. long, required for turfing a lawn which is 65 yd. long and 26 yd. wide.

(13) A water tank is $6\frac{1}{4}$ ft. long, $4\frac{1}{2}$ ft. wide, and 2 ft. deep. How many bucketsful of water will fill the tank if the bucket holds $2\frac{1}{2}$ gall. (1 cub. ft. = $6\frac{1}{4}$ gall)

(14) Two plots of land were offered for sale, and one measuring 1 ac. 25 sq. po. was sold for £408 17s. The other plot was rectangular in shape, measuring 84 yd. in length and 25 yd. 2 ft. in width. How much ought it to bring in if sold at the same rate ?

(15) On a map, the scale of which is 3 in. to the mile, a farm is represented by $8\frac{1}{4}$ sq in. Find the ground rent at £1 7s. 6d. per acre.

(16) A barn is 35 yd. long and 9 yd. wide. The height to the eaves is 20 ft. and to the ridge 28 ft. Find the capacity of the barn in cubic feet.

(17) What is the cost of repairing a path 95 yd. long and $2\frac{1}{2}$ ft. wide at 3s. $1\frac{1}{2}$d. per square yard ?

(18) An iron safe is cubical in shape and measures on the outside 2′ 3″ each way. The sides are $1\frac{1}{4}$″ thick. Find the weight of the safe if a cubic foot of iron weighs 487 lb.

(19) Find the cost of an oblong stack of wood 4 yd. 1 ft. 3 in. long, $3\frac{1}{2}$ yd. wide, and 8 ft. high at $1\frac{1}{4}$d. per cubic foot. How long will the stack last if $2\frac{1}{2}$ cub. ft. are used daily ?

(20) In the plan of a triangular field, drawn to a scale of $\frac{3}{4}$″ to 1 chain, the two shortest sides measure 8 in. and 6 in. respectively, and are at right angles to each other. Draw a plan of the field and find the rent at £3 15s. per acre.

(21) An allotment is $13\frac{3}{4}$ yd. wide. Find the number of each of the following plants required for—

8 rows cauliflower, each 24 in. apart;
5 rows kidney beans, each 12 in. apart;
6 rows cabbages, each 18 in. apart.

The first plant in each row is 6 in. from the edge of the allotment. If the rows are 18 in. from one another, and the end rows 9 in. from the edges, what is the area of the allotment in square poles and square yards ?

(22) A wall 50 yd. long, 8 ft. high, and 9 in. thick, cost £134 15s. Find the cost at the same rate of a wall, of the same thickness and 6 ft. high, which encloses a garden 45 ft. long and 33 ft. 9 in. wide, if a gap 7 ft. wide is left.

(23) Find the cost of carpeting a room 18′ long, 14′ wide, 10′ high with carpet 2 ft. 3 in. wide at 3s. $4\frac{1}{2}$d. per yard, and of papering the walls if the cost per piece of length 12 yd. and width 21 in. be 1s. 8d.

(24) The measurements of a rick are: Length, 28 ft.; width, 17 ft. 6 in.; height to the eaves, 16 ft.; and height to the top, 20 ft. Find the weight of hay, if 1 cub. ft. weighs 6 lb. 14 oz. Find the value of the hay at £7 15s. per ton.

(25) A rectangular field is 150 yd. long and 110 yd. broad. Two paths, each 10 ft. wide, are made across so as to divide the field into 4 equal rectangular plots How much would be realized by selling the plots at £57 15s. per acre ?

(26) A tennis court is 26 yd. long and 12 yd. wide. A man owns a rectangular field 126 yd. long and 95 yd. wide, and decides to mark out as many courts as possible, the netting being 6 yd. from the base lines and 3 yd. from the side lines. If he let all the courts at 7 guineas per season each, what would be the rental of the field per acre ?

(27) A glazier has to put in 15 panes 2′ by 1′ 8″, 9 panes 3′ 9″ by 2′ 9″, and 6 panes 1′ 10″ by 1′ 4″. Find the cost of glass at 7½d. per square foot.

(28) A garden is 60 ft. long and 25 ft. wide. A hole 10 ft. long, 5 ft. wide, and 4 ft. 6 in. deep, is dug and filled in with cinders. The mould excavated is spread evenly over the whole surface of the garden, with the exception of a path 60 ft. long and 3 ft. 6 in. wide. What will be the depth of the layer of mould ?

(29) How many blocks 9″ by 4½″ are required to pave a road 1¾ miles long and 14 yd. wide ?

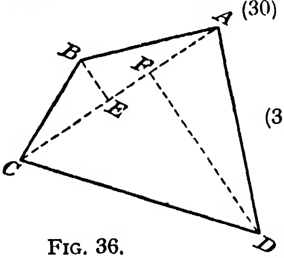


Fig. 36.

(30) $ABCD$ represents the plan of an estate drawn on a map, the scale being 1½ in. to 1 mile.

$AC = 1\frac{1}{4}''$, $BE = \frac{1}{4}''$, and $DF = 1''$. Find the value of the estate at £275 per acre.

(31) A rectangular tank has a base measuring 5′6″ by 3′9″; height is 4′ 6″. It is found that water from a tap fills it in 4 min. 24 sec. What is the supply of water from the tap in gallons per minute ?

(32) A swimming bath is 100 ft. long, 10 yd. wide, and its depth ranges from 3′ 6″ to 6′. It is filled by water from a pipe which supplies 880 gall. per minute. How long does it take to fill the bath ?

(33) It is found that 1 lb. of a certain powder occupies 32½ cub. in. Rectangular tins whose bases measure 2½″ by 1¾″ are to hold 6 oz. of powder each. What depth of powder will there be in each tin ?

(34) Tins of fruit are to be packed in cases, each holding 75 tins. The tins are 3½″ high and 6″ across. The packer contemplates as to whether they shall be packed 3 layers of 25 tins or 5 layers of 15 tins. Calculate the areas of wood (including lid) required in each case.

(35) A rectangular field is 6 chains 20 links by 2 chains 35 links. It is required to divide it into allotments each of area 12 sq. perch by lines parallel to the shorter sides of the field. How many of these allotments will there be ? what will be the width of each allotment ? and what will be the area of the remaining portion ?

(36) A reservoir, shaped as the one in Example xiii, is 175′ by 48′ at the base and 179′ by 52′ at the surface, and its depth is 10′. If full of water, how long would it last a town of population 5,600, allowing 32 gall. of water per inhabitant per day ?

(37) A field having 5 sides, represented by the sketch of paragraph 37, is such that AB is 35° E. of N: AH = 410 links, HC = 620 links, CD = 120 links, DB = 75 links, HG = 325 links, FD = 249 links, CE = 571 links. Enter results as in a field book, draw a plan of the field to a scale of ½″ to 1 chain, and calculate the area of the field.

(38) Find the area of the field represented by Figure 37, given that AB is at right angles to ED: $AH = 15$ yd., $AK = 44$ yd., $AL = 76$ yd., $AB = 115$ yd., $GH = 38$ yd., $FL = 65$ yd., $EB = 52$ yd., $KC = 70$ yd., $BD = 33$ yd.

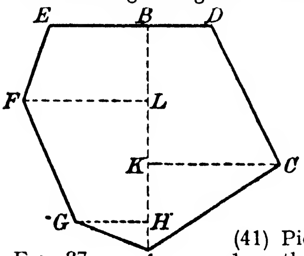


FIG. 37.

(39) How many square tiles of side x in. are required to pave a rectangular courtyard a yd. long and b ft. wide?

(40) How many turfs 18 in. long and x in. wide could be obtained by removing the turf to a width of 5 ft. round the outside of a square lawn whose side is y ft. long?

(41) Pieces of metal, shaped as shown in Figure 38, are $\frac{1}{8}$ in thick. If l, h_1, h_2 are numbers of inches, find the number of pieces of metal that would weigh 1 cwt., given that 1 cub. ft. of metal weighs x lb.

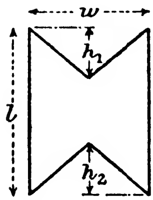


FIG. 38.

(42) The side of a house is represented by Figure 39, d, H, h_1, and h_2 being numbers of feet. Show that its area is

$$\frac{d(2H + h_1 + h_2)}{18} \text{ sq. yd.}$$

(43) A flat roof is a ft. long and b ft. broad. Water due to a rainfall of $\frac{1}{4}$ in. is drained from the roof into a rectangular tank, whose base is s ft. t in. by u ft. What depth of water would there be in the tank?

(44) During a rainfall of $\frac{1}{2}''$, how many gallons of water fall per acre? and how many tons per square mile?

(45) A triangular piece of wood is cut from a plank $\frac{3}{4}$ in. thick; its weight is 3 lb. 6 oz. and one of its sides is 2 ft. 4 in. long. What is the corresponding height, if 1 cub. ft. of wood weighs 57 lb.?

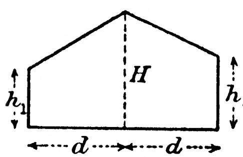
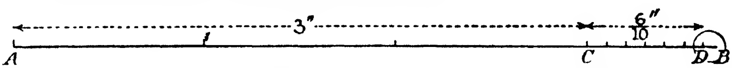


FIG. 39.

CHAPTER VI.

DECIMAL FRACTIONS—APPROXIMATIONS—CONTRACTED METHODS OF MULTIPLICATION, DIVISION, AND SQUARE ROOT—DECIMALIZATION OF MONEY—CALCULATION OF COST—METRIC SYSTEM OF WEIGHTS AND MEASURES.

44. APPROXIMATE MEASUREMENT.



AB is a line drawn without regard to any particular length. AC is 3″ long, but, in general, it is not possible to express the portion CB as an exact fraction of an inch. It is possible, however, to find a fraction of an inch that differs from the exact value of CB by a quantity very small indeed.

$CD = \frac{6}{10}$ inch; and in order to see how to measure DB, suppose it be magnified as shown. XY is $\frac{1}{10}$ inch magnified and, on being

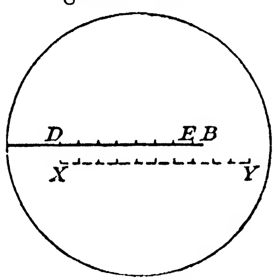


FIG. 40.

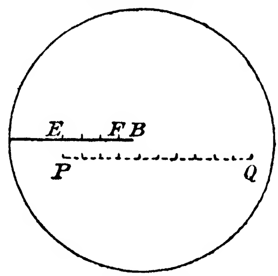


FIG. 41.

divided into 10 equal parts, each part will be $\frac{1}{100}$ inch. DE is thus $\frac{7}{100}$ inch long, and the remaining portion EB is less than $\frac{1}{100}$ inch long. If it be required to measure EB, magnify this portion again.

PQ represents $\frac{1}{100}$ inch, so that on being divided into 10 equal parts, each part will be $\frac{1}{1000}$ inch. EF is $\frac{3}{1000}$ inch long and FB is less than $\frac{1}{1000}$ inch long.

Thus the actual length of AB differs by less than $\frac{1}{1000}$ inch from $3 + \frac{6}{10} + \frac{7}{100} + \frac{3}{1000}$ inches.

45. DECIMAL NOTATION.

$3 + \frac{6}{10} + \frac{7}{100} + \frac{3}{1000}$ is more briefly written 3·673, the units figure being separated from the tenths figure by a dot which is called the **Decimal Point.** Figures to the left of the decimal are

units, tens, hundreds, etc.; and to the right, tenths, hundredths, thousandths, etc.

Thus, ·7 denotes $\frac{7}{10}$ and ·07 denotes $\frac{7}{100}$; also ·17 denotes $\frac{1}{10} + \frac{7}{100}$, which equals $\frac{17}{100}$. Again, ·60 denotes $\frac{6}{10} + \frac{0}{100}$, which is, of course, $\frac{6}{10}$; and thus no alteration in value comes about by writing noughts after the last decimal figure.

Knowing the decimal notation, a decimal fraction can at once be expressed as a vulgar fraction.

e.g., $\cdot 5 = \frac{5}{10} = \frac{1}{2}$ $\qquad$ $\cdot 375 = \frac{375}{1000} = \frac{3}{8}$

$\cdot 032 = \frac{32}{1000} = \frac{4}{125}$

46. ADDITION AND SUBTRACTION.

It has been seen before that the sums and differences of quantities have a useful interpretation only when the units associated with the quantities are the same. Thus, tenths should be added to or subtracted from tenths, hundredths added to or taken from hundredths, etc.

Thus in adding or subtracting decimal quantities, if the numbers are put down in columns, the decimal points should be immediately above and below one another.

47. MULTIPLICATION.

$10 \times 5{\cdot}347 = 10 \times (5 + \frac{3}{10} + \frac{4}{100} + \frac{7}{1000}) = 50 + 3 + \frac{4}{10} + \frac{7}{100} = 53{\cdot}47$

Similarly, $100 \times 5{\cdot}347 = 534{\cdot}7$, $1000 \times 5{\cdot}347 = 5347$

and $10000 \times 5{\cdot}347 = 53{,}470$

Thus when multiplying a decimal quantity by 10, 100, 1,000, etc., the result is obtained by moving the decimal point 1, 2, 3, etc., places to the right respectively.

Proceeding in the reverse way, it is obvious that when dividing a decimal quantity by 10, 100, 1,000, etc., the result is obtained by moving the decimal point 1, 2, 3, etc., places to the left respectively.

Suppose it be required to evaluate 71·3 × ·7 × ·019. This can be expressed as $\frac{713}{10} \times \frac{7}{10} \times \frac{19}{1000}$ and, multiplying according to the rules of the previous chapter, the result is

$$\frac{94{,}829}{100{,}000} \text{ or } \frac{94{,}829}{10^5}.$$

Now, to divide by 10^5, the decimal point must be moved 5 places to the left, and it is seen that this number of places is equal to the total number of decimal places in the quantities that are being multiplied.

Thus, to multiply decimal quantities, multiply the numbers obtained by omitting the decimal points, and then place the decimal point such that the number of places is the same as the total number of places in the quantities being multiplied.

$$e.g., = \cdot011 \times 70 \times \cdot02 \times \cdot5 = \cdot007700 = \underline{\cdot0077}$$

An alternative method, which depends upon the fact that if one number forming part of a product be multiplied by a multiple of 10, while another number of the product be divided by the same multiple of 10, then the value of the product is unaltered, is important.

It is clear that $47\cdot13 \times \cdot057 = \cdot4713 \times 5\cdot7$

and $\cdot137 \times 71\cdot4 = 1\cdot37 \times 7\cdot14$

and it is seen that the last number in each case lies between 1 and 10.

The principle can be extended until the product of several numbers can be expressed as a product of numbers, all but one of which have one figure and one only before the decimal point.

Thus, $71\cdot3 \times \cdot7 \times \cdot019 = \cdot0713 \times 7 \times 1\cdot9$

and $61\cdot3 \times 41\cdot7 \times \cdot0016 \times 23\cdot8 = 6\cdot13 \times 4\cdot17 \times 1\cdot6 \times 2\cdot38$

When this has been done, the multiplication can be performed such that the decimal in any line of figures lies immediately above or below the decimal points in the other lines of figures.

e.g., $203 \times \cdot0079 \times \cdot137$	·0203 7·9	·16037 1·37
$= \cdot0203 \times 7\cdot9 \times 1\cdot37$	·1421 ·01827	·16037 ·048111 ·0112259
$= \cdot2197069$	·16037	·2197069

48. DEGREES OF APPROXIMATION.

The quantity $6\frac{7}{16}$ is nearer to 6 than to 7, and the error is $\frac{7}{16}$, which is less than $\frac{1}{2}$; $6\frac{9}{16}$ is nearer to 7 than to 6, and the error is again $\frac{7}{16}$. Thus if a quantity be expressed to the nearest unit, the error does not exceed a half-unit.

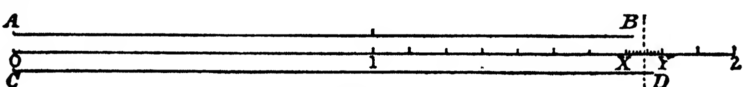


FIG. 42.

Again, suppose the approximate lengths of AB and CD are required to the nearest tenth of a unit. The lengths lie between

1·7 and 1·8 units. The dotted line is midway between the 1·7 and 1·8 marks, and thus the distance from 0 to the dotted line is 1·75, as is seen by subdividing XY into ten equal parts, each part being one-hundredth of a unit. As B falls short of the dotted line, it is nearer to the 1·7 unit mark than the 1·8 unit mark. As D is beyond the dotted line, it is nearer the 1·8 unit mark than the 1·7 unit mark. Thus, in order that the error should be as small as possible, the approximate lengths of AB and CD are 1·7 and 1·8 units respectively, and in each case the error is less than half a tenth.

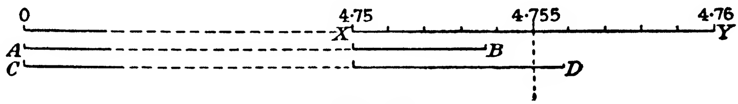


FIG. 43.

Further, suppose it be required to write down the approximate lengths of 4·7537 and 4·7558 correct to the nearest hundredth. It is seen that each lie between 4·75 and 4·76. The dotted line determines the distance 4·755. B and D determine the positions 4·7537 and 4·7558 respectively. B falls short of the dotted line, and so is nearer to X than Y; but D is beyond the dotted line, and so is nearer to Y than X. Thus the approximate lengths are 4·75 and 4·76 respectively, and the errors are each less than a half-hundredth.

From these considerations it is seen that if the approximate value of a quantity be required *correct* to r decimal places, the $(r+1)^{th}$ decimal figure should be found. If this be less than 5, no change must be made in the r^{th} decimal figure; and if it be 5 or more, the r^{th} decimal figure must be increased by 1. When this is done, the error does not exceed $\frac{1}{2}$ of the value of 1 in the r^{th} decimal place.

The following columns show the approximate values of quantities to different degrees of accuracy. The magnitude of the errors given in the brackets should be noted.

Quantity.	Approximate value correct to nearest			
	Integer.	Tenth.	Hundredth.	Thousandth.
7·4153	7 (− ·4153)	7·4 (− ·0153)	7·42 (+ ·0047)	7·415 (− ·0003)
3·7899	4 (+ ·2101)	3·8 (+ ·0101)	3·79 (+ ·0001)	3·790 (+ ·0001)

49. LIMITS WITHIN WHICH AN APPROXIMATED QUANTITY MUST LIE.

Suppose that the exact length of a line is unknown, but that its approximate length to the nearest ·01 in. is 4·26 in.

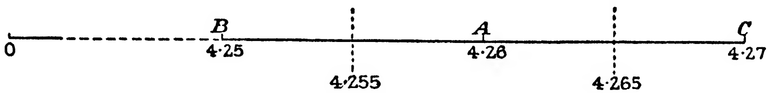


FIG. 44.

In order that this should be the case, the end of the line must be nearer to the mark A than to either of the marks B or C, and this will be the case when the end falls between the dotted lines shown in the figure. Thus, the exact length may be of any magnitude between 4·255 in. and 4·265 in.

Similarly, if the approximate value of a quantity be 2·372 correct to 3 decimal places, its exact value lies between 2·3715 and 2·3725.

50. DIVISION.

Suppose a decimal quantity is being divided by a whole number, then when the integer part has been divided, the remainder is a number of units. On being reduced to tenths and the tenth figure added, it is obvious that the next figure obtained in the quotient will be a number of tenths. Thus the decimal point must be put in the quotient as soon as the first figure (including nought) after the decimal in the dividend has been dealt with.

Whenever possible, short division by factors should be employed in preference to long division. In the case of short division, the decimal points in the various lines lie immediately above and below one another.

Suppose it be required to evaluate 4·7691 ÷ ·084. This value can be expressed $\frac{4{\cdot}7691}{{\cdot}084}$, and it has already been shown that the quotient is unaltered when dividend and divisor are multiplied by the same number. Now, by multiplying the divisor by 1,000, it becomes 84, so that the quotient is $\frac{4769{\cdot}1}{84}$ and the divisor is now a whole number. **Thus, when it is required to divide by a decimal quantity, the decimal point in the divisor is moved one, two, three, etc., places to the right, just sufficient to make it a whole**

umber; then the decimal point in the dividend is moved the same number of places to the right, and thus the division is reduced to the case of division by a whole number.

If the quotient be required correct to, say, three decimal places, the figure that would have come in the fourth place should be considered as regards its being more or less than a five, so that the result can be written down in accordance with the preceding paragraph.

An important alternative method of performing long division is to move the decimal point in the divisor a number of places to the right or left, so that there shall be one figure and only one before the decimal point, and then to move the decimal point in the dividend the same number of places to the right or left respectively. Then the division can be performed such that the decimal point in the quotient comes immediately above that in the dividend. Thus—

$$\cdot 1792 \div \cdot 00931$$
$$= 179\cdot 2 \div 9\cdot 31$$

```
         19·248
9·31)179·2
      93·1
      86·10
      83·79
       2·310
       1·862
        ·4480
         3724
          756
```

$$= 19\cdot 25 \text{ correct to 2 places of decimals}$$

EXAMPLE (i)—

Express $\frac{13}{28}$ as a decimal fraction correct to 3 places.

```
4 | 13·0000
7 |  3·2500
      ·4642
```

Ansr.—·464

EXAMPLE (ii)—

Express 13s. 5½d. as a decimal fraction of £1 correct to 5 places.

$$13/5\tfrac{1}{2} = £\frac{161\cdot 5}{240} = £\cdot 67292$$

```
4 | 16·150000 *
6 |  4·037500
      ·672916
```

EXAMPLE (iii)—

Express £·7134 in shillings and pence to the nearest ¼d.

```
£·7134
    20
14·2680 shillings
    12
 3·216  pence
     4
  ·864  farthing
```

Ansr.—14/3¼

* The shaded "zeros" need not be written down in actual working.

NOTE 1.—In actual practice there is no need to put down 20, 12, and 4.

EXAMPLE (iv)—

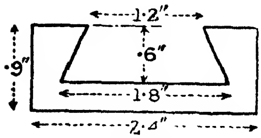


FIG. 45.

A prism of silver is to be made 7·3″ long and its cross-section is as shown. What length must be cut from a prism of silver whose section is a square, side 1·9″, to provide the amount of silver required? (Answer to be correct to ·001 in.)

$$\begin{aligned} \text{Area of section} &= 2{\cdot}4 \times {\cdot}9 - {\cdot}6 \times 1{\cdot}5 \text{ sq. in.} \\ &= 2{\cdot}16 - {\cdot}90 \quad \text{,,} \\ &= 1{\cdot}26 \text{ sq. in.} \\ \therefore \text{ Volume of silver} &= 1{\cdot}26 \times 7{\cdot}3 \text{ cub. in.} \\ \therefore \text{ Length cut from sq. prism} &= \frac{1{\cdot}26 \times 7{\cdot}3}{1{\cdot}9 \times 1{\cdot}9} \text{ in.} \end{aligned}$$

$$\begin{array}{r} 1{\cdot}26 \\ 7{\cdot}3 \\ \hline 8{\cdot}82 \\ {\cdot}378 \\ \hline 9{\cdot}198 \end{array} \qquad \begin{array}{r} 1{\cdot}9 \\ 1{\cdot}9 \\ \hline 1{\cdot}9 \\ 1{\cdot}71 \\ \hline 3{\cdot}61 \end{array} \qquad \begin{array}{r} 2{\cdot}5479 \\ 3{\cdot}61)\overline{9{\cdot}198} \\ 7{\cdot}22 \\ \hline 1{\cdot}978 \\ 1{\cdot}805 \\ \hline {\cdot}1730 \\ {\cdot}1444 \\ \hline 2860 \\ 2527 \\ \hline 3330 \end{array} \qquad \begin{aligned} &= \frac{9{\cdot}198}{3{\cdot}61} \text{ in.} \\ &= 2{\cdot}548 \text{ in.} \end{aligned}$$

NOTE 2.—In any problem involving decimals, it should always be seen that the answer obtained agrees with a rough estimate of the answer, *e.g.*, the rough estimate is $\frac{1 \times 7}{2 \times 2}$ (*i.e.*, 1·75), from which it is obvious there is one figure before the decimal point.

EXAMPLE (v)—

Reduce 17 cwt. 2 qr. 15 lb. 9 oz. to the decimal of 1 ton correct to 5 decimal places.

$$\begin{array}{rr|ll} 16\begin{cases} \\ \\ \end{cases} & \begin{array}{r} 4 \\ 4 \end{array} & \begin{array}{l} 9 \\ \hline 2{\cdot}25 \end{array} & \text{oz.} \\ 28\begin{cases} \\ \\ \end{cases} & \begin{array}{r} 4 \\ 7 \end{array} & \begin{array}{l} 15{\cdot}5625 \\ \hline 3{\cdot}890625 \end{array} & \text{lb.} \\ & 4 & 2{\cdot}555803 & \text{qr.} \\ & 20 & 17{\cdot}638951 & \\ & & {\cdot}881947 & \end{array}$$

Ansr.—·88195 ton.

NOTE 3.—The 15 lb. is added on after the oz. are expressed in lb.; the 2 qr. is added after the lb. are expressed in qr.; the 17 cwt. is added after the qr. are expressed in cwt.

NOTE 4.—Sometimes an approximation is made by stating the number of *significant figures*. This consists in writing down this number of the most important figures, taking care to see that they have their proper value (*e.g.*, the approximate values of 37597·3827 correct to 2, 3, 4, 5, 6, and 7 significant figures are 38,000, 37,600, 37,600, 37,597, 37597·4, 37597·38 respectively).

EXAMPLE (vi)—

In 1929 the births in Great Britain and Northern Ireland numbered 761,963, the rate being 16·7 per 1,000. Find, correct to 3 sig. figs., the population of Great Britain and Northern Ireland in 1929, based on these results.

$$\begin{array}{r} 45620000 \\ 1{\cdot}67\overline{)76196300} \\ \underline{668} \\ 939 \\ \underline{835} \\ 1046 \\ \underline{1002} \\ 443 \end{array}$$

$$\text{Population} = \frac{761{,}963{,}000}{16{\cdot}7}$$
$$= \underline{45{,}600{,}000}, \text{ correct to 3 sig. figs.}$$

51. CONTRACTED METHODS OF ADDITION AND SUBTRACTION.

It is sometimes required to add a series of numbers together and obtain the result correct to a certain number of significant figures. Below are additions obtained by adding the numbers in full and approximating to 4 significant figures afterwards, approximating to nearest 10,000, 1,000, and 100 respectively and then approximating in the latter two cases to 4 significant figures.

3638247	3640000	3638000	3638200
779632	780000	780000	779600
615795	620000	616000	615800
492826	490000	493000	492800
1947569	1950000	1948000	1947600
349594	350000	350000	349600
4070963	4070000	4071000	4071000
11894626	11900000	11896000	11894600
11890000		11900000	11890000

From a study of these results, it is seen that an error arises in the second and third cases. This is due to the fact that in approximating the numbers before adding, errors are introduced and, when added, the $+ve$ and $-ve$ errors do not always neutralize one another; and the total errors become too great to give the correct approximation. Two extra columns of significant figures should, in general, be considered to give a margin of safety; but should it happen that, say, 20 or more numbers are to be added, it may be that 3 or even 4 extra columns are necessary. No definite rules can be given; but if the principle be understood, the student can judge for himself the number of extra columns required by consideration of the problem in question.

As regards subtraction, one extra column of significant figures is sufficient for giving the correct approximation.

*52. INCOMMENSURABLE QUANTITIES.

A large number of quantities dealt with in everyday life are

incommensurable quantities, that is, their magnitudes are such that they cannot be expressed exactly. If a quantity of tea be taken and an accurate balance used to obtain its weight, it would in general be found that its weight is not an exact number of ounces or even grains, and that even the exact fraction of a grain could not be measured. Again, a litre—the metric system unit of capacity—cannot be expressed exactly in gallons; and thus with reference to the British units, a litre is an incommensurable quantity.

In practice, the approximate magnitudes of incommensurable quantities are expressed using the decimal notation, for then it is at once known the limits within which the errors lie.

In dealing with the approximations of incommensurable quantities, consideration to the errors involved must be given. It is known that correct to two decimal places, 1 metre = 39·37 in. Suppose it be required to express 103 metres in inches. Now, $103 \times 39{\cdot}37$ in. = 4055·11 in.; but it is absurd to say that 103 metres = 4055·11 in., for the error in the measurement of 1 metre is any amount up to ·005 in., so that the error in the above measurement for 103 metres is any amount up to $103 \times {\cdot}005$ in., *i.e.*, ·515 in. Therefore, all the data permits one to say is that the exact value of 103 metres lies between 4055·625 in. (*i.e.*, $103 \times 39{\cdot}375$ in.) and 4054·595 in. (*i.e.*, $103 \times 39{\cdot}365$ in.). Thus, ·11 in the quantity 4055·11 in. has no practical meaning at all.

Suppose the sides of a rectangle are measured, and their approximate values are 17·46 in. and 9·513 in. respectively. As regards the area, it could be said that it lies between $17{\cdot}465 \times 9{\cdot}5135$ sq. in. and $17{\cdot}455 \times 9{\cdot}5125$ sq. in.

Considering the general case, suppose that the approximate lengths of the sides are a in. and b in. respectively, and suppose that the maximum errors of these measurements are x in. and y in. respectively. Taking a in. and b in. as being the exact lengths, the area of the rectangle would be ab sq. in. In reality, however, all that can be known concerning the actual area is that it lies between $(a + x)(b + y)$ sq. in. and $(a - x)(b - y)$ sq. in.; that is between $ab + ay + bx + xy$ sq. in. and $ab - ay - bx + xy$ sq. in. Now, if a in. and b in. are close approximations, x in. and y in. are small quantities, so that the quantity xy sq. in. is so small as to be negligible. Thus the area of the rectangle differs from ab sq. in. by a quantity not exceeding $ay + bx$ sq. in.

Suppose that the approximate area of a rectangle is a sq. in. and the approximate length b in., and suppose that the maximum errors (supposed small compared with a and b) of these measurements are x sq. in. and y in. respectively. Taking a sq. in. and b in. as being the exact dimensions, the breadth of the rectangle would be $\frac{a}{b}$ in. In reality, however, all that is known concerning the actual breadth is that it lies between

$$\frac{a+x}{b-y} \text{ in. and } \frac{a-x}{b+y} \text{ in. respectively.}$$

Now it is known that, if z be a small quantity (*e.g.*, ·0005)

$\frac{1}{1-z}$ approximately equals $1+z$ and $\frac{1}{1+z}$ approximately equals $1-z$,

$$\text{so that } \frac{a+x}{b-y} \text{ sq. in.,}$$

$$\textit{i.e.}, \frac{a+x}{b\left(1-\frac{y}{b}\right)} \text{ sq. in. approximately equals } \frac{(a+x)\left(1+\frac{y}{b}\right)}{b} \text{ sq. in.,}$$

$$\textit{i.e.}, \frac{a}{b} + \frac{bx+ay}{b^2} + \frac{xy}{b^2} \text{ sq. in.}$$

$$\text{Also } \frac{a-x}{b+y} \text{ sq. in.,}$$

$$\textit{i.e.}, \frac{a-x}{b\left(1+\frac{y}{b}\right)} \text{ sq. in. approximately equals } \frac{(a-x)\left(1-\frac{y}{b}\right)}{b}$$

$$\textit{i.e.}, \frac{a}{b} - \frac{bx+ay}{b^2} + \frac{xy}{b^2} \text{ sq. in.}$$

As x and y are small quantities,

the quantity $\frac{xy}{b^2}$ is so very small (unless b also is small) as to be negligible.

Thus the breadth of the rectangle differs from $\frac{a}{b}$ in.

$$\text{by a quantity not exceeding } \frac{ay+bx}{b^2} \text{ in.}$$

To sum up—

Let the approximate values of two quantities be a and b respectively.

Let the maximum errors of a and b be x and y respectively (supposed small compared with a and b).

Then the error of ab does not exceed ay + bx.

and " $\frac{a}{b}$ " " $\frac{ay+bx}{b^2}$.

EXAMPLE (vii)—

The dimensions of a square prism are 3·73″ × 3·73″ × 2·45″, each being correct to the nearest $\frac{1}{100}$ in. Find the limits within which the volume of the prism lies.

Area of base lies between $3{\cdot}73 \times 3{\cdot}73 \pm (3{\cdot}73 \times {\cdot}005 + 3{\cdot}73 \times {\cdot}005)$ sq. in.

i.e., „ $13{\cdot}9129 \pm {\cdot}0373$ sq. in.

i.e., „ 13·9502 sq. in. and 13·8756 sq. in.

∴ Max. volume = $13{\cdot}9502 \times 2{\cdot}455$ cub. in.

= 34·248 cub. in. correct to ·001 cub. in.

Also Min. volume = $13{\cdot}8756 \times 2{\cdot}445$ cub. in.

= 33·926 cub. in. correct to ·001 cub. in.

NOTE 5.—Thus from the data the closest approximation as regards the volume of the prism is that it is 34 cub. in. correct to the nearest cubic inch.

* EXAMPLE (viii)—

The diameter of a certain shell is 7·14 in. correct to ·01 in. Given that 1 in. = 2·540 cm. correct to ·001 cm., calculate the diameter in centimetres to as many significant figures as the data allows.

The maximum error of $7{\cdot}14 \times 2{\cdot}540$ is $7{\cdot}14 \times {\cdot}0005 + 2{\cdot}540 \times {\cdot}005$ cm., *i.e.*, approx.: ·016 cm. Thus the result can be determined correct to nearest $\frac{1}{10}$ cm.

Ansr.—18·1 cm.

* EXAMPLE (ix)—

Gold is approx. 19·32 times heavier than water. Given that approx. 277·274 cub. in. of water weighs 70,000 gr., find the weight of 1 cub. in. of gold in oz. Troy to as close an approximation as the data permit.

$$\text{Wt. of 1 cub. in. of gold} = \frac{7000\cancel{0} \times 19{\cdot}32}{277{\cdot}274 \times 48\cancel{0}} \text{ oz. Troy approx.}$$

Now max. error in num. $= 7{,}000 \times {\cdot}005 = 35$

Also „ denom. $= 48 \times {\cdot}0005 = {\cdot}024$

$$\therefore \text{Max. error of quotient approx.} = \frac{7000 \times 20 \times {\cdot}024 + 280 \times 50 \times 35}{280 \times 50 \times 280 \times 50} \text{ oz. Tr.}$$

$$i.e., \quad „ \quad \frac{49}{19600} \text{ oz. Tr.}$$

i.e., „ ·0025 oz. Tr.

Thus the data permit a result correct to ·01 oz. Tr.

Therefore 1 cub. in. of gold weighs 10·16 oz. Tr. correct to ·01 oz. Tr.

53. CONTRACTED METHOD OF MULTIPLICATION.

Suppose it be required to evaluate the approximate value of 517·36477 × ·37294 correct to five significant figures. Of course, the multiplication could be performed in full and the approximation made afterwards, but this would involve more mechanical work than is necessary. It has been pointed out in paragraph 51 that two extra columns of figures should be dealt with so that in

this example the working should involve seven columns of significant figures.

If the approximate value of the product be required correct to three places of decimals, then by making a rough estimate (520 × ·4 = 208) it is clear that the working should involve eight columns of significant figures.

The working to obtain the result correct to five significant figures is as follows—

```
  xxxxxx
51·736477
 3·7294
--------
155·2094
 36·2155
  1·0347
   ·4656
    207
--------
192·9459
```

(*a*) Place the decimals' points so that the number to be written on the second line has one figure before the decimal point.

(*b*) Put down the numbers so that the decimal points lie immediately above or below one another.

(*c*) Multiply first by 3 and start with that figure on the top line which will give the seventh significant figure (2 has been "carried" as a result of multiplication with the last 7 but one, so that the result being 14, the 4 is put down in its proper column and then the decimal point in the third line will fall immediately below that of the second)

(*d*) Multiply the top line by 7, starting with the 6 in order that the first figure of the result to be written down shall be under the 4 to be in its proper column. (The "carrying" figure is 3, because multiplying the 4 by 7, the result is 28, and there is less error in regarding this as 30 than as 20.)

(*e*) Proceed in this way, marking off the figures in the top line to ensure that the right starting figure is used in each multiplication, until either there are no more figures to multiply or there are no figures to be placed in the columns

(*f*) Add and approximate to the required degree of accuracy

The answer is 192·95.

EXAMPLE (x)—

Find the value of 3 ton 17 cwt. 2 qr. 15 lb. 9 oz. at £1 13s. 5½d. per ton.

3 ton 17 cwt. 2 qr. 15 lb. 9 oz. = 3·88195 tons correct to 5 dec. places.
£1 13s. 5½d. = £1·67292 ,, ,, ,, ,,

The maximum error in £3·88195 × 1·67292 is £·000005 × (3·88193 + 1·67292). As this is less than £·00003, which is so small as to be neglected, the answer to the nearest 1d. can be obtained by evaluating £3·88195 × 1·67292 correct to 3 dec. places.

```
 xxxxx
3·88195        ·494
1·67292       9·88
-------       ----
3·88195      10·56
2·32917
 ·27173
    776
    349
      8
-------
6·49418
```

Ansr.—£6 9s. 11d.

54. CONTINUED MULTIPLICATION.

If the value of a product, consisting of more than two numbers to be multiplied together, be required to any specified degree of accuracy, it is necessary to know how many columns of significant figures there should be in each multiplication performed.

Suppose it be required to evaluate 317·534 × 1·735 × ·9386 × ·0738 correct to 5 significant figures.

The product equals (·317534 × 1·735 × 9·386) × 7·38 and, as the final answer is to be correct to 5 significant figures, it follows that 7 columns of significant figures must occur in the last multiplication. Therefore (·317534 × 1·735 × 9·386) must be evaluated correct to 7 significant figures.

Again, as (·317534 × 1·735) × 9·386 is to be evaluated correct to 7 significant figures, it follows by the same argument that (·317534 × 1·735) must be evaluated correct to 9 significant figures.

The minimum amount of working necessary to ensure the result to be of the required degree of accuracy is thus as follows—

	xx	xxx
·317534	·55092149	5·170949
1·735	9·386	7·38
·317534	4·95829341	36·19664
·2222738	·16527645	1·55128
952602	4407372	·41367
1587670	330553	38·16159
·550921490	5·17094911	*Ansr.*—38·162.

To generalize, if the value of $abcde$ be required correct to r significant figures,

$$\left.\begin{array}{llll} \text{then } abcd & \text{must be evaluated correct to} & r+2 \\ \qquad abc & \quad ,, \qquad ,, \qquad ,, & r+4 \\ \text{and } ab & \quad ,, \qquad ,, \qquad ,, & r+6 \end{array}\right\} \begin{array}{c}\text{significant}\\ \text{figures}\end{array}$$

Also if the result be required correct to a specified number of decimal places, the corresponding number of significant figures can easily be obtained by making a rough estimate.

55. CONTRACTED METHOD OF DIVISION.

The method whereby the working can be contracted when the value of a quotient is required to a specified number of significant figures or decimal places is illustrated by the following—

Evaluate 1·1736892 ÷ ·078543264 correct to two places of decimals.

```
     xxxx     \  14·943
 7·85432(6)   )117·36892
              /  78·5433
                 -------
                 38·8256
                 31·4173
                 -------
                  7·4083
                  7·0689
                  ------
                   ·3394
                   ·3142
                   -----
                     252
 Ansr.—14·94.
```

(*a*) Move the decimal points so that the divisor has one figure before the decimal point.

(*b*) Retain in the divisor *two* more sig. figs. (if they occur) than are required in the quotient. (If number of figures is insufficient, the number of sig. figs. can be made up by adding "0" s. The next figure, if any, is put down in a bracket for the sake of the "carrying" figure. The rough estimate is 117 ÷ 8, *i.e.*, 15, so that six sig. figs. are required in the divisor.)

(*c*) After the first subtraction, instead of bringing down the next figure in the dividend, cross out the last figure in the divisor. After the next subtraction, cross out the next figure in the divisor and so on until there is one more figure than the required number in the quotient.

56. MULTIPLICATION AND DIVISION COMBINED.

Suppose it be required to evaluate—

$$\frac{\cdot1473 \times 1\cdot1736 \times 37\cdot54}{17\cdot49 \times 3\cdot67 \times \cdot014372} \text{ to 3 places of decimals.}$$

The rough estimate is $\dfrac{\cdot15 \times 1\cdot2 \times 37}{17 \times 4 \times \cdot014}$ *i.e.*, $\dfrac{6\cdot7}{\cdot9}$ *i.e.*, 7; and in order that the final answer must contain *four* significant figures—

(*a*) 17·49 × 3·67 × ·014372 must be evaluated correct to *six* significant figures;

(*b*) ·1473 × 1·1736 × 37·54 must be evaluated correct to *seven* significant figures.

The minimum working, therefore, is as follows—

```
 1·473           ·1749                         \ 7·0346
 1·1736         3·67               xxxx        )-------
 ------         ------        9·22514(2)      /64·89588
 1·473           ·5247                         64·57600
  ·1473          ·10494                        --------
  ·10311          12243                           31988
    4419         -------                          27675
    8838         ·641883                          -----
 ---------      1·4372                             4313
 1·7287128      --------                           3690
 3·754           ·641883                           ----
 ----------      ·2567532                           623
 5·1861384        1925649
 1·21009896        449318
   8643564          12838
    691485      ---------                   Ansr.—7·035
 ----------      ·92251425
 6·48958785
```

57. SQUARE ROOT.

The quantity which when multiplied by itself gives a result N is denoted by $\sqrt{N}$; thus $\sqrt{81} = 9$, and 9 is called the square root of 81. A quantity whose square root is a commensurable quantity is termed a perfect square; but as comparatively few quantities met with are perfect squares, the square roots of most quantities are incommensurable quantities, so that their values can only be approximated. This can be done, however, to any degree of accuracy which may be required.

* The method is based on the identity $(a + b)^2 = a^2 + 2ab + b^2$, which becomes, on subtracting a^2 from each side, $(a + b)^2 - a^2 = b(2a + b)$.

Suppose it be required to evaluate $\sqrt{2875 \cdot 4}$. As $50^2 = 2{,}500$ and $60^2 = 3{,}600$, it lies between 50 and 60.

Let $\sqrt{2875 \cdot 4} = 50 + x$, so that

$$2875 \cdot 4 = (50 + x)^2$$

Then as $(50 + x)^2 - 50^2 = x(2 \times 50 + x)$

i.e., $2875 \cdot 4 - 2{,}500 = x(2 \times 50 + x)$

i.e., $375 \cdot 4 = x(2 \times 50 + x)$,

it is seen that by dividing both sides by $(2 \times 50 + x)$,

$$x = \frac{375 \cdot 4}{2 \times 50 + x}$$

so that the approx. value of x is obtained by dividing $375 \cdot 4$ by 2×50. Thus x lies between 3 and 4; and on dividing $375 \cdot 4$ by $2 \times 50 + 3$, *i.e.*, 103, the result is 3 and the remainder $66 \cdot 4$.

The working, so far, is shown on the right to line A.

As $\sqrt{2875 \cdot 4}$ lies between 53 and 54, let

$$\sqrt{2875 \cdot 4} = 53 + y$$

Then as $(53 + y)^2 - 53^2 = y(2 \times 53 + y)$

i.e., $2875 \cdot 4 - 2{,}809 = y(2 \times 53 + y)$

i.e., $$y = \frac{66 \cdot 4}{2 \times 53 + y},$$

it is seen that the approx. value of y is obtained by dividing $66 \cdot 4$ by 2×53. Thus y lies between $\cdot 6$ and $\cdot 7$; and on dividing $66 \cdot 4$ by $2 \times 53 + \cdot 6$, *i.e.*, $106 \cdot 6$, the result is $\cdot 6$ and the remainder $2 \cdot 44$.

The working is shown on the right as far as line B.

By denoting $\sqrt{2875 \cdot 4}$ by $53 \cdot 6 + z$, in precisely the same way the next figure in the answer is obtained, and by repeating the process the answer can contain as many significant figures as may be required.

	2875·4 | 50 + 3 + ·6	
	2500	
2 × 50 + 3 = 103	375·4	
	309	
2 × 53 + ·6 = 106·6	66·4	A
	63·96	
	2·44	B

In actual practice, it is usual to proceed as follows—

(*a*) From the decimal point, mark off the figures in pairs to the left and to the right.

(*b*) $\sqrt{28}$ lies between 5 and 6: put up the 5 and place 25 (*i e.*, 5^2) under 28.

(*c*) Subtract and bring down the next pair of figures.

(*d*) Double the number already in the answer and divide the result 10 into 37, which gives 3. Put this after the 5 and the 10.

(*e*) Multiply 103 by 3 and subtract result from 375.

(*f*) Bring down the next pair of figures and place the decimal point in the answer as ·40 is the first pair of decimal figures brought down.

Repeat as in (*d*) and so on.

	2875·40 | 53·6227
	25
103	375
	309
1066	6640
	6396
10722	24400
	21444
107242	295600
	214484
107244	8111600

$\sqrt{2875 \cdot 4} = 53 \cdot 623$, correct to 3 places of decimals.

When the approximation is required to a comparatively large number of significant figures, the working can be contracted by proceeding in the above way until just over half the significant figures are obtained, and then completing by ordinary contracted division.

Example (xi)—

The area of Middlesex is 148,701 ac. If in shape it were a square, what would its perimeter (distance round) be to the nearest foot?

$$\text{Area} = 148701 \times 4840 \times 9 \text{ sq. ft.}$$
$$= 1487010 \times 484 \times 9 \quad ,,$$
$$\therefore \text{Perimeter} = \sqrt{1487010} \times 22 \times 3 \times 4 \text{ ft.}$$

As 22×12 lies between 100 and 1,000, $\sqrt{1,487,010}$ must be evaluated correct to 3 decimal places (*i.e.*, to 7 significant figures).

	1487010·(1219·432
	144
241	470
	241
2429	22910
	21861
	1049
	972
	77
	73
	4

Perimeter = $1219 \cdot 432 \times 22 \times 12$ ft.
= 321,930 ft. to nearest ft.
= 107,310 yd.
= 60 miles 1,710 yd.

NOTE 5.—If $a \times a \times a = N$, a is called the cube root of N, i.e., $a = \sqrt[3]{N}$. Similarly, if $a \times a \times a \times a \times a = N$, a is called the fifth root of N and is written $\sqrt[5]{N}$. The 4th root of a number is obtained by taking the square root of the square root of the number. The cube root of a number is best obtained by the use of logarithms, by means of which any root or power of a quantity can be easily evaluated.

58. DECIMALIZATION OF MONEY.

3d.= £·0125; 6d.= £·025; 9d.= £·0375; 1/- = £·05.

Thus, 13s.= £·65; and it is clear that by multiplying the number of shillings by ,5, the amount is expressed as a decimal of £ by making the result the first two decimal places.

Also 17/9 = £·85 + £·0375 = £·8875.

Thus a sum of money ending in 3d., 6d., or 9d. can be exactly expressed as a decimal of £1 by inspection.

EXAMPLE (xii)—

1,000 articles at £1 11s. 3d. = £1,000 × 1·5625
= £1562·5
= £1,562 10s.

Approximate Decimalization of Money.

Suppose $\frac{1}{1000}$ of a £1 be called a *mil*
Then $\frac{960}{1000}$ of a farthing equals 1 mil, *i.e.*, 1 mil = $\frac{24}{25}$ farthings.
∴ 1 farthing − 1 mil = $\frac{1}{25}$ farthing = $\frac{1}{24}$ mil.

Now if x farthings be written x mils, the error is $-\frac{x}{24}$ mils.

If x is less than 12, the error is less than $\frac{1}{2}$ mil.
" x lies between 12 and 36, the error lies between $\frac{1}{2}$ mil and $1\frac{1}{2}$ mil.
" x " 36 and 48, " " $1\frac{1}{2}$ " 2 mils

$$\text{Thus}\begin{cases} x \text{ farthings} = x \text{ mils to nearest mil., when } x < 12 \\ x \quad " \quad = x + 1 \quad " \quad " \quad " \quad x > 12, \text{ but } < 36 \\ x \quad " \quad = x + 2 \quad " \quad " \quad " \quad x > 36, \text{ but } < 48. \end{cases}$$

This rule enables any sum composing shillings and pence to be at once expressed as a decimal of £ correct to 3 places of decimals.

e.g., 17/7¾ = £·85 + £·032 = £·882.

NOTE 7.—Care must be taken as to when and when not to use this rule. Thus, suppose the cost of 1,000 articles at 17s. 7¾d. be required, the rule could not be applied, as £·882 differs from 17s. 7¾d. by something less than ½ mil.; but on multiplying by 1,000, the error may be anything less than 500 mil. (*i.e.*, anything less than 10s.). On the other hand, if, say, £91 17s. 7¾d. were to be divided by any number greater than 1, or multiplied by a fraction less than 1, the error would be diminished, and thus the result would be correct to the nearest mil.

Reverse Process.—If x mils. be written x farthings, the error is

$$+\frac{x}{25} \text{ farthing.}$$

If x is less than 12·5, the error is less than ½ farthing.
" x lies between 12·5 and 37·5, the error lies between ½ farthing and 1½ farthing.

If x lies between 37·5 and 50, the error lies between 1½ farthings and 2 farthings.

$$\text{Thus}\begin{cases} x \text{ mils} = x \text{ farthings to nearest farthing when } x < 12{\cdot}5 \\ x \quad ,, \quad = x - 1 \quad ,, \quad ,, \quad ,, \quad x > 12{\cdot}5, \text{ but } < 37{\cdot}5 \\ x \quad ,, \quad = x - 2 \quad ,, \quad ,, \quad ,, \quad x > 37{\cdot}5, \text{ but } < 50 \end{cases}$$

Suppose it be required to express £734 in shillings and pence to the nearest farthing—

£·734 = £·70 + £·034 = 14/– + 33 farthings to nearest ¼d. = 14/8¼ to nearest ¼d.
Also £·097 = £·05 + £·047 = 1/– + 45 farthings to nearest ¼d.
= 1/11¼ to nearest ¼d.

Thus, when a sum is expressed exactly in £'s in the form of a decimal not exceeding 3 places, this sum can at once be expressed in £ *s. d.* to the nearest farthing. In actual practice, however, the decimal consists of more than three places, and in approximating correctly to 3 decimal places and then applying the above rule; the result frequently involves an error of ¼d. For example—

£·0314 = £·031 correct to 3 dec. places
= 30 farthings to nearest ¼d. by above rule.
£·0316 = £·032 correct to 3 dec. places
= 31 farthings to nearest ¼d. by above rule.
Now £·0314 = ·0314 × 960 farthings
= 2·512 × 12 farthings = 30·144 farthings
Also £·0316 = ·0316 × 960 farthings
= 2·528 × 12 farthings = 30·336 farthings

Thus, in the case of £·0316, the amount to the nearest farthing is 7½d., not 7¾d. as the above rule gives. This result is due to the fact that two approximations are made; and although each involves an error of less than ½ farthing, the two together involve an error greater than ½ farthing. In the case of £·0314, the two errors involved in the approximations partially neutralize one another.

In order to ensure that a number of £'s involving decimals exceeding 3 places should, when expressed in £ *s. d.*, be correct to the nearest farthing, the result correct to 4 decimal places should be obtained and the decimal part split up into two parts: the first being an exact number of shillings; the second part should be multiplied by 960 and thus expressed in farthings. Thus—

£7·67387 = £7·6739 correct to 4 places of decimals
= £7 + £·65 + ·0239 × 960 farthings
= £7 + 13/– + 22·944 farthings
= £7 13s. 5¾d. correct to nearest farthing.

Note 8.—Under the proposed decimal system of coinage, £1 = 1,000 mils, and the value of each coin would be a whole number of mils. Under this system, sums of money would be expressible *exactly* in £'s by means of 3 decimal places.

59. CALCULATION OF COST.

There are several methods of calculating the cost of a quantity at so much per unit, four of which are tabulated below—

(1) By use of vulgar fractions.

(2) By decimalizing each quantity and applying contracted multiplication.

(3) By applying the rule of approximate decimalization.

(4) By compound practice.

It greatly depends on the problems as to the method it is advisable to use.

Method 1 is illustrated by Example (xv), Chapter IV, and should be used whenever that part of the quantity expressed in the lower units can be easily expressed as a vulgar fraction of the fundamental unit. The advantage of this method is due to the fact that approximations are not involved until right at the end.

Method 2 is illustrated by Example (x) of this chapter. Mistakes are frequently made in using this method owing to the fact that the principles involved in the estimation of the errors resulting from approximations are not thoroughly known. The number of decimal places in which it is necessary for the approximate values to be expressed is determined by consideration of the errors involved as explained in paragraph 52.

Method 3. This is illustrated by the following example—

EXAMPLE (xiii)—

Find the cost of excavating a railway cutting 5 miles 2 furlongs 7 chains 10 yards long at £473 14s. $2\frac{1}{4}$d. per mile.

$$
\begin{aligned}
\text{Cost of 5 miles} &= £2{,}368\ 10\text{s. } 11\tfrac{1}{4}\text{d.}\\
\text{2 fur. 7 ch. 10 yd.} &= 440 + 154 + 10 \text{ yd.} = \tfrac{604}{1760} \text{ mile}\\
\therefore \text{ Cost of }\ ,,\quad ,,\quad ,, &= £\tfrac{151}{440} \times 473{\cdot}709\\
&= £162{\cdot}5683 \qquad \text{correct to 4 dec. places}\\
&= £162\ 11\text{s. } 4\tfrac{1}{2}\text{d.} \qquad ,, \qquad \text{nearest } \tfrac{1}{4}\text{d}\\
\therefore \text{ Total cost} &= £2{,}531\ 2\text{s. } 3\tfrac{3}{4}\text{d.}
\end{aligned}
$$

NOTE 9.—2 fur. 7 chn. 10 yd. = $\frac{151}{440}$ mile *exactly*. £473·709 involves an error less than $\frac{1}{2}$ farthing, so that on multiplying by $\frac{151}{440}$ the result involves an error less than $\frac{1}{2}$ farthing.

Method 4 is illustrated by working the same example.

$$14/2\tfrac{1}{4} = £\frac{14{\cdot}1875}{20} = £{\cdot}709375.$$

	ml.	*fur.*	*ch.*	*yd.*			£
Cost of	1	0	0	0	=		473·70937
,,		2	0	0	=	2 fur. = $\frac{1}{4}$ of 1 ml.	118·42734
,,			5	0	=	5 ch. = $\frac{1}{4}$ of 2 fur.	29·60684
,,			2	0	=	2 ch. = $\frac{1}{10}$ of 2 fur.	11·84273
,,				10	=	10 yd. = $\frac{1}{11}$ of 5 ch.	2·69153
,,		2	7	10	=		162·5684
					=		£162 11s 4½d.
,, 5 miles					=		£2,368 10s 11¼d.
∴ Total cost					=		£2,531 2s 3¾d.

NOTE 10.—As the final error must not exceed £·0005, taking two additional places, the amounts on each line should be correct to 5 dec. places. On adding, express the result correct to 4 dec. places.

60. TABLES OF NINE MULTIPLES.

When a number of problems of the same type are required to be solved, and each requires the decimalizing of quantities, the method of "Nine Multiples" is of great use.

Suppose a number of quantities each consisting of cwt. qr. lb. oz. be required to be expressed as a decimal of a ton correct to 5 decimal places. As 1 ton = 2,240 lb., it may be required to multiply 1 lb. by a number like 2,000, so that the table should be obtained correct to 9 decimal places. The table, and an example showing its use, are as follows—

LBS. to TONS.

lb.	*tons.*
1	·000446429
2	·000892857
3	·001339286
4	·001785714
5	·002232143
6	·002678572
7	·003125
8	·003571429
9	·004017857

EXAMPLE (xiv)—

Express 11 cwt. 2 qr. 25 lb. 6 oz. in tons correct to 5 decimal places.

11 cwt. 2 qr. 25 lb. 6 oz. = 1,313⅜ lb.

1,000 lb.	=	·446429	ton.
300 ,,	=	·1339286	,,
10 ,,	=	·0044643	,,
3 ,,	=	·0013393	,,
⅜ ,,	=	·0001674	,,
1,313⅜ lb.	=	·58633	ton

EXAMPLE (xv)—

A manufacturer supplies to wholesalers, numbers of a certain article at 3s. 7¼d. each. The largest order is 29,715 articles. Write down a table for quickly estimating the costs and use it for finding the value of the largest order.

The final result must be correct to 3 places of decimals at least, so that as the cost of 1 article has to be multiplied by a number between 10,000 and 100,000, the table must contain results correct to 8 places.

$$3/7\tfrac{1}{4} = £\frac{3{\cdot}6041666}{20} = £{\cdot}180208333 \text{ correct to 9 dec. places.}$$

Table for price 3/7¼.

	£
1	·18020833
2	·36041667
3	·54062500
4	·72083333
5	·90104167
6	1·08125000
7	1·26145833
8	1·44166667
9	1·62187500

		£
20,000	articles cost	3604·1667
9,000	,,	1621·87500
700	,,	126·14583
10	,,	1·80208
5	,,	·90104
∴ 29,715	articles cost	£5354·8907
		= £5,354 17s. 9¾d

61. METRIC SYSTEM OF WEIGHTS AND MEASURES.

With the exception of the United Kingdom, all the countries of Europe in general make use of the *Metric System* of weights and measures. In all countries of the world it is permissive to use this system of units in commerce, and many attempts have been made in England to abandon the British system in its favour, but these attempts have met with little success. The Metric System is also employed by almost all the republics of Central and South America. As regards scientific work, the Metric System is universally employed, but the units lb. and foot are still the British fundamental units in practical engineering.

The units of length, weight, and capacity in the Metric System are the *Metre*, *Gram*, and *Litre*, although in many countries their names are modified. The Metre was derived as being 1 ten-millionth of the distance from the North Pole to the Equator. In 1794, a platinum bar was constructed which permanently records this distance, and which is kept in safe custody at the French Record Office. One hundredth part of a metre is called a centimetre, so that the volume of a cube whose edges are 1 centimetre long is 1 cub. cm. The Gram (or gramme) is defined as being the weight of 1 cub. cm. of distilled water at 4° C. The Litre is defined as a capacity of 1,000 cub. cm.

Each unit has multiples of 10, 100, 1,000, and 10,000; and sub-multiples of $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$. The multiples are named by the prefixes Deca., Hecto., Kilo., and Myria., which are Greek for 10, 100, 1,000 and 10,000 respectively. The sub-multiples are named by the prefixes Deci., Centi., and Milli., which are the Latin for $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$ respectively. By means of this system of multiples and sub-multiples, the process of reduction can be

made by inspection, as it involves only a movement of the decimal point, so that the compound rules of addition, subtraction, multiplication, and division are greatly simplified. Another advantage of the Metric System lies in the fact that the units of weight and capacity are derived from that of length, and thus they have a simple relationship (*e.g.*, 1 litre = 1,000 cub. cm.), whereas 1 gall. = 277·274 cub. in.).

The Metric table of length is given below; but by replacing "metre" by "gram" (gm.) and "litre" (l.) respectively, the tables of weight and capacity are obtained, the abbreviations being shown in the brackets.

·001	metre (m.)		=	1 millimetre (mm.)
·01	,, ,,	or 10 mm.	=	1 centimetre (cm.)
·1	,, ,,	,, 10 cm.	=	1 decimetre (dm.)
10	metres		=	1 decametre (Dm.)
100	,,	,, 10 Dm.	=	1 hectometre (Hm.)
1,000	,,	,, 10 Hm.	=	1 kilometre (Km.)
10,000	,,	,, 10 Km.	=	1 myriametre (Mm.)

In addition, as regards the table of weight, 100 Kgm. =1 *quintal* and 1,000 Kgm. or 10 quintals = 1 *metric ton or millier.* The units cm., m., Km., gm., Kgm., and quintal are those in common use. The Livre, which is half a Kgm., is in common use in Belgium and some parts of France.

The tables of square and cubic measure are derived from that of length by squaring and cubing the numbers respectively. Thus, 1 sq. m. = 100 × 100 sq. cm., 1 sq. Km. = 1,000 × 1,000 sq. m., and 1 c. m. = 100 × 100 × 100 c. cm. In dealing with land, the units used are *Are,* which is 100 sq. m.; the *Hectare,* which is 100 ares; and the sq. Km., which is seen to be 100 hectares. Sometimes a volume of 1 c.m. is called a *Stere.*

By combining the above tables, it is seen that—

1 litre of water weighs 1 kilogram
1 cubic metre = 1,000 litres
1 litre = 1 c. decimetre

France, Belgium, Switzerland, Norway, Sweden, Portugal, Serbia, and most of the republics of Central and South America use the Metric System and employ the names as given above, but several European countries which also use the Metric System, give different names to their units.

The following are the tables of equivalents existing between the Imperial and Metric Systems of units—

Metric Units to British Units.			*British Units to Metric Units.*		
1 cm.	·3937079	in.	1 in.	2·5399541	cm.
1 m.	1·093633	yd.	1 yd.	·9143835	m.
1 Km.	·6213824	mile	1 mile	1·609315	Km.
1 sq. m.	1·196033	sq. yd.	1 sq. yd.	·836097	sq. m.
1 hectare	2·47114	acres	1 acre	·40467	hectare
1 c.m.	1·3080215	c. yd.	1 sq. mile	258·98945	hectares
1 gm.	15·4323487	grains	1 c. yd.	·7645134	c.m.
1 Kgm.	2·20462125	lb.	1 lb.	·4535926	Kgm.
1 metric ton	·984206	ton	1 oz.	28·34954	gm.
1 l.	1·7607734	pint	1 oz. Tr.	31·103496	gm.
			1 ton	1·0160475	metric ton
			1 gall.	4·543457	l.

62. The French unit of money is called the **Franc**, which equals **100 Centimes.** Before the Great War, the *Franc* of Belgium, the *Franc* of Switzerland, the *Lire* of Italy, and the *Peseta* of Spain each represented the same amount of gold as the Franc of France. After the War, however, France, Belgium, and Italy adopted lower gold standards differing from one another.

EXAMPLE (xvi)—

A train travelled 52 Km. 560 m. in 1 hour. What was its average speed in metres per second?

$$\text{Speed} = \frac{52{,}560}{60 \times 60} \text{ m. per sec.} = \underline{14{\cdot}6 \text{ m. per sec.}}$$

EXAMPLE (xvii)—

A cistern is 92 cm. long, 85 cm. wide, and 70 cm. deep. How many litres is it capable of holding?

$$\text{Vol. of cistern} = 92 \times 85 \times 70 \text{ c. cm.}$$

$$\therefore \text{Capacity} = \frac{92 \times 85 \times 70}{1000} \text{ l.} = \underline{547{\cdot}4 \text{ l.}}$$

EXAMPLE (xviii)—

Find the cost of turfing a lawn 9 m. 45 cm. long and 7 m. 50 cm. wide at 1 f. 35 per sq. m.

$$\text{Cost of turfing} = 9{\cdot}45 \times 7{\cdot}5 \times 1{\cdot}35 \text{ francs}$$

$$= 95{\cdot}681 \text{ francs}$$

$$= \underline{95 \text{ f. } 70} \text{ to nearest 5 centimes piece } (\textit{sou}).$$

EXAMPLE (xix)—

The weight of a case full of coffee was 58·425 Kgm. If the case when empty weighs 5·775 Kgm., how many packets of coffee each containing 450 gm. could be made up?

$$\text{Weight of coffee} = 52{\cdot}650 \text{ Kgm.}$$
$$= 52650 \text{ gm.}$$
$$\therefore \text{ No. of packets} = \frac{52650}{450} = \underline{117}$$

EXAMPLE (xx)—

A rectangular field is 115·72 m. long and 53·35 m. broad. Find the cost at fcs. 2115·00 per hectare.

$$\text{Area of field} = \frac{115{\cdot}72 \times 53{\cdot}35}{100 \times 100} \text{ hectares}$$
$$\therefore \text{ Cost of field} = 1{\cdot}1572 \times {\cdot}5335 \times 2{,}115 \text{ francs.}$$
$$= \underline{\text{fcs. } 1305{\cdot}75 \text{ to nearest } sou.}$$

EXAMPLE (xxi)—

Express 2s 4d. per ton per mile in francs per metric ton per kilometre correct to the nearest 5-centimes piece, given that £1 = 87·62 francs.

$$\text{Cost of 1 ton for 1 mile} = 2\tfrac{1}{3} \text{ shillings}$$
$$\text{Cost of 1 metric ton for 1 mile} = 2\tfrac{1}{3} \times {\cdot}984 \text{ shillings}$$
$$\text{Cost of 1 metric ton for 1 Km.} = 2\tfrac{1}{3} \times {\cdot}984 \times {\cdot}621 \text{ shillings}$$

$$= 7 \times {\cdot}328 \times {\cdot}621 \times 4{\cdot}381 \text{ francs.}$$

$$= \underline{6{\cdot}25 \text{ f.}}$$

EXAMPLE (xxii)—

Find a multiplier for converting pence per oz. Troy into centimes per gm., and use it to obtain the value in francs and centimes of 1 Kgm. of silver, the price of silver being quoted at 1s. 8¾d. per oz. Troy. (£1 = 87·45 francs and 1 gm. = 15·43 grains.)

$$\text{If cost of 1 oz. Tr.} = 1 \text{ penny}$$

$$\text{then } ,, \text{ 1 grain} = \tfrac{1}{480} \text{ penny}$$

$$,, \quad ,, \text{ 1 gm.} = \frac{15{\cdot}43}{480} \quad ,,$$

$$= \frac{15{\cdot}43 \times 8{,}745}{480 \times 240} \text{ centimes.}$$

Now, as this multiplier will be multiplied by 20¾ × 1,000, in order to obtain the final answer correct to the nearest centime, it must be expressed correct to 5 places of decimals.

1 penny per oz. Tr. = 1·17131 centimes per gm.

∴ $20\frac{7}{8}$ pence „ $= 20\frac{7}{8} \times 1{\cdot}17131$

∴ Cost of 1 Kgm. of silver $= 20\frac{7}{8} \times 1171{\cdot}31$ centimes

$= 20\frac{7}{8} \times 11{\cdot}7131$ francs = 244·51 francs.

Ansr.—1·17131; 244·50 f.

TEST EXERCISES I, 6.

(1) The population of Dover in 1931 was 41,095, and the birth-rate and death-rate were 15·81 and 10·75 per thousand respectively. What were the numbers of births and deaths in the complete year?

(2) In 1929 the population of Great Britain and Northern Ireland was approximately 45,600,000 and the numbers of births, marriages, and deaths were 761,963, 353,741, and 623,231 respectively. Find the rate per thousand in each case correct to one place of decimals.

(3) The population of Birmingham in 1931 was 1,002,413 and the birth-rate was 17·7 per thousand. Determine from this data the limits between which the actual number of births could lie.

(4) Given that the weight of a medal is 176·45 gr., calculate the least number of medals which, together, would weigh more than 1 cwt.

(5) Find the cost, to the nearest 1d., of 13·84 therms of gas at $8\frac{3}{4}$d. per therm.

(6) SPECIAL TRADE (MERCHANDISE)

Year.	*Special Imports.*	*Special Exports.*
	£	£
1913 . . .	659,168,008	525,253,595
1926 . . .	1,115,866,309	653,046,909
1927 . . .	1,095,388,311	709,081,263
1928 . . .	1,075,315,169	723,579,089
1929 . . .	1,111,063,472	729,349,322
1930 . . .	957,860,915	570,552,946

Find the excess of Imports over Exports each year to the nearest £1,000,000.

(7) By paying a yearly premium of £22·873, a man aged 30 can obtain at age 60 an annual pension of £76·815. Find, to the nearest penny, what premium he should pay in order that his pension may be £156 per annum.

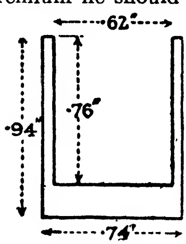


FIG. 46.

(8) A book contains 476 leaves, and the dimensions of each are 4·8″ × 7·45″. Find the total area available for printing matter. If the total thickness of the book without the cover is 1·73″, find the thickness of each leaf correct to $\frac{1}{10000}$ in.

(9) The dimensions of the cross-section of a strip of metal are as shown. If 1 cub. in. of metal weighs 3·47 oz., what is the weight of 1 foot of the strip correct to ·01 oz. Also, if the dimensions given are approximations each correct to two places of decimals, find the greatest and least weight that 1 foot of the strip could possibly be.

(10) Find, correct to 5 decimal places, the value of—

$$1+\frac{1}{3}+\frac{1}{3\times 5}+\frac{1}{3\times 5\times 7}+\frac{1}{3\times 5\times 9}+\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$

(11) Bronze is an alloy of copper 95 parts, tin 4 parts, and zinc 1 part. The weights of a penny, a halfpenny, and a farthing are $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{10}$ oz. Avoirdupois respectively. Calculate the weights in grains of copper, tin, and zinc in a penny, halfpenny, and farthing respectively.

(12) Standard silver consists of $\frac{37}{40}$ fine silver and $\frac{3}{40}$ alloy (*i.e.*, fineness 925). An article made of standard silver weighs 5 oz. Tr. 12 dwt. 13·73 gr. What is the weight of fine silver to the nearest ·01 gr.?

(13) The base of a rectangular tank is 2′ 7·3″ by 1′ 11·4″. What must be the depth to nearest ·01″ in order that it should have a capacity of 400 gall., given that 1 gall. = 277·274 cub. in.? Also find what area of metal is required to make the tank, not counting the lid and allowing one-fifth of the amount for wastage.

(14) A certain kind of gold wire costs 2s. 9½d. per inch. What are the costs of pieces of wire whose lengths are 4·7″, 15·37″, and 9·88″? Also find the length to nearest ·01 in. that could be bought for £5.

(15) What length of wire, to the nearest foot, weighs 15 ton, if 1 yd. of the wire weighs 2·357 oz.?

(16) On a certain day £1 was worth 86·78 francs or 3·40 dollars. On that day how many francs were worth 5 dollars and how many dollars were worth 100 francs?

(17) Express 13 cwt. 3 qr. 17 lb. 13 oz. as a decimal of a ton correct to 7 decimal places.

(18) Express the area of a rectangular field 179 yd. 4 in. long and 113 yd. 2 ft. broad as a decimal of 1 ac. correct to 6 decimal places.

(19) (*a*) Express ·69443 mile in furlongs, poles, yard, feet, inches, correct to nearest inch.

(*b*) Express ·477327 cub. yd. in cubic feet, cubic inches, correct to ·1 cub. in.

(20) The population of a town is 291,118 and the total rateable value is £1,325,761. What sum to the nearest £100 would a rate of 7s. 10¾d. produce?

(21) A tank is 3 ft. 4·7 in. long, 2 ft. 7·3 in. wide, and 1 ft. 10·7 in. deep. Given that 1 gall. = 277·274 cub. in., find to the nearest ¼ pt. the capacity of the tank.

(22) A ship whose average speed was 23·43 knots, took 125¼ hr. on a certain journey. Find to the nearest mile the length of the journey given that 1 knot = 1·15 miles per hour. If another ship took 148 hr. on a journey 95 miles longer, what was its average speed in knots correct to 2 decimal places.

(23) A triangular piece of cardboard whose base is 7·3 cm. and height 3·8 cm. weighs 8·437 gm. What would be the weight of 350 sheets each 1 m. by 78 cm. of the same kind of cardboard to the nearest gram?

(24) Silver is approximately 10·57 times heavier than water. Given that 1 gall. of water weighs 70,000 gr. exactly and that 1 gall. = 277·274 cub. in. approximately, find the weight of 1 cub. in. of silver in ounces Troy, pennyweights, and grains to as close an approximation as the data permits.

*(25) A metre of wire weighs approximately 72·437 gm. Find to as close an approximation as the data allows the weight of a length of wire approximately 57·43 cm. long. Also find, to as close an approximation as the data allows, the length of a coil of the wire which weighs approx. 174·073 gm.

(26) The number of deaths in a town during a certain year was 1,739, and the death-rate was stated to be approximately 15·7 per 1,000. Find, to as many significant figures as the data permits, the population of the town.

*(27) The cost during a certain year of producing rubber on a plantation was stated to be £94,386 to the nearest pound. The average cost per lb. of rubber was stated to be 1s. 9·47d. Find, to as many significant figures as the data permits, the number of lb. of rubber produced.

*(28) On a rubber plantation 978,540 lb. of rubber, to nearest 10 lb., was produced during a certain year, and the average cost per lb. was stated to be 1s. 10·13d. Find the total cost to as close an approximation as the data permits.

*(29) The volume of a rectangular prism is approximately 7·26 cub. in., and the length and breadth are approximately 2·73 in. and 1·96 in. respectively. Find the height of the prism to as close an approximation as the data allows.

*(30) Given that $\pi = 3{\cdot}14159265\ldots$ and $e = 2{\cdot}71828182\ldots$ find (i) to 3 places of decimals, (ii) to 3 significant figures, the values of $\pi + e$, $10\pi - e$, πe, $\frac{\pi}{e}$, $\frac{e}{25\pi}$.

(31) How many gallons of tar per mile will be required to give a road 50 ft. wide a coating of tar of an average thickness of ·07 in.?

(32) The rateable value of a town is £513,747. Find the smallest rate per pound which must be levied in order to raise a sum of not less than £74,750 [Express the answer in shillings, pence, and farthings.]

(33) Find the cost of 5 ton 11 cwt. 3 qr. 17 lb. at £1 17s. 7d. per ton.

(34) Find the cost of 11 ac 7 sq ch 317 sq. yd at £4 17s. 3d. per acre.

(35) An article made of a certain metal weighs 1 oz Tr. 17 dwt. 14·73 gr. What is the value of the metal at £3 17s. 10½d. per ounce Troy?

(36) What is the cost to the nearest shilling of paving a road 7 ch. 53 lk. long at £5,746 15s. per mile?

(37) Find the cost of 5 ton 7 cwt. 57¾ lb. of sugar at £1 19s. 2½d. per cwt.

(38) Find the dividend on £173 8s 6d. at 11s 8¼d. in the £.

(39) What is the rent of a farm 19 ac. 2 rd. 19 sq. pl. at £4 17s. 4½d. per acre?

(40) Find the value of 19·73467 oz. Tr. of metal at £4 4s. 11½d. per ounce Troy.

(41) The area of the Transvaal is 110,450 sq. miles and the European population in 1931 was 695,963. What was the area in acres per person (European)?

(42) The total area of the British Isles is 77,683,084 ac. If the shape were a compact square, what would be the length of a side in miles and yards to the nearest yard?

(43) What is the length to the nearest inch of a fence that would surround a square field whose area is 1 ac.?

(44) What is the height of a square prism the side of whose base is 7·13 cm. and whose volume is equal to that of a cube whose edge is 5·87 cm.

(45) Construct a table of nine multiples of price 4s. 1¾d and calculate the cost at this price of 3,765, 984, 1,407, and 732 articles.

(46) Construct a table of nine multiples expressing in decimals of £ the cost of 1–9 lb. at £1 14s. 8d. per ton Hence find the cost of 13 cwt. 2 qr. 17 lb., 5 ton 7 cwt., and 5 cwt. 3 qr. 23¼ lb. at £1 14s. 8d per ton.

(47) Construct table of nine multiples from the following equivalents—
1 Km. = ·6213824 mile; 1 mile = 1·609315 Km.; 1 millier = ·984206 ton;
1 ton = 1·0160475 millier; 1 hectare = 2·47114 ac.; 1 ac. = ·40467 hect.
Using these tables, express 217·52 Km. in miles, 175·437 miles in Km., 375·47 milliers in tons, 457 tons in milliers, 85 hect. 47 ares in acres and square poles, and 13 ac. 3,500 sq. yd. in hectares and ares.

(48) (a) Express in kilometres, 7 ml. 5 fur 25 per. (1 yd. = ·914 m.).
(b) Express in cwt. qr. lb. oz., 53·76 Kg. (1 Kg. = 2·2046 lb.).

(49) Given that £1 = 89·35 francs and that 1 metre = 39·37 in., express 8·45 f. per metre in shillings and pence per yard.

(50) 2,987,750 ½-kilograms of tobacco are shipped in 16,890 bales. Find to the nearest tenth of a lb. the average weight of a bale, a kilogram being equivalent to 2·2046 lb.

(51) A railway engine running at 45 miles an hour picks up water from a trough ¼ mile long at the rate of 169 gall. per minute. Taking a gallon to be ·16057 cub. ft., find the number of cubic feet of water it picks up during the quarter mile.

CHAPTER VII.

RATIO—PROPORTION—PROPORTIONAL PARTS—PERCENTAGE.

63. WHEN two similar quantities are required to be compared, the symbol ":" is often employed. For example, if A earns £250 per annum and B £300 per annum, their salaries can be compared by 5 : 6, which means that for every 5 units of money earned by A, 6 units are earned by B.

5 : 6 is termed a **ratio**, the first term being called the **antecedent** and the second the **consequent.**

Now for every unit B earns, A will earn $\frac{5}{6}$ of a unit, so that the comparison may be made by the fraction $\frac{5}{6}$, it being understood that if this be regarded as the antecedent, the consequent is unity.

Again, suppose the ratio of the length to the breadth of a rectangle is stated to be 1·753. This means that for every unit in the breadth there will be 1·753 unit in the length. The ratio 1·753 is independent of the units in which the length and breadth are measured; but both measurements must be in the same units in each case, either both in inches or both in feet, etc. Thus, 1·753 is purely a numerical quantity, that is, no unit is associated with it.

It is obvious that it is impossible to compare two quantities when the kinds of unit associated with them are different. For example, 5 oz. cannot be compared with 7 minutes, nor 4 miles per hour with 7 miles: but 5 oz. can be compared with 7 lb., the ratio being $\frac{5}{112}$; and 4 miles per hour can be compared with 80 yds. per minute, the ratio being $\frac{22}{15}$.

Thus, a **Ratio is a numerical quantity which compares the magnitude of two similar quantities,** and to evaluate the ratio of one quantity to a second quantity, it is necessary to express the two quantities in the same units and divide the first by the second.

Suppose a quantity N is to be increased in the ratio $b : a$, b being greater than a, then as for every a units in N the increased quantity will contain b units; the latter will be $N \times \frac{b}{a}\cdot$ Similarly, if N is to be decreased in the ratio $b : a$, a being greater than b, the decreased quantity will be $N \times \frac{b}{a}$.

Example (i)—

A destroyer can travel at 40 knots and an express train at 60 miles per hour. What is the ratio of the speed of the former to that of the latter? (1 knot = 6,080 ft. per hour.)

Speed of destroyer = 40 × 6,080 ft. per hour.

" train = 60 × 1,760 × 3 ft. per hour

$$\therefore \text{ Ratio } = \frac{\overset{2}{\cancel{40}} \times \overset{76}{\cancel{6080}}}{\underset{3}{\cancel{60}} \times \underset{\underset{11}{\cancel{22}}}{\cancel{1,760}} \times 3} = \frac{76}{99} = \cdot 768, \text{ correct to 3 places of decs.}$$

Example (ii)—

A man whose salary was £175 per annum was given an increase of 5s. per week. By what ratio was his salary increased? If another man who earned £200 per annum had £15 per annum increase, whose increase was the greater *pro rata?*

Increased salary of 1st man = £175 + £13 = £188

Ratio of increase to initial salary, 1st man = $\frac{13}{175}$ = ·074 . . .

Also " " " 2nd " = $\frac{15}{200}$ = ·075

∴ 2nd man had the greater increase *pro rata.*

Example (iii)—

The price of an article was £10 10s.: it was increased in the ratio 9.7 and afterwards reduced in the ratio 3:5. What was the final price?

$$\text{Final price} = £\frac{\overset{3}{\cancel{21}}}{2} \times \frac{9}{\cancel{7}} \times \frac{3}{5}$$

$$= £8\ 2s.\ 0d.$$

Example (iv)—

The length and breadth of a rectangle are increased in the ratio 4:3 and 8:5 respectively. By what ratio is the area increased?

Let l, b ≡ original length and breadth respectively

then $\frac{4l}{3}, \frac{8b}{5}$ ≡ increased " "

$$\text{Then area is increased in the ratio } \frac{\frac{4l}{3} \cdot \frac{8b}{5}}{lb} = \frac{32}{15}$$

Example (v)—

A mixture consists of three substances whose volumes are in the ratio 5 : 6 : 8. The weights of equal volumes of the substances are in the ratio 4 : 5 : 3. What is the ratio of the weights of the substances composing the mixture?

Suppose the volumes are 5, 6, and 8 units of volumes respectively. Also that the weights of 1 of these units of volumes are 4, 5, and 3 units of weights respectively.

Then in mixture there are 5 × 4 units of wt. of 1st substance

" " 6 × 5 " 2nd "

" " 8 × 3 " 3rd "

∴ Ratio of weights is 20 : 30 : 24, *i.e.*, 10 : 15 : 12

Note 1.—The specific gravity of a substance is the ratio of the weight of a volume of the substance to the weight of an equal volume of water.

64. SIMPLE PROPORTION.

Suppose that a man walks uniformly at the rate of 4 miles an hour. Then if any number of miles be considered, there is a corresponding number of hours to conform with the given speed.

Let $m \equiv$ number of miles and $h \equiv$ number of hours; and suppose m_1, h_1; m_2, h_2; m_3, h_3, etc., denote particular corresponding values of m and h respectively.

m	h
If $m_1 = 3$	$h_1 = \frac{3}{4}$
„ $m_2 = 2\frac{1}{2}$	$h_2 = \frac{5}{8}$
„ $m_3 = 7$	$h_3 = 1\frac{3}{4}$
„ $m_4 = 5\frac{1}{3}$	$h_4 = 1\frac{1}{3}$
. . . . . .	. . . .

By giving particular values to m corresponding values of h are obtained and sets of corresponding values tabulated.

Now $\dfrac{m_1}{m_2} = \dfrac{3}{2\frac{1}{2}} = \dfrac{6}{5}$

and $\dfrac{h_1}{h_2} = \dfrac{\frac{3}{4}}{\frac{5}{8}} = \dfrac{6}{5} \quad \therefore \dfrac{m_1}{m_2} = \dfrac{h_1}{h_2}$

Again $\dfrac{m_3}{m_2} = \dfrac{7}{2\frac{1}{2}} = \dfrac{14}{5}$ and $\dfrac{h_3}{h_2} = \dfrac{1\frac{3}{4}}{\frac{5}{8}} = \dfrac{14}{5} \quad \therefore \dfrac{m_3}{m_2} = \dfrac{h_3}{h_2}$

Also $\dfrac{m_2}{m_4} = \dfrac{2\frac{1}{2}}{5\frac{1}{3}} = \dfrac{15}{32}$ and $\dfrac{h_2}{h_4} = \dfrac{\frac{5}{8}}{1\frac{1}{3}} = \dfrac{15}{32} \quad \therefore \dfrac{m_2}{m_4} = \dfrac{h_2}{h_4}$ and so on.

It is seen that the ratio of any two values of m is equal to the ratio of the corresponding two values of h. As this is the case, the sets of quantities m and h are said to be proportional to one another.

Two sets of quantities are proportional to one another, when the ratio of any two values of the one is equal to the ratio of the corresponding two values of the other.

If a and b denote two sets of quantities and a is proportional to b, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}.$$

Note 2.—In practice, to find whether one quantity is proportional to another, change one quantity in the simple ratios 1 : 2, 1 : 3, and see whether the other quantity is changed in the same ratios.

Note 3.—If two quantities are equal and each be multiplied by the same number, the results will obviously be equal. Thus, if $\dfrac{x}{1\cdot7} = \dfrac{3\cdot5}{9\cdot2}$ and $\dfrac{x}{1\cdot7}$ and $\dfrac{3\cdot5}{9\cdot2}$ **be multiplied by 1·7, it is clear that** $x = \dfrac{3\cdot5 \times 1\cdot7}{9\cdot2}$

Example (vi)—

If a square carpet, side $3\frac{1}{2}$ yd., cost £5 5s., what should be the price of a carpet 4 yd. 9 in. by 3 yd. 2 ft. 3 in. at the same price per square yard ?

Let x shillings be the cost of $4\frac{1}{4} \times 3\frac{3}{4}$ sq. yd. of carpet, then

$$\frac{x}{105} = \frac{4\frac{1}{4} \times 3\frac{3}{4}}{3\frac{1}{2} \times 3\frac{1}{2}}$$

$$\therefore x = \frac{\overset{15}{\cancel{105}} \times 17 \times 15 \times \cancel{2} \times \cancel{2}}{4 \times \cancel{4} \times \cancel{7} \times 7}$$

$$= 136\tfrac{17}{28}$$

Ansr.—£6 16s. $7\frac{2}{7}$d.

EXAMPLE (vii)—

A bankrupt owed a creditor £875, but the latter received only £492 3s. 9d. How much should be paid to a second creditor to whom the bankrupt owed £384 ?

Let £x be the amount paid to second creditor, then

$$\frac{x}{492\frac{3}{16}} = \frac{384}{875}$$

$$\therefore x = \frac{\overset{9}{\cancel{7875}} \times \overset{24}{\cancel{384}}}{\cancel{16} \times \cancel{875}}$$

$$= 216$$

Ansr.—£216.

65. INVERSE PROPORTION.

Suppose two ports are 120 miles apart, and that a number of ships travel from one to the other at uniform speeds. Then if any number of miles per hour be considered, there is a corresponding number of hours to conform with the given distance.

Let $s \equiv$ number of miles per hour and $t \equiv$ number of hours; and suppose s_1, t_1; s_2, t_2; s_3, t_3, etc., denote particular corresponding values of s and t respectively.

s	t
If $s_1 = 10$	$t_1 = 12$
,, $s_2 = 14$	$t_2 = 8\frac{4}{7}$
,, $s_3 = 16$	$t_3 = 7\frac{1}{2}$
,, $s_4 = 17\frac{1}{2}$	$t_4 = 6\frac{6}{7}$
,, $s_5 = 18$	$t_5 = 6\frac{2}{3}$
. . . . .	. . . .

By giving particular values to s, corresponding values of t are obtained and the results tabulated.

Now $\dfrac{s_1}{s_4} = \dfrac{10}{17\frac{1}{2}} = \dfrac{20}{35} = \dfrac{4}{7}$

and $\dfrac{t_1}{t_4} = \dfrac{12}{6\frac{6}{7}} = \dfrac{12 \times 7}{48} = \dfrac{7}{4} \therefore \dfrac{s_1}{s_4} = \dfrac{t_4}{t_1}$

Again $\dfrac{s_5}{s_3} = \dfrac{18}{16} = \dfrac{9}{8}$ and $\dfrac{t_5}{t_3} = \dfrac{6\frac{2}{3}}{7\frac{1}{2}} = \dfrac{40}{45} = \dfrac{8}{9} \quad \therefore \dfrac{s_5}{s_3} = \dfrac{t_3}{t_5}$

Also $\dfrac{s_2}{s_4} = \dfrac{14}{17\frac{1}{2}} = \dfrac{28}{35} = \dfrac{4}{5}$ and $\dfrac{t_2}{t_4} = \dfrac{8\frac{4}{7}}{6\frac{6}{7}} = \dfrac{60}{48} = \dfrac{5}{4} \quad \therefore \dfrac{s_2}{s_4} = \dfrac{t_4}{t_2}$ and so on.

It is seen that the ratio of any two values of s is equal to the reciprocal of the ratio of the corresponding two values of t.

As this is the case, s and t are said to be inversely proportional to one another.

Two sets of quantities are inversely proportional when the ratio of any two values of the one is equal to the reciprocal of the ratio of the corresponding two values of the other.

If a and b denote two sets of quantities, and a is inversely proportional to b, then

$$\frac{a_1}{a_2} = \frac{b_2}{b_1}.$$

NOTE 4.—In practice, to find whether one quantity is inversely proportional to another, change one quantity in the simple ratios 1 : 2, 1 : 3, and see whether the other is changed in the ratio 2 : 1, 3 : 1 respectively.

EXAMPLE (viii)—

In a building, 84 gas-burners were used when the cost of gas was 2s. 2d. per 1,000 cub. ft. If the cost per 1,000 cub. ft. were to be increased by 2d., how many gas-burners should be used in order that the total cost should be the same?

Let $x \equiv$ no. of burners used when gas is 2s. 4d. per 1,000 cub. ft., then

$$\frac{x}{84} = \frac{26}{28}$$

$$\therefore\ x = \frac{\overset{3}{\cancel{84}} \times 26}{\cancel{28}} = 78.$$

Ansr.—78 burners.

EXAMPLE (ix)—

To print a novel it is found that if 42 lines are printed per page, 635 pages are required. What is the least number of lines per page in order that the number of pages shall not exceed 400?

Let $x \equiv$ no. of lines per page when no. of pages is 400; then

$$\frac{x}{42} = \frac{635}{400}$$

$$\therefore\ x = \frac{\overset{21}{\cancel{42}} \times \overset{127}{\cancel{635}}}{40\cancel{0}}$$

$$= 66\tfrac{27}{40}$$

Ansr.—67 lines.

66. VARIATION.

Consider a uniform sheet of metal which is such that 1 sq. in. weighs $\frac{1}{2}$ oz. Then if the number of inches in the side of a square be given, there will be a corresponding number of ounces giving the weight of the square of the given side.

Let $x \equiv$ number of inches in side of square and $y \equiv$ number of ounces in weight of the square; and suppose that x_1, y_1; x_2, y_2, x_3, y_3; etc., denote particular corresponding values of x and y respectively.

x	y
If $x_1 = 2$	$y_1 = 2$
,, $x_2 = 3$	$y_2 = 4\frac{1}{2}$
,, $x_3 = 5$	$y_3 = 12\frac{1}{2}$
,, x_4 $4\frac{1}{2}$	$y_4 = 10\frac{1}{8}$
. . . . .	. . . . .

By giving particular values to x, corresponding values of y are obtained and the results tabulated.

Now $\frac{x_1}{x_2} = \frac{2}{3}$ and $\frac{y_1}{y_2} = \frac{2}{4\frac{1}{2}} = \frac{4}{9}$

$\therefore \frac{y_1}{y_2} = \frac{x_1^2}{x_2^2}$ or $\frac{x_1}{x_2} = \frac{\sqrt{y_1}}{\sqrt{y_2}}$

Again $\frac{x_2}{x_3} = \frac{3}{5}$ and $\frac{y_2}{y_3} = \frac{4\frac{1}{2}}{12\frac{1}{2}} = \frac{9}{25}$ $\quad\therefore \frac{y_2}{y_3} = \frac{x_2^2}{x_3^2}$ or $\frac{x_2}{x_3} = \frac{\sqrt{y_2}}{\sqrt{y_3}}$

Also $\frac{x_4}{x_1} = \frac{4\frac{1}{2}}{2} = \frac{9}{4}$ and $\frac{y_4}{y_1} = \frac{10\frac{1}{8}}{2} = \frac{81}{16}$ $\quad\therefore \frac{y_4}{y_1} = \frac{x_4^2}{x_1^2}$ or $\frac{x_4}{x_1} = \frac{\sqrt{y_4}}{\sqrt{y_1}}$

It is seen that the ratio of any two values of y is equal to the ratio of the squares of the corresponding two values of x. As this is the case, y is said to be proportional to the square of x or, in other words, y varies as the square of x. The symbol $\propto$ denotes "varies as," so that the latter statement can be written $y \propto x^2$ Similarly, $x \propto \sqrt{y}$.

Similarly, $a \propto \frac{1}{b}$ denotes a varies inversely as (or is inversely proportional to) b and $a \propto \frac{1}{b^2}$ denotes a varies inversely as the square of b, etc.

(1) If $a \propto b$, then $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ $\qquad$ (5) If $a \propto \sqrt{b}$, then $\frac{a_1}{a_2} = \frac{\sqrt{b_1}}{\sqrt{b_2}}$

(2) ,, $a \propto \frac{1}{b}$, ,, $\frac{a_1}{a_2} = \frac{b_2}{b_1}$ $\qquad$ (6) ,, $a \propto \frac{1}{\sqrt{b}}$, ,, $\frac{a_1}{a_2} = \frac{\sqrt{b_2}}{\sqrt{b_1}}$

(3) ,, $a \propto b^2$, ,, $\frac{a_1}{a_2} = \frac{b_1^2}{b_2^2}$ $\qquad$ (7) ,, $a \propto b^3$, ,, $\frac{a_1}{a_2} = \frac{b_1^3}{b_2^3}$

(4) ,, $a \propto \frac{1}{b^2}$, ,, $\frac{a_1}{a_2} = \frac{b_2^2}{b_1^2}$ $\qquad$ (8) ,, $a \propto \frac{1}{b^3}$,, $\frac{a_1}{a_2} = \frac{b_2^3}{b_1^3}$

Example (x)—

Given that the number of gallons supplied per minute by two pipes is proportional to the square of their diameters, find the time taken to fill a reservoir with water from two pipes whose diameters are 13·5 in. and 2 ft. respectively, if the second alone can fill it in 2 hr. 50 min.

The time taken to fill the cistern is inversely proportional to the square of the diameter of the supply pipe.

Let x minutes be the time to fill the reservoir by the pipe, 13·5″ diameter, alone;

$$\text{then } \frac{x}{170} = \frac{24 \times 24}{13{\cdot}5 \times 13{\cdot}5}$$

$$\therefore \; x = \frac{170 \times \overset{8}{\cancel{24}} \times \overset{8}{\cancel{24}} \times 4}{\underset{9}{\cancel{27}} \times \underset{9}{\cancel{27}}}$$

$\therefore$ In 1 min., both taps together fill $\frac{1}{170} + \frac{81}{170 \times 256}$ of the reservoir

i.e., $\frac{256 + 81}{170 \times 256}$ " "

$\therefore$ To fill the reservoir, they take $\frac{170 \times 256}{337}$ min.

$= 129\frac{47}{337}$ " "

Ansr.—2 hr. 9 min. to nearest minute.

EXAMPLE (xi)—

The weight of a sphere is proportional to the cube of the radius. A sphere of metal, radius 1·4″ weighs 3 lb. 2¾ oz.; find the weight of a sphere of the same material, radius 1″.

Let x oz. $\equiv$ wt. of sphere, radius 1″,

$$\text{Then } \frac{x}{50\frac{3}{4}} = \frac{1 \times 1 \times 1}{1{\cdot}4 \times 1{\cdot}4 \times 1{\cdot}4}$$

$$\therefore \; x = \frac{\overset{29}{\cancel{203}}}{4 \times \underset{\cdot 2}{\cancel{1{\cdot}4}} \times 1{\cdot}4 \times 1{\cdot}4} = 18{\cdot}5, \text{ correct to 1 dec. place}$$

Ansr.—1 lb. 2·5 oz.

67. COMPOUND PROPORTION.

Let l, b, and h denote the length, breadth, and height in inches of a rectangular prism of a substance, 1 cub. in. of which weighs k ounces, and let W denote the weight of the prism. Then $W = klbh$. Keeping b and h the same, it is clear that W is proportional to l, for if the latter be increased in any ratio, W is increased in the same ratio. Similarly, if l and h are constant, W is proportional to b, and keeping l and b constant, W is proportional to h. W is thus proportional to l, b, and h.

Suppose that when the dimensions are l_1, b_1, and h_1, the weight is W_1, then $W_1 = kl_1b_1h_1$: also that when the dimensions are l_2, b_2, and h_2, the weight is W_2, then $W_2 = kl_2b_2h_2$. Thus—

$$\frac{W_1}{W_2} = \frac{kl_1b_1h_1}{kl_2b_2h_2} = \frac{l_1}{l_2} \times \frac{b_1}{b_2} \times \frac{h_1}{h_2}$$

which shows that the ratio of any two values of W equals the product of the corresponding ratios of l, b, and h.

Suppose that $Q = k\,\frac{xy}{z^2}$, where k is a constant : then Q depends for its value on the values given to x, y, and z. It is clear that Q is proportional to x, proportional to y, and inversely proportional to the square of z. Suppose that when $x = x_1$, $y = y_1$, and $z = z_1$, $Q = Q_1$, and that when $x = x_2$, $y = y_2$, and $z = z_2$, $Q = Q_2$, then

$$Q_1 = \frac{kx_1y_1}{z_1^2} \text{ and } Q_2 = \frac{kx_2y_2}{z_2^2}$$

$$\text{Thus } \frac{Q_1}{Q_2} = \frac{\dfrac{kx_1y_1}{z_1^2}}{\dfrac{kx_2y_2}{x_2^2}} = \frac{kx_1y_1z_2^2}{kx_2y_2z_1^2} = \frac{x_1}{x_2} \times \frac{y_1}{y_2} \times \frac{z_2^2}{z_1^2}.$$

Briefly this result can be stated as follows—

$$\text{If } Q \propto \frac{xy}{z_2}, \text{ then } \frac{Q_1}{Q_2} = \frac{x_1}{x_2} \times \frac{y_1}{y_2} \times \frac{z_2^2}{z_1^2}$$

Similarly, if a quantity R varies inversely as a and directly as the cube of b,

$$i.e., R \propto \frac{b^3}{a}, \text{ then } \frac{R_1}{R_2} = \frac{a_2}{a_1} \times \frac{b_1^3}{b_2^3}$$

EXAMPLE (XII)—

If 5 men can plant 675 rows of potatoes in 3 days of 9 hours each, how many women will be required to plant 1,500 rows in 5 days of 8 hours, if 4 women can do as much as 3 men in the same time?

The number of men is proportional to the number of rows, inversely proportional to the number of days, and inversely proportional to the number of hours per day.

Let $x \equiv$ no. of *men* required for 1,500 rows, in 5 days of 8 hours.

$$\text{Then } \frac{x}{5} = \frac{1500}{675} \times \frac{3}{5} \times \frac{9}{8}$$

$$\therefore x = \frac{\overset{\overset{15}{\cancel{60}}}{\cancel{1500}} \times \cancel{3} \times \cancel{9} \times \cancel{5}}{\underset{\underset{\cancel{25}}{\cancel{75}}}{\cancel{675}} \times \cancel{5} \times \underset{2}{\cancel{8}}} = \frac{15}{2}$$

$$\therefore \text{Number of women required} = \frac{\overset{5}{\cancel{15}}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{3}}$$

$= 10$ *Ansr.*—**10 women.**

EXAMPLE (xiii)—

The electric current passing through a wire is for any constant voltage proportional to the square of the diameter and inversely proportional to the length. If the current passing through 2 miles of wire diameter ·2 in. be 450 amperes, what current will pass through a wire 3 miles 250 yd. of diameter ·35 in.?

Let x amperes $\equiv$ current when wire has length 5,530 yd. and diameter ·35 in.

$$\text{then } \frac{x}{450} = \frac{\cdot 35 \times \cdot 35}{\cdot 2 \times \cdot 2} \times \frac{3520}{5530}$$

$$\therefore x = \frac{\overset{\cdot 05}{\cancel{\cdot 35}} \times \cdot 35 \times \overset{88}{\cancel{3520}} \times 450}{\underset{\cdot 1}{\cancel{\cdot 2}} \times \underset{\cdot 1}{\cancel{\cdot 2}} \times \underset{79}{\cancel{5530}}}$$

$$= \frac{693}{\cdot 79} = 877{\cdot}2 \text{ correct to 1 dec. place.}$$

Ansr.—877·2 ampères.

68. Sometimes in a problem the quantities that occur are such that one quantity is proportional to part of another quantity. For example, if £C is the total cost of a gramophone and n records and £x is the cost of the gramophone, then n is proportional to $(C - x)$.

EXAMPLE (xiv)—

A commercial traveller was in receipt of a fixed sum per annum plus a fixed percentage of the value of orders obtained. In 1929 he obtained orders to the value of £1,750 and his salary was £193 15s. In 1930, the value of his orders amounted to £2,500 and his salary was £212 10s. What was his salary in 1931 when he obtained orders to the value of £4,500?

Income due to orders, value £750 = £18 15s.

Let £x $\equiv$ income due to orders, value £2,000,

$$\text{then } \frac{x}{18\frac{3}{4}} = \frac{2000}{750}$$

$$\therefore x = \frac{75 \times 2000}{4 \times 750} = 50$$

$\therefore$ Income in 1931 = £212 10s. + £50
= £262 10s.

69. PROPORTIONAL PARTS.

Suppose a quantity is to be divided into two portions in the ratio 3 : 5. Then if the quantity be divided into 8 equal parts, the first portion will contain 3 of these parts, and so will be $\frac{3}{8}$ of

the whole quantity. Similarly, the second portion will be $\frac{5}{8}$ of the whole quantity.

Similarly, if N be divided into, say, 3 portions in the ratios $a : b : c$, the three portions will be

$$\frac{aN}{a+b+c}, \frac{bN}{a+b+c} \text{ and } \frac{cN}{a+b+c} \text{ respectively.}$$

EXAMPLE (xv)—

Three men rent a field jointly for £90. One puts in 20 sheep for 25 weeks, another 68 sheep for 20 weeks, and the third 75 sheep, 50 of which are put in for 24 weeks and the remainder for 30 weeks. What should each pay towards the rent?

They should pay rent in the ratios 500 : 1360 : 1200 + 750
i.e., 50 : 136 : 195.

$\therefore$ 1st man's share $= £\frac{50}{381} \times 90 = \underline{£11\ 16s.\ 3d.}$ to nearest penny.

Also 2nd „ „ $= £\frac{136}{381} \times 90 = \underline{£32\ 2s.\ 6d.}$ „ „

$\therefore$ 3rd „ „ $= £90 - £43$ 18s. 9d. $= \underline{£46\ 1s.\ 3d.}$

NOTE 5.—Suppose m lb. of a substance at a shillings per lb. be mixed with n lb. of another substance at b shillings per lb. Suppose a is greater than b; then if x shillings per lb. be the cost of the mixture, x is less than a but greater than b. The amount lost on the dearer substance is $ma - mx = m(a - x)$ shillings, while the amount gained on the cheaper is $nx - nb = n(x - b)$ shillings. Thus, in order that there should be neither gain nor loss, $m(a - x) = n(x - b)$, so that on dividing both sides by $n(a - x)$, $\frac{m}{n} = \frac{x - b}{a - x}$. **Thus the ratio of the quantities to be mixed equals the inverse ratio of the differences of their costs per unit from the cost per unit of the mixture**

EXAMPLE (xvi)—

How many lb. of tea costing 2s. $1\frac{1}{4}$d. per lb. and 1s. $9\frac{1}{2}$d. per lb. respectively must be mixed together in order to obtain 100 lb. of mixture costing 1s. 11d. per lb.?

$1/11 - 1/9\frac{1}{2} = 1\frac{1}{2}$d. and $2/1\frac{1}{4} - 1/11 = 2\frac{1}{4}$d.

$\therefore$ $2/1\frac{1}{4}$d. per lb. tea must be mixed with $1/9\frac{1}{2}$d. per lb. tea in ratio 6 : 9
i.e., 2 : 3

$\therefore$ Amount of tea, $2/1\frac{1}{4}$d. per lb. $= \frac{2}{5} \times 100$ lb. $= \underline{40 \text{ lb.}}$

and „ „ $1/9\frac{1}{2}$d. „ $= \frac{3}{5} \times 100$ lb. $= \underline{60 \text{ lb.}}$

70. PERCENTAGE.

It has been seen that a ratio compares the relative values of two quantities. Now, suppose the ratio of A to B is $\frac{5}{8}$, and the ratio of C to D is $\frac{13}{20}$: it cannot be seen at a glance which ratio is the bigger; but if the ratios be expressed having their

consequents each 100, then by comparing the antecedents the ratios could be at once compared. Thus,

$$\frac{5}{8} = \frac{62\cdot5}{100} \text{ and } \frac{13}{20} = \frac{65}{100},$$

so that the ratio $\frac{13}{20}$ is greater than the ratio $\frac{5}{8}$. **The antecedent of a ratio whose consequent is 100 is called a Percentage.**

Thus, if it be required to evaluate x per cent. (or $x\%$) of a quantity N, the result is $N \times \frac{x}{100}$

Suppose it be required to find what percentage a quantity A units is of a quantity B of the same units.

The ratio of the 1st quantity to the second is $\frac{A}{B}$, so that if

$x \equiv$ percentage, it follows from the definition that $\frac{x}{100} = \frac{A}{B}$;

so that, multiplying both sides by 100, $x = \frac{A}{B} \times 100$.

NOTE 6.—$5\% = \frac{1}{20}$, $2\frac{1}{2}\% = \frac{1}{40}$, $7\frac{1}{2}\% = \frac{3}{40}$, $12\frac{1}{2}\% = \frac{1}{8}$, $15\% = \frac{3}{20}$; so that 5%, $2\frac{1}{2}\%$, $7\frac{1}{2}\%$, $12\frac{1}{2}\%$, and 15% of £1 are 1s., 6d., 1s. 6d., 2s. 6d., and 3s. respectively.

EXAMPLE (xvii)—

A man bought a farm of 90 ac. for £2,418 15s After repairing the buildings, he let the farm at 27s 6d. per acre, thereby getting a return of 3 per cent. on his money. How much did he spend on repairs?

$$\text{Rent received for farm} = \pounds1\tfrac{3}{8} \times 90$$
$$= \pounds\tfrac{495}{4}$$

$\pounds\frac{495}{4}$ is $\frac{3}{100}$ of the total outlay.

$$\therefore \text{ Total outlay } = \frac{\pounds\overset{165}{\cancel{495}} \times 100}{4 \times \cancel{3}}$$
$$= \pounds4,125.$$

$$\therefore \text{ Amount spent on repairs} = \pounds1,706 \text{ 5s.}$$

EXAMPLE (xviii)—

Calculate 7 per cent. of £3 14s 7¼d. to the nearest farthing.

7% of £3 14s. 7¼d. $= \pounds3\cdot730 \times \frac{7}{100} = \pounds\cdot03730 \times 7 = \pounds\cdot2612$
$= 5\text{s. } 2\tfrac{3}{4}\text{d.}$

NOTE 7.—In finding so much per cent. (less than 100%) of a sum of money involving shillings and pence, the shillings and pence should be expressed as a decimal of £1 by inspection.

EXAMPLE (xix)—

A retailer bought a number of articles for £57 13s. 4d. and sold them for £98 16s. 8d. What was his percentage profit? If he based his percentage profit on the selling price, what would be his percentage profit?

$$\text{Profit} = £41\ 3s.\ 4d.$$

$$\therefore \text{ Percentage profit } = \frac{41\frac{1}{6}}{57\frac{2}{3}} \times 100 = \frac{247 \times \cancel{3} \times 100}{\cancel{6} \times 173}_{2}$$

Ans. 71·39%

$$\text{Percentage profit based on selling price } = \frac{41\frac{1}{6}}{98\frac{5}{6}} \times 100 = \frac{247 \times 100}{593}$$

Ans. 41·65%

EXAMPLE (xx)—

A dealer bought an article for 5 guineas. For what must he sell it to gain (1) 15 per cent. on his outlay, (2) 15 per cent. on the returns?

(1) $$\text{Profit} = £\frac{\overset{3}{\cancel{15}}}{\underset{20}{\cancel{100}}} \times \frac{21}{4} = 15/9$$

$\therefore$ Selling price = £6 0s. 9d.

(2) Suppose the selling price be 100 units of money
then the profit is 15 ,, ,,
$\therefore$ buying price is 85 ,, ,,
$\therefore$ the ratio of buying price to selling price is 85 : 100

$$\therefore \text{ Selling price} = £\tfrac{21}{4} \times \tfrac{100}{85}$$
$$= £6\ 3s.\ 6d. \text{ to nearest 1d.}$$

71. To increase a quantity A by $r\%$, $\frac{r}{100}$ of A is obtained and added to the original quantity A. The increased value is, therefore,

$$A + \frac{rA}{100} = A\left(1 + \frac{r}{100}\right) = A \times \frac{100 + r}{100}$$

Similarly, the value obtained by decreasing A by $r\%$ is

$$A \times \frac{100 - r}{100}.$$

For example, a quantity when multiplied by $\frac{107}{100}$ is increased by 7% and a quantity when multiplied by $\frac{93}{100}$ is decreased by 7%.

NOTE 8.—The same result is obtained also as follows: 100 increased by $r\%$ is $100 + r$, so that a quantity to be increased by $r\%$ must be increased in the ratio $100 + r : 100$: i.e., the quantity must be multiplied by $\frac{100 + r}{100}$

EXAMPLE (xxi)—

A manufacturer sells an article to a merchant thereby gaining 5 per cent. profit on his cost price. The merchant sells it to a retailer, thereby gaining 20 per cent. profit on his outlay. The retailer sells it to a customer, thereby gaining 15 per cent. profit on his cost price. How much per cent. is the customer's price greater than the cost price to the manufacturer?

Let cost price to manufacturer $= 100$ units

then „ merchant $= 105$ „

„ „ retailer $= 105 \times \frac{120}{100}$ units

„ „ customer $= 105 \times \frac{120}{100} \times \frac{115}{100}$ units

$= 144{\cdot}9$ units

∴ Increase of cost $= 44{\cdot}9\%$.

EXAMPLE (xxii)—

A man saved 7 per cent. of his income. If his income be increased by $12\frac{1}{2}$ per cent. and he saves the same amount as before, by how much per cent. does his expenditure increase?

Let his original income be 100 units

Then the amount saved $=$ 7 „

∴ his expenditure $=$ 93 „

Increase of expenditure $= 12\frac{1}{2}$ „

∴ Increase per cent. $= \dfrac{12\frac{1}{2}}{93} \times 100$

Ans. $13{\cdot}44\%$.

EXAMPLE (xxiii)—

By what per cent. would the cost of a square carpet be above or below that of a second carpet if the side of the former is 8 per cent. greater than that of the latter, but the cost per square yard 20 per cent. less than that of the latter?

Let cost of second carpet $= 100$ units

Then „ first „ $= 100 \times \frac{108}{100} \times \frac{108}{100} \times \frac{80}{100}$ units

$= 93{\cdot}312$ units

∴ Cost of first carpet is $6{\cdot}688\%$ less than that of second carpet.

EXAMPLE (xxiv)—

A retailer bought certain articles at 18s. 9d. each and sold them for £1 4s. Owing to taxation, the cost price is increased 2s. per article. By how much to the nearest halfpenny should he increase the retail price so as to gain the same percentage profit on his outlay?

The additional outlay of 2/– should give a return of $\dfrac{2 \times 24}{18\frac{3}{4}}$ shillings

in order for the percentage profit to remain the same,

∴ Increase of retail price $= 2\frac{14}{25}$ shillings

Ans. $2/6\frac{1}{2}$.

72. SIMPLE INTEREST.

If an amount of money be deposited at a bank, at the end of a year a certain percentage of this amount, called "the interest," is paid to the person in whose name the money, or principal, is deposited. If the principal remains for a number of years and the interest be paid out (not added to the principal) at the end of each year, and if the rate of interest remain the same, the yearly interest will remain the same. If the principal be deposited only for a fraction of a year, the amount of interest is this fraction of the interest that would accrue if the principal remained deposited for a complete year. The interest may be looked upon as a payment made by the bank for the advantage of using the money deposited.

Let £P = principal

" £I = total interest for n years at r% per annum.

Then $\frac{r}{100}$ = interest on £1 for 1 year

$\therefore \frac{Pr}{100} =$ " £P "

$\therefore I = \frac{Prn}{100}$

Multiplying both sides of formula by 100, it is seen that $100I = Prn$ and, dividing by rn, Pn, and Pr respectively, the formulae

$$P = \frac{100I}{rn},\ r = \frac{100I}{Pn},\ n = \frac{100I}{Pr} \text{ are established.}$$

EXAMPLE (xxv)—

Find the simple interest on £325 16s. for the period 10th April to 7th July at $4\frac{1}{2}$ per cent. per annum.

Number of days = 88

$$\therefore \text{Interest} = £\frac{325\frac{4}{5} \times 4\frac{1}{2} \times \frac{88}{365}}{100}$$

$$£\frac{1629 \times 9 \times 88}{5 \times 2 \times 365 \times 100}$$

$$= £3\ 10\text{s.}\ 8\text{d.}$$

EXAMPLE (xxvi)—

For a loan of 60,000 francs for 3 months, interest to the value of 750 francs was charged. What was the rate per cent. per annum?

$$\text{Rate per cent. per annum} = \frac{100 \times 750 \times 4}{60,000}$$

Ans. 5% per annum.

EXAMPLE (xxvii)—

What sum now is equivalent to £750 in 95 days' time, if the rate of interest be $4\frac{1}{2}$ per cent. per annum ?

$$\text{In 95 days at } 4\tfrac{1}{2}\% \text{ per annum, £100 amounts to £100} + £\frac{9 \times 95}{2 \times 365}$$

$$\textit{i.e.}, \text{ to £}101\tfrac{25}{146}$$

$$\therefore \text{ Equivalent sum at present} = £\frac{750 \times 100}{101\frac{25}{146}}$$

$$= £\frac{75000 \times 146}{14771}$$

$$= £741 \text{ 6s. 4d.}$$

NOTE 9—The ratio of the present value to the value in 95 days time is $100 : 101\frac{25}{146}$, so that to obtain the present value the amount is reduced in this ratio.

EXAMPLE (xxviii)—

£12,000 is borrowed on 4th June: until 10th August, the rate of interest is 4 per cent.; from 11th August to 25th September, $4\frac{1}{2}$ per cent.; and $4\frac{3}{4}$ per cent. from 26th September to 18th December, when the money is repaid. What is the total interest ?

$$\text{Interest for 1st period} = £120 \times 4 \times \tfrac{67}{365}$$

$$\text{,, 2nd ,,} = £120 \times \tfrac{9}{2} \times \tfrac{46}{365}$$

$$\text{,, 3rd ,,} = £120 \times \tfrac{19}{4} \times \tfrac{84}{365}$$

$$\therefore \text{ Total interest} = £\tfrac{120}{365} \times (268 + 207 + 399)$$

$$= \frac{£120 \times 874}{365}$$

$$= £287 \text{ 6s. 10d.}$$

73. When a considerable number of calculations require to be performed in order to find the interest, it is advisable to construct a table giving the interest expressed as a decimal of a £ on £1 at 1% per annum for one up to nine days. Thus, supposing that the greatest sum involved is £70,000, that the greatest rate is 7% per annum, and the number of days does not exceed 200, then as the interest on £1 for 1 day at 1% per annum will have to be multiplied by a number not exceeding $70{,}000 \times 200 \times 7$ (*i.e.*, by 98,000,000), and as the result must be correct to 3 decimal places in order to obtain the interest to the nearest penny, the value of the interest on £1 for 1 day at 1% per annum must be correct to 11 decimal places. As the use of the table will involve a considerable amount of addition, two additional places should be used.

The table then will be as shown—

No. of days	Interest on £1 at 1% per annum.
	£
1	·0000273972603
2	·0000547945205
3	·0000821917808
4	·0001095890411
5	·0001369863014
6	·0001643835616
7	·0001917808219
8	·0002191780822
9	·0002465753425

The right-hand column also gives the interest on £1 to £9 for 1 day at 1 per cent. per annum.

EXAMPLE (xxix)—

Find the interest on £65,000 for 137 days at 4½ per cent. per annum.

	Interest on	£1	for	100 days	at	1 %	per annum	=	£·00273972603
	,,	£1	,,	30	,,	1 %	,,	=	£·00082191781
	,,	£1	,,	7	,,	1 %	,,	=	£·00019178082
	,,	£1	,,	137	,,	1 %	,,	=	£·00375342466
∴	,,	£1	,,	137	,,	4 %	,,	=	£·01501369864
∴	,,	£1	,,	137	,,	½%	,,	=	£·00187671233
	,,	£1	,,	137	,,	4½%	,,	=	£·01689041097
	,,	£60,000	,,	137	,,	4½%	,,	=	£1013·4246582
	,,	£5,000	,,	137	,,	4½%	,,	=	£ 84·4520548
	,,	£65,000	,,	137	,,	4½%	,,	=	£1097·8767
								=	£1,097 17s. 6d.

EXAMPLE (xxx)—

Find the interest on £97 16s. for 60 days at 5¼ per cent. per annum.

Interest on	£90	for	1 day	at	1 %	per annum	=	£·002465753
,,	£ 7	,,	1	,, ,,	1 %	,,	=	£·000191781
,,	£ ·8	,,	1	,, ,,	1 %	,,	=	£·000021918
,,	£97·8	,,	1	,, ,,	1 %	,,	=	£·00267945
,,	£97·8	,,	60 days	,,	1 %	,,	=	£·160767
,,	£97·8	,,	60	,, ,,	5 %	,,	=	£·803835
,,	£97·8	,,	60	,, ,,	¼%	,,	=	£·040192
,,	£97·8	,,	60	,, ,,	5¼%	,,	=	£·8440
							=	16/11

74. COMPOUND INTEREST.

If money is deposited on the understanding that each instalment of interest as it becomes due shall be added to the principal, so that the principal for any period of time is greater than that of the former period, then the difference between the initial sum of money and the amount after a number of years is called the **Compound Interest** on the former.

For example, suppose £100 be placed out at compound interest at 5% per annum payable yearly. At the end of the 1st year the interest is £5, so that the principal during the 2nd year is £105. At the end of the 2nd year the interest is £5 5s., so that the principal during the 3rd year is £110 5s. If the interest were 5% per annum payable half-yearly, at the end of the 1st half-year the interest would be £2 10s., so that the principal during the 2nd half-year would be £102 10s. At the end of the 2nd half-year the interest would be £2 11s. 3d., so that the principal for the 3rd half-year would be £105 1s. 3d. and so on. Thus, the interest for the complete year is £5 1s. 3d. as against £5 in the case when the interest is payable yearly.

Let $£P \equiv$ principal at beginning.
,, $£i \equiv$ interest on £1 for 1 year, payable yearly.
,, $n \equiv$ number of years.
,, $£A \equiv$ amount after n years.

Then, as during each year £1 amounts to $£(1 + i)$,

The amount at end of 1st year $= £P(1 + i)$
,, ,, 2nd ,, $= £P(1 + i) \times (1 + i) = £P(1 + i)^2$
,, ,, nth ,, $= £P(1 + i)^n$
thus $A = P(1 + i)^n$

If the interest were payable half-yearly, the number of periods would be $2n$, and during each period £1 would amount to $£\left(1+\frac{i}{2}\right)$ so that the amount after n years would be obtained by multiplying the initial principal by $\left(1+\frac{i}{2}\right)^{2n}$.

The compound interest on £575 for 4 years at 3½% per annum payable yearly, would be given by $£575(1{\cdot}035)^4 - £575$; but unless logarithms are to be used, it is better for the calculations to be made step by step as shown by the following examples.

EXAMPLE (xxxi)—

Find the compound interest on £715 15s. 3d. for $3\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. per annum payable yearly

		£
Principal, 1st year	=	715·76250
Interest, 1st ,,	=	28·63050 3·57881
Principal, 2nd ,,	=	747·97181
Interest, 2nd ,,	=	29·91887 3·73986
Principal, 3rd ,,	=	781·63054
Interest, 3rd ,,	=	31·26522 3·90815
Principal, 4th ,,	=	816·80391
Interest next half-year	=	16·33608 2·04201
Amount after $3\frac{1}{2}$ years	=	835·18200
Principal, 1st year	=	715·76250
∴ Compound interest	=	119·41950
	=	£119 8s. 5d.

NOTE 10.—To obtain 4% of the principal, the decimal point is moved two places to the left and the result is multiplied by 4, the decimal figures beyond the fifth place being omitted, but due regard is paid to the carrying figure: $\frac{1}{2}$% is obtained by moving the decimal point two places to the left and dividing by 2.

EXAMPLE (xxxii)—

What sum will amount to £745 16s. in 2 years at 5 per cent. per annum, compound interest, payable half-yearly ?

		£
Let principal, 1st half-year	=	1
then interest, 1st ,,	=	·025
,, principal, 2nd ,,	=	1·025
,, interest, 2nd ,,	=	·025625
,, principal, 3rd ,,	=	1·050625
,, interest, 3rd ,,	=	·026265625
,, principal, 4th ,,	=	1·076890625
,, interest, 4th ,,	=	·026922266
,, amount after 2 years	=	1·1038129

The present value will be obtained by decreasing £745·8 in ratio 1 : 1·1038129

$$\therefore \text{ Present value} = £\frac{745{\cdot}8}{1{\cdot}1038129} = £675 \text{ 13s. 2d. to nearest penny.}$$

NOTE 11.—To obtain result correct to nearest penny, three places of decimals will be required, so that the result in £'s must obviously be correct to 6 significant figures; therefore the divisor, using contracted method of division, should be correct to 8 significant figures.

TEST EXERCISES, I, 7.

(1) A train travelled 2 miles in 2 min. 20 sec.; a motor-car travelled 3 miles 740 yd. in 4 min. Find the ratio of their speeds.

(2) A earned 1s. 1d. per hour and worked 48 hours per week, but lost time during the year amounting to 65 hr. B earned £120 per annum. Find the ratio of their earnings for the year.

(3) Find the ratio of the speed of a train and a destroyer, if the former travels at 60 miles per hour and the latter at 35 knots (1 knot = 6,080 ft. per hour).

(4) Two farmers own land rectangular in shape. That of the first is 8 ch. 73 lk. long and 3 ch. 45 lk. wide, while that of the second is 7 ch. 12 lk. long and 4 ch. 92 lk. wide. Find the ratio of the areas of the land.

(5) The size of foolscap paper is $17'' \times 13\frac{1}{2}''$ and that of crown paper $20'' \times 15''$. What is the ratio of the areas of 1 page of each?

(6) A photograph, whose whole dimensions are $7\frac{1}{2}'' \times 4\frac{1}{2}''$, is enlarged, so that the length is 1 foot. What will be the enlarged breadth? and what will be the ratio of the areas of the photograph?

(7) A and B, whose salaries were £240 and £150, were given an increase, after which A received £265. If B's salary was increased in the same ratio as A's, what should be B's increased salary?

(8) A retailer decreased the profit on each of a certain article in the ratio 9 : 10 and found that the number of articles sold per week increased in the ratio 6 : 5. By what ratio did his weekly profit increase or decrease?

(9) Bronze is an alloy of copper, tin, and zinc in the ratio of 95 : 4 : 1. What weight of tin and zinc must be used with 1 ton of copper to form bronze, and how many pennies could be made with the bronze, the weight of 1 penny being $\frac{1}{3}$ oz. Avoirdupois?

(10) In 1931 the population in England and Wales was: Males, 19,138,844; and females, 20,809,087. What was the ratio of the number of females to the number of males? Assuming the ratio to be the same in Scotland, calculate the number of females to nearest thousand in Scotland, if the number of males was 2,325,867.

(11) In 1929, the amount of wheat imported into Great Britain and Northern Ireland from the Colonies was 40,285,347 cwt., and that from foreign countries was 71,482,051 cwt. Find the ratio of the amount of wheat obtained from the colonies to that obtained from foreign countries.

(12) Three steamers travel at 10, 12, and 15 knots respectively. Show that the times taken by them to travel a certain distance are in the ratio 6 : 5 : 4.

(13) The weights of equal volumes of three substances are in the ratios 3 : 4 : 5. What are the ratios of the weights of cubes of these substances, if the lengths of the edges are in the ratios 4 : 3 : 2?

(14) A bankrupt paid £145 to a creditor to whom he owed £204 9s. What should be paid to a creditor to whom he owes £973 5s.? If he paid another creditor £235 10s., how much did he owe him?

(15) If 150 yd. of cloth 21 in. wide were bought for £13 10s., what should be paid for 90 yd. of the same kind of cloth 27 in. wide?

(16) A man was engaged on 5th March at a salary of £260 per annum. If he left on 20th August of the same year: what sum should he have received?

(17) If the freight on goods weighing 1 ton 13 cwt. 3 qr. be £1 3s. 3d., what would be the cost of carriage of 4 tons 1 cwt. for the same distance?

(18) A field of 4 ac. 5 sq. ch. 88 sq. yd. is rented at £15 4s., what should be the rent of a field whose area is 3 ac. 2 sq. ch. 150 sq. yd.? Also what area could be rented for £20?

(19) 148·75 tons of tin oxide valued at £16,832 were obtained by crushing 14,472 tons of ore. How many more tons of ore must be crushed to obtain 200 tons of oxide altogether, and what would be the value?

(20) A ship travelling at 22·3 knots took 3 dy. 14 hr. to steam a certain distance. How long would another ship take to steam the same distance at 17·7 knots? What speed must be attained to perform the journey in 3 dy. 6 hr.?

(21) 5 ton 14 cwt. of coal cost £7 12s. 6d. What is the cost of 25,000 Kgm. at the same rate in francs if £1 is equivalent to 89·64 francs? Also what weight in kilograms could be bought for 10,000 francs?

(22) A farmer has sufficient food for 15 cows for 50 days. After 14 days, 4 cows are sold. For how many more days would the food last for the remaining cows?

(23) If a farmer laid $1\frac{3}{4}$ tons of lime on an acre of land, how many kilograms is this per are? (Assume 1 metre = $39\frac{3}{8}$ in.; 1 kgm. = $2\frac{1}{5}$ lb.)

(24) If 980 square tiles of side $5\frac{1}{2}''$ are required to pave a courtyard, how many square tiles of side $7''$ would be required to pave it?

(25) If 5 men could mow $33\frac{1}{3}$ ac. in 8 days, how long would 472 men take to mow 1 square mile? Also how many acres could be mown by 18 men in 7 days?

(26) A farmer found that 9 horses consumed 140 bush of oats in a week. What quantity of oats would be sufficient for 15 horses for 18 days? Also how long would 20 horses take to consume 350 bush. of oats?

(27) The wages of a boy, woman, and a man are in the ratio 3 : 5 : 8 The total wages of 15 men, 20 women, and 6 boys was £83 6s. What would be the total wages of 13 men, 24 women, and 8 boys?

(28) A contractor undertakes to complete a certain amount of work by Saturday, 28th November. He starts on Monday, 11th May, and employs 35 men, who work on week days $9\frac{1}{2}$ hr, except Saturday, when 5 hrs'. work is done. By 12th September only half the work is accomplished. If the men work an additional hour every day, how many more men should be engaged in order to fulfil the contract?

(29) The proprietor of a boarding-house charged £23 2s. for boarding and lodging a party of 11 people for 8 days. What should be the bill in the case of a party of 10 people who stayed for 22 days?

(30) The gas for 7 burners, 5 hr. per day for 20 days, was 8s. 3d. For how many hours and minutes every day may 10 burners be lighted for 31 days for 12s.?

(31) The number of cylindrical tins required to contain a quantity of powder is inversely proportional to the height of the tin and inversely proportional to the square of the diameter. 350 cylindrical tins, height $7\frac{1}{4}''$, diameter $4\frac{1}{2}''$, are required for a certain amount of a powder. How many tins, height $8''$, diameter $5''$, would be required for the same amount of the powder?

(32) An engine while driving machinery burns coal at the rate of $1\frac{3}{4}$ ton in 8 hr. When the machinery is not in motion the consumption of coal is $\frac{5}{12}$ of this rate. How much coal would the engine consume in 1,635 hr. during $\frac{1}{8}$ of which the machinery is not in motion?

(33) For 25 days a shopkeeper used 15 electric lights of 32 candle-power for an average of $4\frac{1}{2}$ hr. per day. For the next 27 days he desires to use 12 lights of 50 candle-power, so that the total cost is the same as during the first period. What is the average duration of time per day that the lamps should be glowing, assuming the cost of light per hour for a lamp is proportional to its candle-power?

(34) If the cost of conveying 250 tons of merchandise for 76 miles be £66 10s., what would it cost to convey 145 tons for 112 miles (*a*) at the same rate, (*b*) if the rate be increased by 20 per cent.?

(35) (*a*) What length of copper wire, diameter ·15 in., has the same weight as 1 mile of copper wire, diameter ·22 in.?

(*b*) What diameter should copper wire be if its length is to be 1,600 yd. and its weight the same as above?

(When weights are equal, the length is inversely proportional to the square of the diameter; that is, the diameter is inversely proportional to the square root of the length.)

(36) a varies directly as b, and inversely as the square of c. When $b = 1 \cdot 15$ and $c = 2 \cdot 3$, $a = 4 \cdot 91$. Find the value of a when $b = 4$ and $c = 3 \cdot 2$.

(37) When the price of butter is 1s. 10d. per lb., find the cost in francs of 22 Kg. Also if the price per lb. be increased by 1½d., how much less butter can be bought for the same number of francs? (1 lb. = 453·6 gm.; 90 francs = £1.)

(38) A tank measuring 9″ by 11″ by 18″ is to be filled with a liquid which is to be sold at 15·60 francs per litre. Find the cost of the liquid to the nearest shilling, assuming that £1 = 90 francs, 1 in. = 2·54 cm.

(39) Find the dividend at the rate of 11s. 10½d. in the £ on (*a*) 74 francs 35 centimes, (*b*) 522 dollars 40 cents.

(40) Using the data of Question (9), find the value of metal contained in 240 pennies, if copper cost £66 per ton, tin £170, and zinc £25.

(41) If 8,450 francs earn 65 francs 40 centimes in 2 months, what sum will earn £4 12s. 10d. in 5 months? and in what time will 7,500 dollars earn 44 dollars 20 cents at the same rate?

(42) The salary of a traveller consisted partly of a fixed sum and partly of commission, which was proportional to the value of the orders obtained. In two consecutive years he obtained orders to the value of £3,150 and £4,500 respectively, and he received £254 10s. and £295 respectively. If his salary for the next year was £320, what was the value of the orders obtained? If he had obtained orders to the value of £600, what would have been his salary?

(43) The total weight of a waggon and 17 cases was 52 cwt. 44 lb., while the weight of the waggon and 25 cases was 68 cwt. 12 lb. Assuming that the weight of each case was the same, find the weight of the waggon and 30 cases.

(44) A photographer offered to supply 6 copies of a photograph for 15s. and 12 copies for 1 guinea, and he estimated his profit would be 5s. and 7s. respectively. For what sum should he supply 30 copies in order to gain 10s. profit?

(45) An author received a sum of money for a book and, in addition, a fixed amount for every copy sold. When 500 copies had been sold, he had received £85 16s. 8d., and when 1,350 had been sold he had received altogether £142 10s. How much will he have received when 10,000 copies have been sold?

(46) A, B, and C had £5,250, £4,500, and £3,125 invested in a business. If their total profit was £1,047 15s., what should each receive?

(47) A sum of £7 15s. was paid to convey goods by rail a distance of 103 miles and the rails of two companies were used, the mileage on the first company's track being 68½. How should the sum be divided among the two companies?

(48) A, B, C, D, and E club together and rent an acre of land for £5 12s. 6d. for the purpose of forming allotments. The sizes of the allotments are 45 sq. pl., 37 sq. pl., 34½ sq. pl., 30 sq. pl., and 13½ sq. pl. respectively. What amounts should they subscribe for the rent?

(49) If A, B, and C of Question (46) are working partners and decide to divide half the total profits equally among them and the other half in the ratios of the amounts invested, what should each have received had the profit been £1,114 12s.?

(50) X, Y, and Z enter into partnership and bring capital to the amounts

of £1,600, £4,200, and £7,800 respectively. They agree that only one of them need attend work at any one time and that the number of days worked by each shall be inversely proportional to the capital subscribed. Find on how many of the 305 working days each should attend.

(51) A ship with its cargo is worth £140,360 and is insured at 7½ per cent. by its owners A, B, and C for such an amount that in case of loss the value of the ship and the premiums will be recovered. A has an interest in the ship of £81,416, B of £37,112, and C of the remainder. How much, to the nearest shilling, will each have to pay for premium?

(52) A, B, and C form a business with a capital of £10,500. Of this, £4,400 is contributed by A, £3,700 by B, and the rest by C. After 5 months, C withdraws £800 capital, whilst A and B each add £400. At the end of the year the profit to be divided is 13¾ per cent. of the total capital. What should each receive, assuming the profits to be divided in proportion to capital?

(53) A man left ⅔ of his property to his wife, ⅙ to his son, and the remainder was divided among his two daughters in the ratio of 6 : 5. If the younger daughter received £235, what did each of the others receive?

(54) In what ratio should tea costing £6 10s. a cwt. be mixed with tea costing £7 15s. a cwt., so that by selling the mixture at 1s. 6d. per lb. a profit of 12 per cent. may be obtained?

(55) Evaluate 3½ per cent., 5 per cent., 12½ per cent., 14·7 per cent., and 119 per cent. of each of the following amounts: £715 13s. 6d., £4,536 11s 7d., 175 francs 75 centimes, 4,265 francs, 714 dollars 35 cents, and 4,937 dollars.

(56) Fill in the columns left vacant, and also find the total gain per cent. based on (a) cost price, (b) selling price.

Cost Price.	Selling Price.	Gain or Loss.	% gain or loss based on C.P.	% gain or loss based on S.P.
£ s. d.	£ s. d.			
374 13 9	437 14 –			
175 15 4	247 13 4			
504 – –	428 11 6			
237 12 6	245 – –			

(57) Find the gain or loss per cent. based on cost price in the following cases: C.P. = £14 per ton, S.P. = 1¾d. per lb.; C.P. = £6 7s. 6d. per quarter, S.P. = 3½d. per pint; C.P. = £2 8s. 4d. per gross; S P. = 3½d each.

(58) The output and value of coal produced in the five years 1925–1929 is as follows—

Year.	*Tons.*	*Value at Pit.*
		£
1925 . . .	243,176,000	198,978,000
1926 . . .	162,279,000	123,384,000
1927 . . .	252,252,000	183,544,000
1928 . . .	237,472,000	152,516,000
1929 . . .	257,907,000	173,233,000

Calculate by what percentage the value per ton in 1929 is greater or less than the average value per ton for the period 1925–29.

(59) Fill in the columns left vacant.

Cost Price.	Selling Price so as to gain $7\frac{1}{2}$% on C.P.	Selling Price so as to gain $7\frac{1}{2}$% on S.P.	Selling Price so as to lose 3% on C.P.	Selling Price so as to lose 3% on S.P.
£793 15s. 6d.				
£248 3s. 9d.				
4716·45 francs				
3109·85 dollars				

(*Note.*—The ratios of C.P. to S.P. are 100: $107\frac{1}{2}$, $92\frac{1}{2}$: 100, 100 : 97, and 103: 100 respectively.)

(60) A dealer bought 160 sheep at £2 5s. 6d. each. The cost of sending them to market was £6 17s. 6d., and he obtained £420 by selling them. Find his gain per cent.

(61) A merchant bought goods at 1050 francs per metric ton. He sold 72 per cent. of the goods at 14s. per cwt. and the remainder at 15s. 6d. per cwt. Assuming 1 Kg. = 2·205 lb. and £1 = 90·50 francs, find his percentage gain or loss.

(*Note.*—Suppose 100 metric tons be bought, then C.P. = 105,000 francs.

$$\text{Then S.P.} = \frac{72 \times 2205 \times 14 \times 90{\cdot}5}{112 \times 20} + \frac{28 \times 2205 \times 15{\cdot}5 \times 90{\cdot}5}{112 \times 20} \text{ francs}$$

$$= \frac{2205 \times 14 \times 90{\cdot}5 \times (72 + 31)}{112 \times 20} \text{ francs.})$$

(62) A dealer bought a quantity of goods at a certain price. He sold 55 per cent. of the quantity, thereby gaining 26 per cent. profit; 36 per cent., thereby gaining 15 per cent.; and the remainder, which were damaged, he sold at a loss of $22\frac{1}{2}$ per cent. What was his percentage profit on the whole quantity?

(63) In order to express in acres an area of land given in hectares, a clerk multiplied by $2\frac{1}{2}$. What was the error per cent. to two places of decimals in his result, given that 1 hectare = 2·47114 ac.

(*Note.*—Error on 1 hectare = ·02886 ac.

$$\therefore \text{ Error per cent.} = \frac{{\cdot}02886 \times 100}{2{\cdot}47114})$$

(64) A publisher sells books to a bookseller at 30 per cent. below the published price and allows him $7\frac{1}{2}$ per cent. discount off his bill. The bookseller retails the books at 3d. in the shilling below the published price. What is his profit per cent.?

(65) At an election 4,731 persons voted out of a total of 5,370 electors. 2,947 voted for A and the remaining votes were recorded for B. What percentage of electors did not vote and what percentage of the total votes did A receive?

(66) A wholesale paper merchant supplies the trade with paper of various

descriptions at the following prices per ream: 3s. 10d., 4s. 2d., and 4s. 6d. The retailer sells the three kinds of paper respectively at 4s. 11d., 5s. 3d., 5s. 9d. per ream, but allows $4\frac{1}{2}$ per cent. discount for cash. What is his net profit per cent. on the cost price in each case?

(67) The cost of manufacturing cycles was £4 17s. 6d., and the sale price to the retailer was £5 15s. Afterwards the cost was increased by 18s., and the sale price was increased by 25s. If two-thirds as many cycles were sold as formerly, find the increase or decrease per cent. in the total profits.

(68) A manufacturer sold an article to a merchant, thereby gaining $7\frac{1}{2}$ per cent. The latter sold it to a retailer, thereby gaining $12\frac{1}{2}$ per cent.; and the retailer sold it to a customer, thereby gaining 22 per cent. These percentages being based on cost price in each case, find how much per cent. the customer's price was greater than the cost price to the manufacturer. If the customer's price was 7 guineas, what was the cost of production?

(69) A mixture of teas was made as follows: 35 lb. at 2s. 8d. per lb.; 25 lb. at 2s. 2d. per lb.; and 55 lb. at 1s. $10\frac{1}{2}$d. per lb. What is the lowest price per lb. to the nearest penny, in order that by selling the mixture a profit of at least $12\frac{1}{2}$ per cent. of cost price will be made?

(70) The price of an article was increased by r per cent. Afterwards the latter price was reduced by r per cent. Show that the final price is $\frac{r^2}{100}$% less than the initial price.

(71) A man builds a block of 20 flats on land for which he pays £234 ground rent. The contractor's price is £19,500 and the architect's fee 5 per cent. of that. The building is assessed at £1,800 and the yearly rates are 8s 3d. in the £ Reckoning 6 per cent. of total initial cost annually for repairs and allowing for two of the flats on an average to be unoccupied, what rent to the nearest shilling must he charge on each flat to bring him in a clear $7\frac{1}{2}$ per cent on the money he has sunk?

(72) There are two fields for sale: the length and breadth of the first are respectively $27\frac{1}{2}$ per cent and 15 per cent. greater than those of the second. If the cost per acre of the first is 10 per cent. less than that of the second, how much per cent. is the total cost of the former greater than that of the latter?

(73) A retailer bought a commodity at £1 10s. per cwt and retailed it at $4\frac{1}{2}$d. a lb. If the cost price be increased to £1 18s. per cwt., what should the minimum retail price per lb be, in order that he may obtain at least the same percentage profit as before?

(74) Assuming 1 metre to be 39·3708 in , find the percentage error made in assuming 1 Km. to be equal to $\frac{5}{8}$ mile.

(75) A mixture of wine and water contains 35 per cent. water. If the mixture be further diluted by the addition of 1 pint of water to every gallon of mixture, by how much per cent. should the price per gallon of the latter mixture be less than that of the former?

(*Note* —Obtain in each case the quantity of wine in 100 gall. of mixture.)

(76) 42 per cent. of the workers in a factory are men getting an average wage of £2 12s. a week each, and the rest are women getting an average wage of £1 5s a week each. Find what percentage of the total wages earned is paid to (*a*) men, (*b*) women

(77) A manufacturer buys a machine for use in his factory, and estimates that its value decreases each year 15 per cent. of its value at the beginning of that year. If he sells it after three years for half its original cost, what percentage of the estimated value at the time of sale does he gain or lose?

(78) Find the simple interest in the following cases.

Principal.	*Rate per Cent. per annum.*	*Time.*
£ 715 14s.	4½	95 days
£1,174 13s. 10d.	5¼	2½ years
40,755 francs	3½	114 days
24,364 dollars	5½	1 year 45 days

(79) A bank pays 3 per cent. per annum on sums deposited. What sum must be deposited to provide an income of £240 a year?

(80) What sum would amount to £1,000 in 85 days' time at 5½ per cent. per annum?

(81) A moneylender asks for £10 for the loan of £150 from 20th March to 19th September of the same year. What rate per cent. interest is he charging?

(82) For how many days could a sum of £540 be borrowed, so that the interest should not exceed £5? Reckon interest at 5 per cent. per annum.

(83) Draw up a table giving the interest on £1 up to £9 at 4½ per cent. per annum for 1 day. Use it to calculate the interest on (*a*) £4,625 for 97 days; (*b*) £2,947 13s. for 127 days; (*c*) £14,723 17s. 2d. for 70 days, the interest in each case being 4½ per cent. per annum.

(84) Construct a table correct to 10 places of decimals, giving the interest on £1 for 1 day at 4, 4¼, 4½, 4¾, 5 per cent. per annum. Use it to calculate the interest on £715 for 90 days at 4¼ per cent., 4½ per cent., and 4¾ per cent.

(85) Find the difference between the simple and compound interest on £1,500 for 3 years at 4½ per cent. per annum.

(86) £7,500 was borrowed on 10th May: until 15th July, the rate of interest was 4½ per cent. per annum; from 16th July to 20th August, 5 per cent. per annum; and from 21st August to 3rd December, 5½ per cent. per annum. What sum should be paid on the latter date to repay the debt?

(87) Find the compound interest in each of the following cases—

Principal.	*Rate.*	*Time.*
£ 945 13s. 6d.	4 % per annum, payable yearly	4 years
£2,738 14s. 10d.	5 % ,, ,, half-yearly	3 ,,
£ 420 3s. 7d.	4½% ,, ,, ,,	2½ ,,
43,342 francs	6 % ,, ,, ,,	2¾ ,,
11,246·75 dollars	3¾% ,, ,, yearly	2½ ,,

(88) The difference between the simple interest and the compound interest for 1 year on a sum of money is £5 2s. 10d. The rate is 5 per cent., and in the latter case the interest is payable half-yearly. What is the sum of money?

(*Note.*—Find the difference on £100, then increase £100 in the ratio of £5 2s. 10d. to this amount.)

(89) Find the difference in the interest on £1,000 for 2 years at 3½ per cent. per annum, when the interest is payable yearly and when payable quarterly.

(90) Find the compound interest on £695 for 3 years, if the interest is payable yearly, the rate for the first 2 years being 3 per cent. and for the third year 4 per cent. per annum.

(91) What sum will amount to £2,500 in 3 years at 4 per cent. per annum payable yearly, compound interest?

(92) A owes B £2,000, but it is not due for payment till the end of three years from now. How much ought B to be willing to accept now in order to clear off the debt, (*a*) taking money to be worth 4 per cent. per annum simple interest, (*b*) taking it to be worth 4 per cent. per annum, compound interest payable yearly?

(93) A sum of money is put out at compound interest at 5 per cent. per annum, reckoned quarterly. What, to the nearest hundredth, is the equivalent rate per cent. per annum if reckoned annually?

CHAPTER VIII.

MENSURATION—

SIMPLE EQUATIONS AND THEIR APPLICATION TO INVERSE PROBLEMS

75. SIMILAR FIGURES.

IN map-drawing, it is required to represent a given surface by a figure which shall have the same *shape* as the given surface. The scale of the map is really the ratio of a given length on the map to the corresponding length of the surface. For example, a scale of $2'$ to 1 mile means that the ratio of a length on the map to the corresponding length on the surface is $2:5280$, that is, $\frac{1}{2640}$, which is termed the representative fraction. The question arises then as to whether, if all the lengths in connection with the surface be reduced in the same ratio, the figure obtained will be similar to the surface.

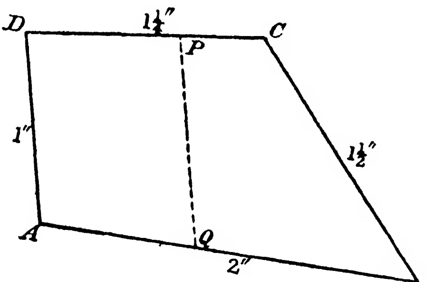


FIG. 47.

The sides of $ABCD$ are $2''$, $1\frac{1}{2}''$, $1\frac{1}{4}''$, $1''$, and it is required to reduce these sides in the ratio $2:1$, so that the sides of the second figure will be $1''$, $\frac{3}{4}''$, $\frac{5}{8}''$, $\frac{1}{2}''$.

$A_1B_1C_1D_1$ is a figure which has these latter measurements, but it is seen that it is not similar to $ABCD$, as the angles are not equal to those of $ABCD$.

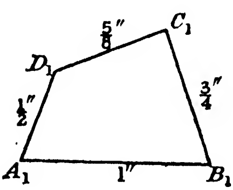


FIG. 48.

Again, it does not follow that if the angles of two figures (not triangles) are equal, they are similar: for if PQ be any line parallel to AD, the figures $ABCD$ and $QBCP$ are equi-angular, but they are obviously not similar.

Figures are similar when corresponding sides or lengths are proportional and corresponding angles are equal.

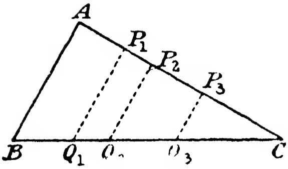


FIG. 49.

76. ABC is a triangle, P_1Q_1, P_2Q_2, P_3Q_3 are any lines parallel to AB. The figures ABC, P_1Q_1C, P_2Q_2C, and P_3Q_3C are equi-angular; and it can be proved that the ratios $AB : BC : CA$, $P_1Q_1 : Q_1C : CP_1$, $P_2Q_2 : Q_2C : CP_2$, and $P_3Q_3 : Q_3C : CP_3$ are equal.

Thus, **all equi-angular triangles are similar.**

The converse, that all triangles whose corresponding sides and lengths are proportional are similar, is true and should be known.

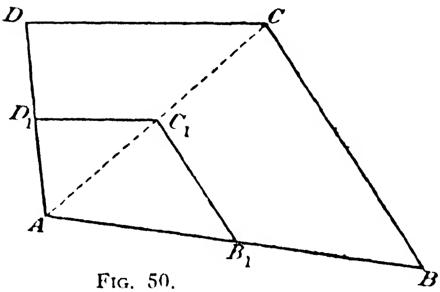


FIG. 50.

By dividing the figure $ABCD$ of paragraph 75 into triangles and bisecting AB at B_1, by drawing B_1C_1 and C_1D_1 parallel to BC and CD respectively, the figure $AB_1C_1D_1$ is obtained, and this is similar to $ABCD$, as the angles are equal to the corresponding angles of $ABCD$ and the distances involved in $AB_1C_1D_1$ are each one-half the corresponding distances in $ABCD$.

In a similar way a rectilinear figure of any number of sides can be divided into triangles by joining one angular point to the remaining angular points, and a similar figure be drawn whose sides bear any given ratio to the sides of the original figure.

77.

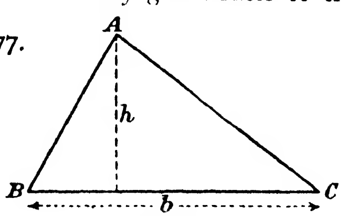


FIG. 51.

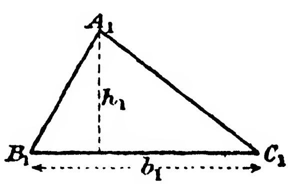


FIG. 52.

ABC and $A_1B_1C_1$ are similar triangles. Let x and x_1 denote their areas respectively, then $x = \frac{1}{2}bh$ and $x_1 = \frac{1}{2}b_1h_1$.

$$\therefore \frac{x_1}{x} = \frac{b_1h_1}{bh} = \frac{b}{b_1} \times \frac{h}{h_1}$$

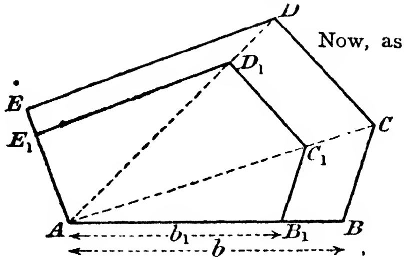


FIG. 53.

Now, as the triangles are similar, $\frac{b}{b_1} = \frac{h}{h_1}$

$$\therefore \frac{x}{x_1} = \frac{b^2}{b_1^2} \text{ or } \frac{h^2}{h_1^2}$$

Thus the areas of similar triangles are proportional to the squares of corresponding distances.

If $r \equiv$ the ratio $\frac{b}{b_1}$ or $\frac{h}{h_1}$, then $\frac{x}{x_1} = r^2$, so that $x = r^2 x_1$

$ABCDE$ and $AB_1C_1D_1E_1$ are similar figures.

Let r denote the ratio $\frac{AB}{AB_1}$ or $\left(\frac{BC}{B_1C_1} \text{ or } \frac{AC}{AC_1}, \text{ etc.}\right)$

" $x \equiv$ area of $ABCDE$

" $x_1 \equiv$ " $AB_1C_1D_1E_1$

" $u,v,w \equiv$ " triangles AB_1C_1, AC_1D_1, AD_1E_1 respectively;

then $x_1 = u + v + w$

and the areas of ABC, ACD, ADE are r^2u, r^2v, and r^2w respectively

$$\begin{aligned} \therefore\ x &= r^2u + r^2v + r^2w \\ &= r^2(u + v + w) \\ &= r^2x_1 \end{aligned}$$

By dividing both sides by x_1,

$$\frac{x}{x_1} = r^2 = \frac{b^2}{b_1^2},$$

so that the areas of the similar figures are proportional to the squares of corresponding distances.

It is clear that the same result holds whatever the number of sides the figure has.

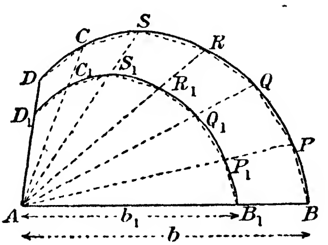


FIG. 54.

$ABCD$ and $AB_1C_1D_1$ are similar figures which are not rectilinear. By taking points $PQRS$ on the curved portion and joining the rectilinear figure $ABPQRSCD$ is obtained. By the previous method the similar rectilinear figure $AB_1P_1Q_1R_1S_1C_1D_1$ is obtained, so that

$$\frac{\text{Area of } ABPQRSCD}{\text{Area of } AB_1P_1Q_1R_1S_1C_1D_1} = \frac{b^2}{b_1^2}$$

Now, if instead of the four points $P_1Q_1R_1S$ being taken on the curved portion, thousands of points be taken, say, $\frac{1}{1000}$ in. apart,

the rectilinear figure so obtained will almost coincide with the figure $ABCD$, and the difference in area will be negligible. Similarly the difference of the area of the corresponding rectilinear figure and that of the figure AB_1C_1D will be negligible.

$$\text{Thus, } \frac{\text{Area of } ABCD}{\text{Area of } AB_1C_1D_1} = \frac{b^2}{b_1^2}.$$

It can be proved that the same result holds in the case when the similar surfaces are not plane surfaces.

Thus, **the areas of similar figures are proportional to the square of corresponding distances.**

78. **Similar Solid Figures** are such that their corresponding faces are similar, and the lengths of corresponding edges are proportional.

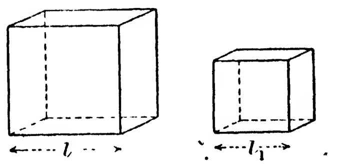


FIG. 55.

Cubes are similar solid figures. Let V and V_1 be the volumes of cubes whose edges are l and l_1 respectively.

Then $V = l^3$ and $V_1 = l_1^3$,

$$\text{so that } \frac{V}{V_1} = \frac{l^3}{l_1^3}.$$

$$\text{If } r \equiv \text{the ratio } \frac{l}{l_1}, \quad \frac{V}{V_1} = r^3 \qquad \therefore\ V = r^3V_1$$

The figures below represent two solid similar figures, the length of two corresponding edges being l and l_1 respectively. Let V and V_1 denote their volumes respectively, and let $r \equiv$ the ratio $\frac{l}{l_1}$

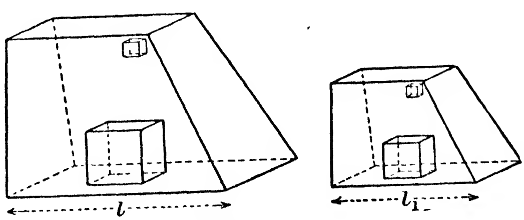


FIG. 56.

Now the first figure may be considered as being built up of a number of cubes, some of which are infinitesimally small, in order

to fill up the irregular spaces. Let x and y be the volumes of two such cubes.

As the second figure is similar, for every cube in the first there will be a cube in the second the length of whose edges will be in the ratio $\frac{l_1}{l}$ to that of the edges of those in the first. If x_1 and y_1 denote the volumes of the cubes in the second which correspond to the cubes whose volumes are x and y in the first respectively,

then $x = r^3 x_1$ and $y = r^3 y_1$ where $r = \frac{l}{l_1}$

$$
\begin{aligned}
\text{Now } V_1 &= x_1 + y_1 + \quad . \quad . \quad . \quad . \quad . \\
\text{and } V &= x + y + \quad . \quad . \quad . \quad . \quad . \\
&= r^3 x_1 + r^3 y_1 + \quad . \quad . \quad . \quad . \\
&= r^3(x_1 + y_1 + \quad . \quad . \quad . \quad . \quad . \;) \\
&= r^3 V_1
\end{aligned}
$$

By dividing both sides by V_1, $\frac{V}{V_1} = r^3 = \frac{l^3}{l_1^3}$

Thus the volumes of similar solid figures are proportional to the cubes of corresponding distances.

EXAMPLE (i)—

A boy whose eyes are 4′ 6″ from the ground erects a pole in a vertical position so that the top is 8′ 6″ from the ground. On standing 5′ 3″ away from the pole he looks towards the top of a tower which is 50 yd from the place where he is standing. If the top of the pole appears to the boy to coincide with the top of the tower, find the height of the latter

Let h feet $\equiv$ ht. of tower above the boy's eyes; then, as ABC and DBE are similar triangles,

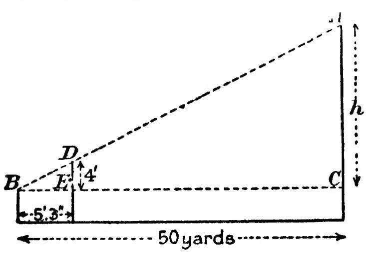


FIG. 57.

$$\frac{h}{150} = \frac{4}{5\frac{1}{4}}.$$

$$\therefore h = \frac{4 \times \overset{50}{\cancel{150}} \times 4}{\underset{7}{\cancel{21}}} = 114\tfrac{2}{7}$$

$$
\begin{aligned}
\therefore \text{Ht. of tower} &= 114'\,3\tfrac{3}{7}'' + 4'\,6'' \\
&= \underline{118'\,9\tfrac{3}{7}''}
\end{aligned}
$$

EXAMPLE (ii)—

The area of the Isle of Wight is 94,145 ac.

(1) If it be required to construct a map such that a square inch will represent 400 ac., what must be the scale of the map?

(2) If a map be constructed whose representative fraction is $\frac{1}{27500}$, what would be the area of the map?

(1) Let $x \equiv$ number of inches to a mile,

$$\text{then } \frac{x \times x}{5280 \times 12 \times 5280 \times 12} = \frac{1}{400 \times 4840 \times 9 \times 144}$$

$$\therefore x^2 = \frac{5280 \times 12 \times 5280 \times 12}{400 \times 4840 \times 9 \times 144} = 1{\cdot}6$$

$$\therefore x = \sqrt{1{\cdot}6} = 1{\cdot}265, \text{ correct to 3 dec. places.}$$

(2) Let x sq. inches $\equiv$ area of map,

$$\text{then } \frac{x}{94145 \times 4840 \times 9 \times 144} = \frac{1 \times 1}{27500 \times 27500}$$

$$\therefore x = \frac{94145 \times 4840 \times 9 \times 144}{27500 \times 27500}$$

$$= 781, \text{ correct to nearest integer.}$$

Ansr.—(1) 1·265″ to 1 mile; (2) 781 sq. inches.

EXAMPLE (iii)—

A cylindrical tin whose diameter is 2¾″ holds ½ lb. of a certain commodity. If it be required to make tins having the same height, but to hold ¼ lb. of the commodity, what should the diameter be?

As the heights are the same, the volume of the tins will be proportional to the square of the diameters.

Let x inches $\equiv$ diameter of ¼ lb. tin,

$$\text{then } \frac{x \times x}{2\frac{3}{4} \times 2\frac{3}{4}} = \frac{\frac{1}{4}}{\frac{1}{2}}$$

$$\therefore x^2 = \frac{11 \times 11 \times 2}{4 \times 4 \times 4} = \frac{242}{64}$$

$$\therefore x = \frac{15{\cdot}556}{8} = 1{\cdot}94 \text{ correct to 2 dec. places.}$$

Ansr.—1·94 inches.

EXAMPLE (iv)—

How many times greater is the volume of a 15-inch shell than that of a 13·5-inch shell, assuming their shapes are similar?

$$\text{Ratio of volumes} = \frac{15 \times 15 \times 15}{13{\cdot}5 \times 13{\cdot}5 \times 13{\cdot}5} = 1{\cdot}37 \text{ correct to 2 dec. places.}$$

EXAMPLE (v)—

36 screws, 1¾ in. long, weigh 1 lb. How many screws of similar material and shape, but 1½ in. long, would weigh just over 1 lb.?

The weight of a screw is proportional to the cube of its length, so that the number that weigh 1 lb. is inversely proportional to the cube of the length.

Let $x \equiv$ number of screws, $1\frac{1}{4}''$ long, which weigh 1 lb.

$$\text{then } \frac{x}{36} = \frac{1\frac{7}{8} \times 1\frac{7}{8} \times 1\frac{7}{8}}{1\frac{1}{4} \times 1\frac{1}{4} \times 1\frac{1}{4}}$$

$$\therefore x = \frac{36 \times 15 \times 15 \times 15 \times 4 \times 4 \times 4}{8 \times 8 \times 8 \times 5 \times 5 \times 5} = 121\tfrac{1}{2}.$$

Ansr.—122 screws

79. CIRCLES, whatever their sizes, are similar figures: the circumferences are corresponding lengths, and so also are the diameters. If $c \equiv$ the circumference of a circle and $d \equiv$ the diameter, then the ratio $\frac{c}{d}$ is the same, whatever the size of the circle, and is denoted by the symbol π (Greek letter). The numerical value of π is an incommensurable quantity whose approximate value is 3·141593. For certain problems where a very close approximation is not required, the value of π can be taken as 3·1416, 3·14, or $3\frac{1}{7}$ according to the numbers occurring in the problem and the degree of accuracy required.

If $r \equiv$ radius of the circle, then $\mathbf{d} = 2\mathbf{r}$,

so that $\frac{\mathbf{c}}{2\mathbf{r}} = \pi$, from which $\mathbf{c} = 2\pi\mathbf{r}$ and $r = \frac{\mathbf{c}}{2\pi}$.

NOTE 1.—The value of π can be calculated correct to any number of decimal places by means of Trigonometrical series. The approximate value can be verified by winding thread, say, 20 times, round a cylinder, and thus the length of the circumference can be obtained; and this, on being divided by the length of a diameter, gives an approximation to the ratio π. By repeating the same with cylinder of different radii, the fact that the ratio $\frac{c}{d}$ is constant can also be verified.

80. AREA OF A CIRCLE.

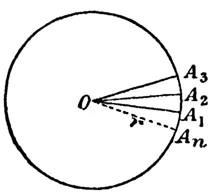


FIG. 58.

Suppose the area of the circle centre O, and radius r to consist of a very large number of portions like OA_1A_2 and OA_2A_3. The bases A_1A_2 and A_2A_3 are so small that they can be considered as straight lines, so that the portions OA_1A_2 and OA_2A_3 can be regarded as triangles whose heights are r.

$$\text{The area of the circle} = \frac{r \cdot \overline{A_1A_2}}{2} + \frac{r \cdot \overline{A_2A_3}}{2} + \quad . \quad . \quad . \quad \frac{r \cdot \overline{A_nA_1}}{2}$$

$$= \frac{r}{2}\left(\overline{A_1A_2} + \overline{A_2A_3} + \quad . \quad . \quad . \quad . \overline{A_nA_1}\right)$$

But $(\overline{A_1A_2} + \overline{A_2A_3} + \ldots + \overline{A_nA_1})$ is the distance round the complete circle and is therefore equal to $2\pi r$.

$$\therefore \textbf{ Area of Circle} = \frac{\mathbf{r}}{2} \times 2\pi\mathbf{r} = \underline{\pi\mathbf{r}^2}$$

81. LENGTH OF ARC AND AREA OF SECTOR.

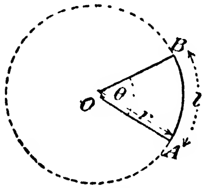


FIG. 59.

A sector of a circle is a portion of a circle bounded by two radii.

Let $l \equiv$ length of arc
,, $a \equiv$ area of sector
,, $\theta \equiv$ number of degrees in angle AOB

It is clear that the length of arc and the area of the sector are each proportional to the angle of the sector. Regarding the complete circle as a sector, the length of the arc is $2\pi r$, the area is πr^2, and the angle is four right angles or 360°.

$$\therefore \frac{l}{2\pi\mathbf{r}} = \frac{\theta}{360} \text{ and } \frac{\mathbf{a}}{\pi\mathbf{r}^2} = \frac{\theta}{360}. \quad \text{Also } \frac{l}{2\pi\mathbf{r}} = \frac{\mathbf{a}}{\pi\mathbf{r}^2}$$

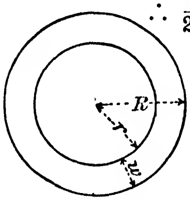


FIG. 60.

82. AREA OF CIRCULAR RING.

Let $R, r \equiv$ outer and inner radii respectively
,, $a \equiv$ area of circular ring

then $a = \pi R^2 - \pi r^2$
$= \pi(R^2 - r^2)$
$= \pi(\mathbf{R} + \mathbf{r})(\mathbf{R} - \mathbf{r})$ by § 39

Let $w \equiv$ width of the ring,
then $a = \pi w(2\mathbf{r} + w)$ or $\pi w(2\mathbf{R} - w)$

Also, difference of circumferences $= 2\pi R - 2\pi r$
$= 2\pi(R - r)$
$= 2\pi w.$

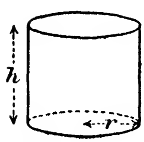


FIG. 61.

83. A CYLINDER can be regarded as a prism whose cross-section is a circle.

Let $V \equiv$ vol. of cylinder, radius r and perp. height h,
then $\mathbf{V} = \pi\mathbf{r}^2\mathbf{h}$

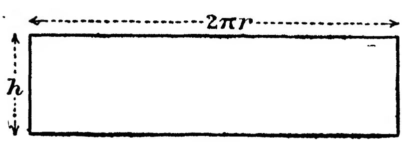


FIG. 62.

If the cylinder be hollow, and the outer and inner radii be R and r respectively, then

$$\mathbf{V} = \pi\mathbf{h}(\mathbf{R} + \mathbf{r})(\mathbf{R} - \mathbf{r}).$$

If the curved surface be opened out, it will become rectangular in shape and its dimensions will be $2\pi r$ by h.

$$\therefore \text{ Area of curved surface} = \mathbf{2\pi rh}$$
$$\text{Also total area of surface} = 2\pi rh + 2\pi r^2 = \mathbf{2\pi r(h + r)}$$
$$\text{Also area of surface without lid} = 2\pi rh + \pi r^2 = \mathbf{\pi r(2r} + r)$$

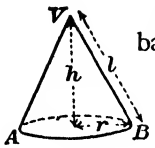


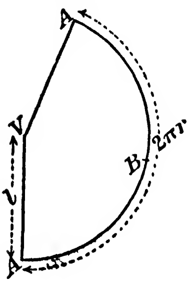
FIG. 63.

84. A CONE can be regarded as a pyramid whose base is a circle.

Let $V \equiv$ vol. of cone, radius r and perp. height h,

then $\mathbf{V = \dfrac{\pi r^2 h}{3}}$.

If the cone be a right cone, the curved surface will, on being opened out, become a sector of a circle whose radius is l, the slant height of the cone, and whose arc is the circumference of the base of the cone, so that the length of the arc is $2\pi r$.



Let $a \equiv$ area of curved surface of cone

then $\dfrac{a}{\pi l^2} = \dfrac{2\pi r}{2\pi l}$

$$\therefore a = \frac{\pi l^2 \times 2\pi r}{2\pi l} = \mathbf{\pi rl}$$

FIG. 64.

$$\text{Total area of surface of cone} = \pi rl + \pi r^2 = \mathbf{\pi r(l + r)}.$$

FIG. 65.

85. SPHERE.

Let $V \equiv$ volume of a sphere, radius r

" $a \equiv$ area of surface of the sphere, then it can be proved (the proof is beyond the scope of this book) that

$$V = \frac{4\pi r^3}{3} \text{ and } a = 4\pi r^2.$$

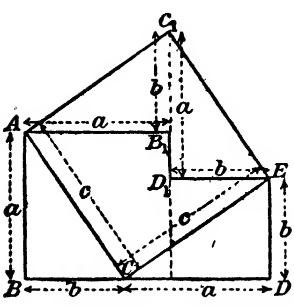


FIG. 66.

86. AB_1D_1EDB is of such a shape that it is made up of two squares, whose sides are a and b respectively. If the triangles ABC and CDE are cut off, they can, for all numerical values of a and b, be placed in the positions AB_1C_1 and C_1D_1E, and thus form the figure $ACEC_1$, which is a square whose side is of length c.

Now the area of $AB_1D_1EDB = a^2 + b^2$

Also " $ACEC_1 = c^2$

$$\therefore \mathbf{c^2 = a^2 + b^2}.$$

Now ABC (or CDE) is a right-angled triangle, the side, AC, opposite the right angle being the hypotenuse: thus **the area of the square on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares on the other two sides.**

EXAMPLE (vi)—

The driving wheel of a locomotive is 6′ 6″ in diameter. If it make 210 revolutions per minute, what will be the speed of the locomotive in miles per hour?

$$\text{Distance moved in 1 hour} = \pi \times \frac{13}{2} \times \frac{210 \times 60}{3 \times 1760} \text{ miles}$$

$$\therefore \text{ Speed of train} = \frac{3{\cdot}1416 \times 13 \times 210}{176} \text{ ml. per hr.}$$

$$= 48{\cdot}73 \text{ ml. per hr. correct to 2 dec. places.}$$

EXAMPLE (vii)—

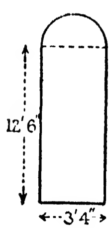


FIG. 67.

A window, whose shape is that composed of a rectangle and a semicircle, has dimensions as shown. What would it cost to glaze it at 4s. 2d. per square foot? ($\pi = 3\frac{1}{7}$)

$$\text{Area of window} = 12\tfrac{1}{2} \times 3\tfrac{1}{3} + \tfrac{1}{2} \times 3\tfrac{1}{7} \times 1\tfrac{2}{3} \times 1\tfrac{2}{3} \text{ sq. ft.}$$

$$= 1\tfrac{2}{3} \times \left(25 + \frac{22 \times 5}{2 \times 7 \times 3}\right) \quad ,,$$

$$\therefore \text{ Cost of glazing} = \tfrac{5}{3} \times \tfrac{580}{21} \times \tfrac{25}{6} \text{ shillings}$$

$$= 191\tfrac{151}{189} \text{ shillings}$$

$$= £9 \text{ 11s. 10d.}$$

EXAMPLE (viii)—

A hollow metal pipe has an outer diameter of 3·2 in. and the thickness of metal is $\frac{1}{4}$ in. If 1 cub. ft. of the metal weighs 756 lb., what is the weight of 1 yd. of the pipe to the nearest ounce?

$$\text{Area of section} = \pi \times \tfrac{1}{4} \times (3\tfrac{1}{5} - \tfrac{1}{4}) \text{ sq. in.}$$

$$\therefore \text{ Vol. of 1 yd. of pipe} = \pi \times \frac{1 \times 59}{4 \times 20} \times 36 \text{ cub. in.}$$

$$\therefore \text{ Wt.} \quad ,, \quad ,, \quad = \pi \times \frac{59 \times 9 \times 756 \times 16}{20 \times 12 \times 12 \times 12} \text{ oz.}$$

To obtain result correct to nearest oz., the value of π must be correct to ·0001.

$$\therefore \text{ Wt. of 1 yd. of pipe} = \frac{3{\cdot}1416 \times 59 \times 9 \times 756 \times 16}{20 \times 12 \times 12 \times 12} \text{ oz.}$$

$$= 584 \text{ oz. correct to nearest oz.}$$
$$= 36 \text{ lb. } 8 \text{ oz.}$$

EXAMPLE (ix)—

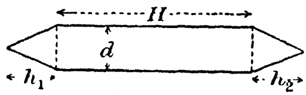


FIG. 68.

Find a formula most suitable for calculation of the cubic capacity of an airship whose shape is composed of a cylinder and two cones, the dimensions being as shown.

$$\text{Volume of airship} = \pi\frac{d^2}{4}H + \frac{1}{3}\pi\frac{d^2}{4}.h_1 + \frac{1}{3}\pi\frac{d^2}{4}.h_2$$

$$= \frac{\pi d^2}{12}\left(3H + h_1 + h_2\right)$$

EXAMPLE (x)—

Assuming the Earth is a sphere of radius 3,960 miles, find the area of its surface to 2 significant figures.

Area of surface of earth $= 4 \times 3\frac{1}{7} \times 3{,}960 \times 3{,}960$ sq. miles.
$= 200{,}000{,}000$ sq. miles to 2 sig. figs.

EXAMPLE (xi)—

What is the distance from one corner of a rectangular field 227 ft. long and 147 ft. broad, to the opposite corner ?

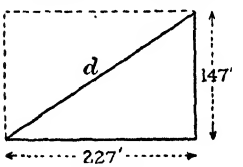


FIG. 69.

Let d feet $\equiv$ distance from one corner to opposite corner,

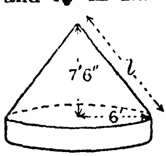
then $d^2 = 227^2 + 147^2$
$= 51529 + 21609$

$\therefore\ d = \sqrt{73138} = 270{\cdot}44$ correct to 2 dec. places.

Ansr.—270 ft. 5 in.

EXAMPLE (xii)—

A tent, of canvas, is in the form of a cone resting on a cylinder 1′ 9″ high and 12′ in diameter, and the vertex of the cone is 9′ 3″ above the ground. Find the cost of canvas at 5s. 6d. per square yard and find the volume of air the tent can hold.



Let l feet $\equiv$ slant height of cone,

then $l^2 = 7{\cdot}5^2 + 6^2$

$\therefore\ l = \sqrt{92{\cdot}25} = 9{\cdot}605.$

FIG. 70. Total area of canvas $= \pi \times 6 \times 9{\cdot}605 + 2 \times \pi \times 6 \times 1{\cdot}75$ sq. ft.
$= \pi \times 6 \times (9{\cdot}605 + 3{\cdot}5)$ sq. ft.

$$\therefore \text{ Cost of canvas} = \frac{3{\cdot}1416 \times 6 \times 13{\cdot}105 \times 11}{9 \times 2} \text{ shillings.}$$

$= 150{\cdot}96$ shillings correct to 2 dec. places
$= £7$ 11s. 0d. to nearest penny.

Vol. of air $= \pi\,.\,6.^2\,\frac{7}{4} + \frac{1}{3}\pi\,.\,6^2\,.\,\frac{15}{2}$ cub. ft. $= 36\pi(\frac{7}{4} + \frac{5}{2})$ cub. ft.
$= 36 + 3{\cdot}1416 \times \frac{17}{4}$ cub. ft. $= 480{\cdot}7$ cub. ft.

EXAMPLE (xiii)—

A bucket 1′ 3″ high is in the shape of a frustum of a cone, the diameters of the top and bottom being 1′ 4″ and 1′ respectively. What is the capacity of the bucket in gallons and pints?

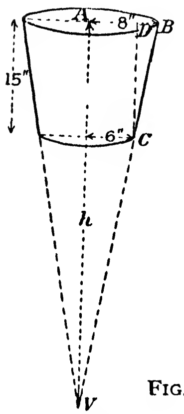


FIG. 71.

Let h inches $\equiv$ height of completed cone,
then as VAB and CDB are similar triangles,

$$\frac{h}{8} = \frac{15}{2}$$

$$\therefore\ h = 60$$

$$\text{Vol. of bucket} = \tfrac{1}{3}\pi \times 64 \times 60 - \tfrac{1}{3}\pi \times 36 \times 45 \text{ c. in.}$$

$$= \frac{\pi \times 4 \times 15}{3} \times (16 \times 4 - 9 \times 3) \text{ c. in.}$$

$$\therefore \text{Cap. ,, ,,} = \frac{3{\cdot}1416 \times 20 \times 37 \times 8}{277{\cdot}274} \text{ pints.}$$

$$= 67{\cdot}1 \text{ pints, correct to 1 dec. place.}$$

Ansr.—8 gallons 3·1 pints.

87. SIMPLE EQUATIONS.

Many problems are such that the Arithmetic principles already dealt with will only with difficulty give a means of finding the result, owing to the fact that these principles have to be applied in an inverse way. By an extremely elementary knowledge of simple equations, however, these difficulties are easily overcome.

It is obvious that if two quantities are known to be equal—

(1) the results, by adding the same quantity to each, will be equal;

(2) the results, by subtracting the same quantity from each, will be equal;

(3) the results, by multiplying each by the same quantity, will be equal;

(4) the results, by dividing each by the same quantity, will be equal.

By applying these three axioms in turn, any simple equation can be at once solved. For example, suppose x be such a number that, on multiplying it by 5 and adding 7, the result is the same as that obtained by multiplying it by 7 and subtracting 9;

then $7x - 9 = 5x + 7$

taking $5x$ from each, then $7x - 5x - 9 = 7$

adding 9 to each, then $7x - 5x = 7 + 9$

i.e., $2x = 16$

dividing both sides by 2, $x = 8$

Again, suppose that x is such that

$$3(x-2)-\frac{x}{4} = 1\tfrac{2}{3}-2(2-3x)$$

multiplying by 12, which is the least common denominator, then

$$36(x-2)-3x = 20-24(2-3x)$$

removing brackets, then $36x-72-3x = 20-48+72x$

subt. $72x$ from and adding 72 to each, then

$$36x-3x-72x = 20-48+72$$

$$i.e., \quad -39x = 44$$

dividing by -39, then $\quad x = -\frac{44}{39} = -1\frac{5}{39}$

The object of adding or subtracting quantities to or from each side is to obtain all terms involving the unknown quantity on one side of the equation and all independent terms on the other side. It will be noticed that this process causes the quantities which are transposed from one side of the equation to the other to have their signs changed from $-$ to $+$ or $+$ to $-$. Thus the complete rules for solving any simple equation are—

1. Multiply both sides by the L.C.M. of the denominators, to clear of fractions.
2. Remove brackets.
3. Transpose terms so that all involving the unknown quantity are on one side and all independent terms on the other.
4. Collect up terms on each side.
5. Divide both sides by the coefficient of the unknown quantity.

EXAMPLE (xiv)—

What is the length of the side of an equilateral triangle to nearest ·1″ which has the same area as a rectangle 7·4″ × 4·8″?

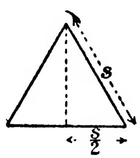


FIG. 72.

Let s inches $\equiv$ length of side of equilateral triangle,

then perp. ht. of triangle

$$= \sqrt{s^2-\frac{s^2}{4}}\text{ in.} = \frac{\sqrt{3}\,.\,s}{2}\text{ in.}$$

$$\therefore \text{ area of triangle } = \frac{\sqrt{3}\,.\,s^2}{4}\text{ sq. in.}$$

$$\therefore \frac{\sqrt{3}\,s^2}{4} = 7{\cdot}4 \times 4{\cdot}8$$

$$\therefore s^2 = \frac{7{\cdot}4 \times 4{\cdot}8 \times 4}{\sqrt{3}} = \frac{7{\cdot}4 \times 4{\cdot}8 \times 4 \times \sqrt{3}}{3}$$

$$= 7{\cdot}4 \times 1{\cdot}6 \times 4 \times 1{\cdot}732 = 82{\cdot}03$$

$$\therefore s = \sqrt{82{\cdot}03} = 9{\cdot}1 \text{ correct to one dec. place}$$

Ansr., 9·1 in. to nearest ·1 in.

NOTE 2.—If $h \equiv$ ht. of triangle, then $h^2 + \frac{s^2}{4} = s^2$ from paragraph 86: transposing the term $\frac{s^2}{4}$, then $h^2 = s^2 - \frac{s^2}{4}$. To evaluate $\frac{1}{\sqrt{3}}$, it is better to multiply numerator and denominator by $\sqrt{3}$, so that the fraction becomes $\frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$, *i.e.*, $\frac{\sqrt{3}}{3}$, and a long division is avoided.

EXAMPLE (xv)—

A horse is tethered in the middle of a field. What should the length of the rope be in order that he should be able to graze over $\frac{1}{2}$ ac. of land?

The horse can graze over land enclosed by a circle whose centre is the fixed end of the rope and whose radius is the length of the rope.

Let l yards $\equiv$ length of rope,

$$\text{then } \pi l^2 = \frac{4840}{2}$$

$$\therefore \quad l^2 = \frac{2420}{3 \cdot 1416} = 770 \cdot 3$$

$$\therefore \quad l = \sqrt{770 \cdot 3} = 27 \cdot 75 \text{ correct to 2 places of decs.}$$

Ansr.—27 yards 2 feet 3 inches to nearest inch

EXAMPLE (xvi)—

An enclosure in the shape of a semicircle is such that the perimeter is 485 yd. What is the area of the enclosure?

Let r yards $\equiv$ radius of semicircle,
then $(\pi r + 2r)$ yards $\equiv$ perimeter of semicircle.

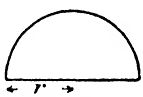


FIG. 73.

$$\therefore \quad r(\pi + 2) = 485$$

$$\therefore \quad r = \frac{485}{\pi + 2} = \frac{485}{5 \cdot 1416}$$

$$\therefore \text{ Area of enclosure} = \frac{3 \cdot 1416 \times 485 \times 485}{2 \times 5 \cdot 1416 \times 5 \cdot 1416} \text{ sq. yd.}$$

$$= 13{,}977 \text{ sq. yd. to nearest square yard.}$$

Ansr.—2 acres 8 sq. chains 425 sq. yards.

EXAMPLE (xvii)—

A cylinder of copper, height 8″, diameter 5″, is drawn into wire, whose diameter is $\frac{3}{32}$″. What length of wire will be obtained?

Let l inches $\equiv$ length of wire,

$$\text{then } \pi \times \tfrac{3}{64} \times \tfrac{3}{64} \times l = \pi \times \tfrac{5}{2} \times \tfrac{5}{2} \times 8$$

$$\therefore \quad l = \frac{\cancel{\pi} \times 25 \times 2 \times 64 \times 64}{\cancel{\pi} \times 9} = 22{,}755\tfrac{5}{9}$$

Ansr.—632 yards $3\frac{5}{9}$ inches.

EXAMPLE (xviii)—

A circular pond is 30 yd. across. If a path be made all round so that the cost at $7\frac{1}{2}$d. per square yard would not exceed £10, what, to the nearest inch, would be the widest path possible?

Let R yd. $\equiv$ distance from centre of pond to outer edge of path,

$$\text{then } \pi(R^2 - 15^2) = \frac{200 \times 8}{5}$$

$$\therefore R^2 - 225 = \frac{320}{3{\cdot}1416}$$

$$\therefore R^2 = \frac{320}{3{\cdot}1416} + 225$$

$$= 101{\cdot}86 + 225$$

$$\therefore R = \sqrt{326{\cdot}86} = 18{\cdot}08 \text{ correct to 2 dec. places.}$$

$$\therefore \text{Width of path} = 18{\cdot}08 - 15 \text{ yards}$$

$$= 3 \text{ yards } 3 \text{ inches.}$$

EXAMPLE (xix)—

A merchant bought 500 qr. of wheat. He sold 240 qr. at $8\frac{3}{4}$ per cent. profit, 150 qr. at 7 per cent. profit, and the remainder at $12\frac{1}{2}$ per cent. profit. The total profit was £190 1s. At what price per quarter did he buy?

Let £$x \equiv$ cost per quarter,

$$\text{then } \frac{240x \times 35}{400} + \frac{150x \times 7}{100} + \frac{110x \times 25}{200} = 190\tfrac{1}{20}$$

$$\therefore 240 \times 35x + 600 \times 7x + 220 \times 25x = 76020$$

$$\therefore 8400x + 4200x + 5500x = 76020$$

$$\therefore 18100x = 76020$$

$$\therefore x = 4\tfrac{1}{5}$$

Ansr.—£4 4s. per quarter.

EXAMPLE (xx)—

The profits made by the manufacturers, wholesale dealers, and retail dealers are 8, 10, and 12 per cent. of their respective outlays. Goods are sold to the consumers by the retailers for £1,247 8s. Find the respective profits of the retailer, wholesale dealer, and manufacturer of these goods.

Let £ $x \equiv$ cost of goods to manufacturers, in

then £ $\frac{108x}{100} \equiv$ „ „ wholesale dealers

then £ $\frac{110 \times 108x}{100 \times 100} \equiv$ „ „ retail „

then £ $\frac{112 \times 110 \times 108x}{100 \times 100 \times 100} \equiv$ „ „ consumers

$$\therefore \frac{112 \times 110 \times 108x}{100 \times 100 \times 100} = 1247\tfrac{3}{8}$$

$$\therefore x = \frac{6237 \times 100 \times 100 \times 100}{5 \times 112 \times 110 \times 108}$$

$$= 937\tfrac{1}{2}$$

$$\therefore \quad \text{Manufacturer's profit} = £\frac{8}{100} \times \frac{1875}{2} = £75$$

$$\therefore \text{Wholesale dealer's profit} = £\tfrac{1}{10} \times 1012\tfrac{1}{2} = £101 \text{ 5s.}$$

$$\therefore \quad \text{Retail} \quad ,, \quad ,, \quad = £\tfrac{12}{100} \times 1113\tfrac{3}{4} = £133 \text{ 13s.}$$

Example (xxi)—

A bought an article and sold it to B, gaining 12 per cent. on his outlay. B sold it to C, gaining 10 per cent. on his outlay. If C paid £8 14s. more for it than A did, what did A pay for it?

Let $£ x \equiv$ cost of the article to A,

then $£ \frac{112 \times 110x}{100 \times 100} \equiv$,, ,, C,

$$\therefore \frac{112 \times 110x}{100 \times 100} - x = 8\tfrac{7}{10}$$

$$\therefore 12320x - 10000x = 87000$$

$$\therefore 2320x = 87000$$

$$\therefore x = 37\tfrac{1}{2}$$

Ansr.—£37 10s.

Example (xxii)—

A man has a sum of £2,500 deposited partly in one bank, which pays $3\frac{1}{2}$ per cent. interest, and the remainder in another which pays $2\frac{1}{2}$ per cent. interest. At the end of the year he received £76 5s. interest. What sums respectively were deposited in the banks?

Let $£x \equiv$ sum deposited in bank paying $3\frac{1}{2}\%$ interest,
then $£(2500 - x) \equiv$,, ,, $2\frac{1}{2}\%$,,

$$\therefore \frac{7x}{200} + \frac{5(2500 - x)}{200} = 76\tfrac{1}{4}$$

$$\therefore 7x + 12500 - 5x = 15250$$

$$\therefore 2x = 2750$$

$$\therefore x = 1375$$

Ansr.—£1,375 and £1,125 in 1st and 2nd banks respectively.

Example (xxiii)—

When a certain coal merchant raises his prices, he alters the price by some multiple of 6d. His price per ton in September was more than £1 5s.; in October he raised it, and in November he raised it again by an equal amount A man bought 2 tons in September, 3 in October, and 5 tons in November. The bill for the whole was £14 14s. 6d. What was the price per ton in September, October, and November?

Let x sixpences $\equiv$ cost of 1 ton of coal in September,
,, y ,, $\equiv$ each increase in price per ton,
then $(x + y)$,, $\equiv$ cost of 1 ton of coal in October,
and $(x + 2y)$,, $\equiv$,, ,, ,, November,

$$\therefore\ 2x + 3(x + y) + 5(x + 2y) = 589$$
$$\therefore\ 2x + 3x + 3y + 5x + 10y = 589$$
$$\therefore\ 10x + 13y = 589$$
$$13y = 589 - 10x$$

From data, x is more than 50 : also as y cannot be a negative quantity, x must be less than 59. Thus, x must be a number from 51 to 58 that causes y to be an integer (from data). By trial, $589 - 10x$ is divisible by 13 when $x = 55$ and equation becomes $13y = 39$, so that $y = 3$.

Ansr.—27/6, 29/–, 30/6.

EXAMPLE (xxiv)—

A, B, and C together do a piece of work in h hours. A and B separately take a and b hours respectively. How long would C take working by himself?

Let x hours $\equiv$ time taken by C,

then $\frac{1}{x} + \frac{1}{a} + \frac{1}{b} \equiv$ fraction of the work done in 1 hour, when working together

$$\therefore\ \frac{1}{h} = \frac{1}{x} + \frac{1}{a} + \frac{1}{b}$$

Mult. both sides by $hxab$, then $xab = hab + xhb + xha$

$$\therefore\ xab - xhb - xha = hab$$
$$\therefore\ x(ab - hb - ha) = hab$$

Ansr.—$\dfrac{hab}{ab - h(a + b)}$ hours.

TEST EXERCISES I, 8.

(1) A field having four sides of lengths 475′, 412′, 374′, and 215′ respectively is represented on a plan such that the longest side is represented by a line 2′ 4½″ in length. What is the scale of the map? What are the lengths of the lines representing the other sides? If the area of the field be 21¾ ac., what would be the area of the plan to the nearest tenth of a square inch?

(2) A wire rope is stretched from the top of a mast to a point on the ground 22′ 6″ from the mast. A man, whose height is 5′ 8½″, stands upright underneath the wire, so that his head just touches it. If his feet are 2′ 2½″ from the point where the rope is fastened to the ground, find the height to the nearest inch of the mast.

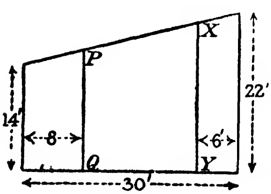


FIG. 74.

(3) Figure 74 represents the side of a shed. Find the length of the supports PQ and XY, using the dimensions given.

(4) From a uniform sheet of metal, two pieces of similar shape are cut out, the lengths being 7·3 cm. and 4·1 cm. respectively. If the weight of the larger be 43·623 gm., what would be the weight of the smaller? Also what would the length of another similar piece be, in order that it should weigh 30 gm.?

(5) A boy measured the length of the

shadow of a poplar and found it to be 52′ 4″ long. He then fixed a pole vertically in the ground and found that the height of the pole was 5′ 9″ and the length of its shadow 4′ 10″. What was the height of the poplar ?

(6) The weights of a penny, a halfpenny, and a farthing are $\frac{1}{3}$ oz. (Avoirdupois), $\frac{1}{6}$ oz., and $\frac{1}{10}$ oz. respectively. The diameter of a halfpenny is 1 in. If the thickness be the same in each case, what should the diameters of a penny and a farthing be ?

(7) On a certain map, $4\frac{1}{2}$ miles is represented by $1\frac{1}{16}$ in. What is the representative fraction, and what area in square miles and acres to the nearest acre is represented by 1 sq. in. ?

(8) A map of France is drawn on a scale of $\frac{1}{20000}$. How many inches to 100 miles is this ? The approximate area of France being 240,000 square miles, what would be the area of the map and what area would be represented by a square on the map whose side was $2\frac{1}{2}$ in. ?

(9) A cylindrical bucket whose interior diameter is 1′ 2″ holds $10\frac{1}{2}$ gall. What should be the diameter of a bucket whose height is the same, but which is required to hold 12 gall ?

(10) The heights of two men of the same build are 5′ $3\frac{1}{2}$″ and 5′ $8\frac{1}{2}$″. If the weight of the former be 9 st. 4 lb., what should be the weight of the latter to the nearest lb. ?

(11) A model of an engine is such that its length is $\frac{1}{36}$ of the true length. If the model weighs 12 lb. 5 oz., what is the weight of the engine ?

(12) A model of a bronze statue is made on a scale of $1\frac{3}{4}$″ to 1 foot. If the cost of the bronze for the model was £4 15s. 6d., what would be the cost of the bronze for the statue ?

(13) The lengths of two potatoes of similar shape are $2\frac{1}{4}$″ and $7\frac{1}{2}$″ respectively. How many times is the weight of the latter greater than that of the former ?

(14) A precious stone worth £600 is dropped and broken into 3 portions whose weights are approximately in the ratio of 3 : 5 : 7. Assuming the value to be proportional to the square of the weight, calculate the loss incurred by the breakage.

(*Note.*—The ratio of the values after and before breakage is—
$(3^2 + 5^2 + 7^2) : 15^2$.)

(15) Two oil cans of similar shape are 1′ 6″ and 1′ 10″ high respectively. If the first holds $2\frac{1}{2}$ gall , what is the capacity of the second ?

(16) A pyramid is 10″ high and weighs 5 lb. 14 oz. If a portion be cut off by a plane parallel to the base and 7″ from the vertex, what would be the weight of the remaining portion ?

(17) The diameter of a bicycle wheel is 28 in. How many revolutions does it make in going 1 mile ? ($\pi = 3\frac{1}{7}$.)

(18) It is required to make a circular piece of metal which is such that it could have 100 cogs of $\frac{1}{8}$″ pitch round its circumference. What should the diameter be to the nearest $\frac{1}{64}$ in. ? ($\pi = 3\frac{1}{7}$.)

(19) An archway is in the form of an arc of a circle radius 26 ft. and the angle subtended is 50°. How many blocks of stone 9″ wide will be required for building the lower edge of the arch ? ($\pi = 3\frac{1}{7}$.)

(20) A horse is tethered by a rope 40 ft. long, which is fastened to the ground at a corner of a field where the angle is 105°. Over what area can he graze ? ($\pi = 3\frac{1}{7}$.)

(21) A circular pond 70 ft. across has a path 7 ft. wide three-quarters of the way round. Find the cost to gravel the path at $7\frac{1}{2}$d. per square yard. ($\pi = 3\frac{1}{7}$.)

(22) A rectangular plot of ground was extended by the addition of a semicircular plot, the diameter of which was one of the shorter sides The rectangle measured 58 yd. × 35 yd. What was the total area? ($\pi = 3\frac{1}{7}$.)

(23) How many cylindrical jars of internal diameter $3\frac{1}{2}$ in. and height $4\frac{1}{2}$ in. can be filled from a cylindrical cask of internal diameter 2 ft. 3 in. containing oil to a depth of 2 ft. 9 in.?

(24) A cylindrical corn bin is 4 ft. 6 in. high and 3 ft. 8 in. in diameter How many bushels of corn will it hold if 1 bush. occupies $1\frac{3}{8}$ cub. ft.? ($\pi = 3\frac{1}{7}$)

(25) A circular top to a table has a diameter of $3\frac{1}{2}$ ft. The inner portion is covered with leather, leaving a margin within the circumference of 4 in. in depth. What is the price of the leather at $1\frac{3}{4}$d. a square inch, and what is the area of the uncovered margin? ($\pi = 3{\cdot}1416$.)

(26) A cylindrical cistern is 7 ft. 6 in. in diameter. How many gallons (to nearest gallon) are there in the cistern when the depth of the water is 4 ft. 3 in.? (1 gall. = 277·274 cub. in. $\pi = 3{\cdot}1416$.)

(27) Find the number of loads of earth that must be removed in making a circular well 215 ft. deep and 5 ft. 6 in. in diameter, if $1\frac{1}{4}$ cub. yd. go to a load. ($\pi = 3\frac{1}{7}$.)

(28) Find the total area of surface of a cylindrical boiler whose length is 9 ft. 6 in. and diameter 5 ft. 9 in. ($\pi = 3\frac{1}{7}$)

(29) The internal diameter of a metal pipe is $2\frac{1}{4}$ in. and the thickness of metal is $\frac{3}{16}$ in. What is the weight of 1 mile of this pipe, if 1 cub. ft. of the metal weighs 535 lb.? ($\pi = 3\frac{1}{7}$.)

(30) A plot of land has the shape formed by a square having a semicircle about each of the four sides as diameter. If the length of the side of the square be 54 ft., what ought the rent of the land to be at £75 an acre? Give the answer to the nearest shilling.

(31) What is the length of a straight path from one corner of a rectangular field 95 yd. long and 72 yd. broad to the opposite corner? What distance is saved by using this path instead of going round the edge?

(32) Show that the length of the diagonal of a square is obtained by multiplying the length of the side by $\sqrt{2}$.

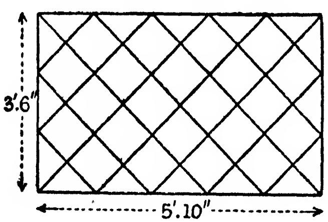


FIG. 75.

The diagram represents an ornamental door of a piece of furniture, and the diagonal lines represent strips of mahogany. Find (1) the length of wood to nearest inch required (perform one multiplication only); (2) the cost of the wood at $6\frac{1}{2}$d. a foot. In both cases the outside framework is not included.

(33) The height of a conical tent is 12 ft. and the diameter of the base is 10 ft. Find the volume of air enclosed and also find the area of canvas.

(34) A ladder 45 ft. long has its lower end 13 ft. from a wall, and its upper end rests against the wall. How high is the upper end from the ground?

If the lower end be pushed 8 ft. nearer the wall, by what distance will the upper end be raised?

(35) A stone column consists of a cylinder surmounted by a hemisphere. If the total height is 10 ft. and the diameter 1′ 9″, what is the weight of the column if 1 cub. ft. of stone weighs 356 lb.?

(36) An earthenware bottle, in the form of a cylinder surmounted by a hemisphere, holds 1 qt. Its external diameter is $3\frac{7}{8}$″ and its external height

(not counting the neck) is $8\frac{1}{2}''$, and its weight when empty is 2 lb. 9 oz. Find the weight of the earthenware per cubic foot. (1 gall. $= 277$ cub. in., $\pi = 3\frac{1}{7}$.)

(37) A solid figure consists of a hemisphere and cone whose diameters are the same. Obtain formulae for the total volume and area of surface.

(38) A bucket, having the shape of a frustum of a cone, has interior diameters $1'\ 3''$ and $1'\ 9''$, and it is $1'\ 4''$ deep. How many pints does it hold when full? What is the area of the curved surface? What would be the capacity and area of curved surface of a bucket of similar shape whose depth is $1'\ 8''$?

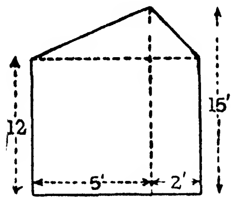


FIG. 76.

(39) Figure 76 shows the section of a hut which is 25 ft. long. Find the cost to cover the roof with tarpaulin at 3s. 6d. per square yard.

(40) Figure 77 represents a section of the interior of a bottle whose shape consists of a cylinder and hemisphere having a cone removed from the bottom. Find a formula for the total volume, neglecting neck, and evaluate when $H = 11''$, $h = 3''$, $r = 2''$, $\pi = 3\frac{1}{7}$.

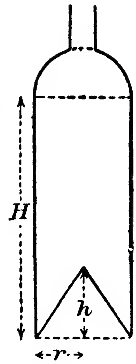


FIG. 77.

(41) A dome in the form of a hemisphere has a diameter of 35 ft. What would be the cost of covering it with lead at 5s. 9d. per square yard?

(42) A circular pond in a park is 75 ft. in diameter and is surrounded by a gravel path 12 ft. wide. Find the cost to the nearest shilling at 8s. 6d. per cubic yard of fresh gravel for the path to cover it to a depth of 3 in.

(43) A tent consists of a cone resting on a cylinder $2'\ 3''$ high and $15'$ in diameter, and the vertex of the cone is $14'\ 6''$ high. Find the cost of canvas at 4s. 9d. per square yard and the volume of air enclosed by the tent.

(44) The length of a rectangular field is three times the breadth and the area is $1\frac{1}{4}$ ac. Find the dimensions of the field.

(45) Coal is stacked in the form of a frustum of a rectangular pyramid, the dimensions of the base being 35 ft. $\times$ 28 ft. and those of the top 20 ft. $\times$ 16 ft., while the height of the stack is 6 ft. Given that a ton of coal occupies 30 cub. ft., find to the nearest ton the weight of coal in the stack.

(46) What is the radius of a circle which has the same area as an equilateral triangle whose side is 10 ft. long?

(47) A cylindrical vessel whose internal diameter is 17·7 cm. contains oil to a depth of 16·4 cm. If this be poured into another cylindrical vessel whose internal diameter was 20 cm., what would be the depth?

(48) A cylindrical vessel whose depth is 5 in. is to be such as to hold $\frac{1}{2}$ pt. What must be the internal diameter to the nearest $\frac{1}{10}''$?

(49) Given that 1 cub. ft. of bronze weighs approximately 8,400 oz., find the average thickness of a halfpenny. (*See* Question (6).)

(50) A building is to be erected whose plan has the shape of a square with a semicircle on one side as diameter. If the site is not to exceed $\frac{1}{4}$ ac., what is the greatest number of feet the frontage of the building could be?

(51) It is required to store petrol in 4-gall. tins which have the shape of a rectangular prism and are 1 ft. high. If the length of the base be twice the breadth, what must be the dimensions? What will be the total weight, if the tin, when empty, weighs 1 lb. 6 oz.? (The specific gravity of petrol is ·61 lb.)

(52) What must be the depth of a conical funnel whose internal diameter is 8″ and which is to be capable of holding 1 qt. ?

(53) A sphere, radius 4·3″, of copper is to be melted down and drawn out into wire so as to obtain two equal lengths of wire of diameters ·15 in. and ·2 in. respectively. What will be the length of each wire ?

(54) Rectangular pieces of paper 1′ 4″ long and 5″ wide are used to completely cover the curved surface of cylindrical tins of condensed milk. Allowing $\frac{1}{2}$″ for overlapping, calculate the volume of the tins in cubic inches.

(55) A pair of wheels connected by an axle 6 ft. long have diameters 4 ft. 11 in. and 5 ft. respectively. They are rolled on the ground. Find the diameter to the nearest foot of the circle the outer wheel will describe.

(56) A length of 30 ft. of lead pipe weighs 115 lb., and a gallon of water will fill it. Given that 1 cub. ft. of lead weighs 704 lb., and that a gallon is approximately 277 cub. in., find the thickness of the lead.

(57) Brass tubing of external diameter $\frac{1}{2}$ in. is sold at $4\frac{1}{2}$d per foot or 1s. 7d. per lb. Given that 1 cub. ft. of brass weighs 524 lb , find the thickness of metal in the tube. ($\pi = 3\frac{1}{7}$.)

(58) £365 is divided among A, B, and C in such a way that B receives two-thirds as much as A and C receives £15 more than B. What do they receive respectively?

(59) A building consists of 4 floors and is let in flats, the rent of each being $\frac{9}{10}$ that of the floor below. If the owner wishes to obtain at least £200 per annum for letting the flats, what is the least amount in pounds and shillings he could accept for the ground floor?

(60) A bought an article for £85 and sold it to B, who sold it to C for £122 8s. If A and B gained the same percentage profit on their respective outlays, what did B pay for the article?

(61) A woman bought a number of oranges at 7d. a dozen. She sold one-third at 1d. each and the remainder, with the exception of 6, which were bad, she sold at 3 for 2d. She made a profit of 2s. 7d. How many oranges did she buy ?

(62) A sum of £500 was partly deposited at 4 per cent per annum and partly at 5 per cent. per annum. The yearly income was £22 5s. Find how much was deposited at each rate.

(63) A, B, and C are in partnership, contributing £6,000, £4,500, and £3,500 respectively on the understanding that after allowing C 15 per cent. of the total profits for his services as manager, the remainder shall be divided in proportion to the capital contributed. At the end of the year, C received £447 5s. What should A and B receive ?

(64) A man spent 20 per cent. of his money, then £50, and then 15 per cent. of the remainder. If he had £1,698 6s. left, what was his original money ?

(65) At a factory, 12 men, 30 women, and 25 boys are employed. Each woman earns 9s. more than the wages of each boy, and each man earns 6d. more than twice the amount earned by each woman. The weekly wages' bill is £91 15s. What are the weekly wages of each ?

(66) Divide £1,221 among X, Y, and Z so that Y receives 20 per cent. more than X and Z 25 per cent. more than Y.

(67) A man estimated that by letting a house for £90 per annum and allowing a certain amount annually for repairs, he would obtain 8 per cent. on the purchase money. He let the house for £84 per annum, and had to allow £4 more annually than he intended for repairs, and gained 6 per cent. on the purchase money. Find the estimated yearly sum for repairs, and hence find what he paid for the house.

(68) A grocer added 18 lb. of coffee and 15 lb. of chicory to 1 cwt. of a mixture of coffee and chicory, and by so doing obtained a mixture consisting of 4 parts of coffee to 1 part of chicory. What was the ratio of the amounts of coffee to chicory in the original mixture ?

(69) A fruit merchant bought 20 tons of apples. He sold 8 tons, gaining 10 per cent. profit; 7 tons at $12\frac{1}{2}$ per cent. profit and 5 tons at 15 per cent. profit, each percentage being based on purchase price. Find the amount per ton he paid for the apples if his total profit was £31 10s. 6d. Also find the percentage profits in each case based on the selling prices.

(70) A retail tobacconist bought tobacco at a certain price per lb. and sold it, thereby gaining $16\frac{2}{3}$ per cent. profit. If he had to pay 1s. 4d. per lb. more by selling at 1s. 4d. per lb. more, he estimated that he would gain $14\frac{2}{7}$ per cent. profit. What did he pay per lb. for the tobacco?

(71) A retailer bought a number of eggs at 13s. 6d. per 100. He sold all except 65, which were bad, at 5 for 1s., and thereby gained £1 15s. 9d. How many eggs did he buy?

(72) A sum of money was borrowed for a year. The rate for 7 months was 4 per cent. per annum and for the remaining time 5 per cent. per annum. The total interest was £26 10s. What was the sum borrowed?

(73) A fruiterer sold a certain number of pears for £1 3s. 9d. If he had given one more pear for 1s., the amount realized would have been only 19s. How many pears did he sell?

(74) A dealer marked an article 25 per cent. above cost price, but allowed a customer 10 per cent. discount for cash. He gained £1 11s. 6d. What did the article cost him?

(75) A dealer bought a total of 100 articles of three different qualities priced at 16s., 17s. 6d., and 20s. 6d. each respectively, for which he paid £86 15s. The number of the cheapest kind was an exact number of dozens, and between 30 and 40 of the articles of medium quality were bought. How many of each kind did he buy?

(76) The prices of cycles of grades I, II, III, and IV are multiples of half a guinea, and the differences in price between grades I and II, II and III are the same, this difference being $\frac{1}{2}$ guinea less than that between grades III and IV. The total cost of 500, 220, 80, and 200 cycles of grades I, II, III, and IV respectively is £6,898 10s. What is the cost of a cycle of each grade?

CHAPTER IX.

USE OF SQUARED PAPER.

88. AREA OF IRREGULAR FIGURES.

AREAS of figures composed of simple geometric shapes can be readily calculated, but in actual practice it is often necessary to find the approximate area of irregular figures. The method by which squared paper can be put to use for this purpose is illustrated by the following example—

EXAMPLE (i)—

The figure represents the plan of an estate drawn on a scale of 3 in. to a mile. What is the area of the estate in acres?

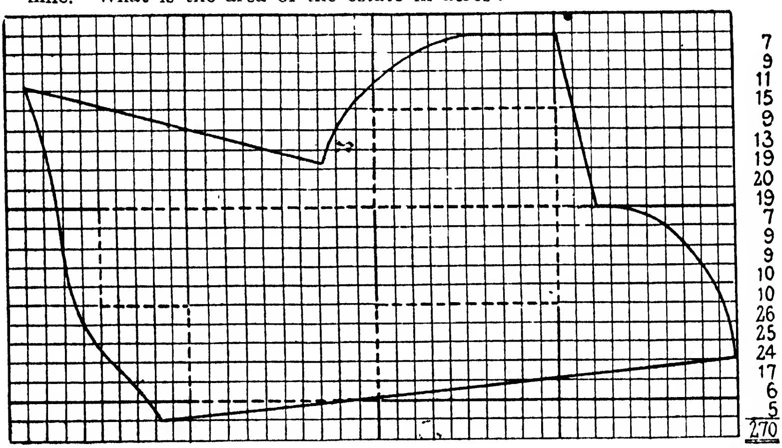


FIG. 78.

$$\text{Total number of small squares enclosed} = 9 \times 25 + 270$$
$$= 495$$
$$\text{Area of plan} = 495 \times \cdot 01 \text{ sq. in.}$$
$$= 4\cdot 95 \text{ sq. in.}$$

Now 1 sq. in. on plan represents $\frac{640}{9}$ acres

$$\therefore \text{ Area of estate} = \frac{4\cdot 95 \times 640}{9} \text{ acres}$$
$$= 352 \text{ acres.}$$

NOTE 1.—Including as many large squares as the figure will permit by dotted line, the number of small squares remaining should be added up in rows, the results being put down in their proper places. In determining the number in a row, a general rule is to consider portions of small squares greater than one-half as whole squares and to neglect portions less than one-half; but should there be, say, three

portions each nearly one-half of a small square, these together might be taken as a whole small square, etc.

With care, the error ought not to exceed ·05 sq. in., so that the resulting percentage error ought not to exceed $\frac{5}{575} \times 100$, *i.e.*, less than 1%.

89. COMPARISONS.

A comparison between quantities taken from statistics can be graphically represented very simply, as the following example shows—

EXAMPLE (ii)—

The chief exports from India during a certain year were—

Jute Manufactures	£25,319,000
Raw Cotton	£16,523,000
Tea	£13,321,000
Raw Jute	£10,428,000
Rice	£10,304,000
Seeds	£ 6,748,000
Hides and Skins	£ 6,529,000
Cotton Yarn and Manufactures	£ 6,404,000
Wheat and Flour	£ 6,379,000
Leather	£ 3,759,000

Represent these values to scale.

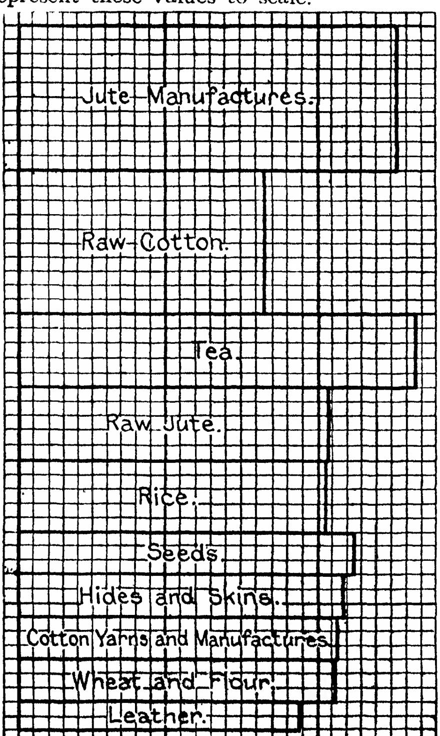


FIG. 79.

Area of 1 small square represents £100,000.

NOTE 2.—The values of the imports in £100,000 can be expressed approx. as follows: 10 × 25·3, 10 × 16·5, 5 × 26·6, 5 × 20·8, 5 × 20·6, 3 × 22·5, 3 × 21·8, 3 × 21·3, 3 × 21·3, and 2 × 18·8. Thus rectangles having their lengths and breadths these numbers of units respectively have areas which show the comparison of the values of the exported goods.

NOTE 3.—If the breadth of the rectangles had been made the same, their lengths would be proportional to the values represented.

90. PROPORTIONAL QUANTITIES.

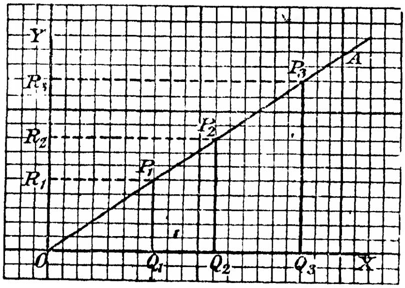


FIG. 80.

It has already been seen that if P_1Q_1, P_2Q_2, and P_3Q_3 are parallel, then

$$\frac{P_1Q_1}{OQ_1} = \frac{P_2Q_2}{OQ_2} = \frac{P_3Q_3}{OQ_3} = \text{etc.}$$

If P_1R_1, P_2R_2, and P_3R_3 be drawn parallel to OX, then $OR_1 = P_1Q_1$, $OR_2 = P_2Q_2$, and $OR_3 = P_3Q_3$

$$\therefore \frac{OR_1}{OQ_1} = \frac{OR_2}{OQ_2} = \frac{OR_3}{OQ_3} = \text{etc.}$$

Thus if a line be drawn passing through the origin O, the distances of points on the line from OX are proportional to the corresponding distances from OY. In order to make use of squared paper, the lines OX and OY are usually drawn at right angles.

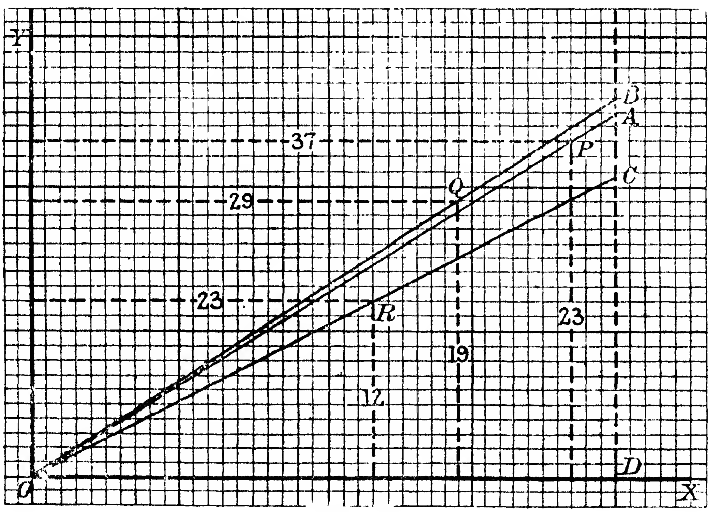


FIG. 81.

Conversely, if two quantities are proportional to one another, and if one value of the one be represented by a distance from O along OX and the corresponding value of the other by a distance

from O along O,Y then by drawing perpendiculars to OX and OY, a point of intersection is obtained. If other sets of corresponding values be likewise dealt with, all the points obtained lie on a straight line passing through O. In other words, the graph connecting two proportional quantities is a straight line through the origin.

EXAMPLE (iii)—

Write down the fractions $\frac{23}{37}$, $\frac{19}{29}$, and $\frac{12}{23}$ in order of magnitude.

Referring to Figure 81, it is seen that the fractions in order of magnitude are $\frac{19}{29}$, $\frac{23}{37}$, $\frac{12}{23}$.

NOTE 4.—The points P, Q, and R are obtained as indicated in Figure 81, and lines are drawn connecting them with the origin.

Now $\frac{BD}{40} = \frac{19}{29}$, $\frac{AD}{40} = \frac{23}{37}$ and $\frac{CD}{40} = \frac{12}{23}$ and as $BD > AD > CD$, $\frac{19}{29} > \frac{23}{37} > \frac{12}{23}$

It should be noted that the greater the slope of the line, the greater the value of the fraction represented.

EXAMPLE (iv)—

A man invested £80, £135, and £180 respectively in three concerns, A, B, and C, and at the end of a year the interest received was £3 12s., £4 19s., and £9 9s. respectively. Find the rate per cent. in each case.

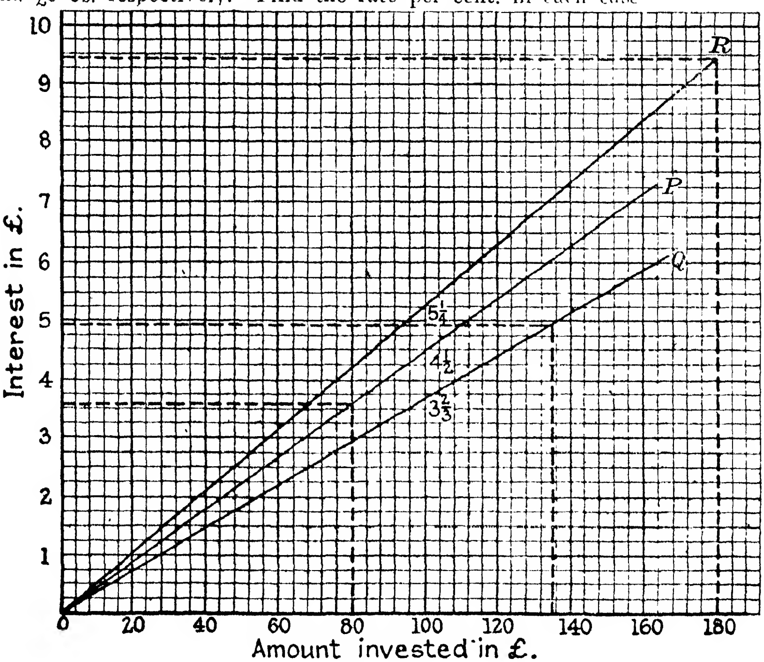


FIG. 82.

The rates of interest are $4\frac{1}{2}$%, $3\frac{2}{3}$%, $5\frac{1}{4}$%.

NOTE 5.—From consideration of the lines OP, OQ, and OR, it is seen that $\frac{3\frac{3}{5}}{80} = \frac{4\frac{1}{2}}{100}$, $\frac{4\frac{19}{20}}{135} = \frac{3\frac{2}{3}}{100}$ and $\frac{9\frac{9}{20}}{180} = \frac{5\frac{1}{4}}{100}$. It is sometimes difficult to judge fractions of units; so that the larger the graph, the less the error is likely to be. Thus units along the axes should be chosen as big as space permits.

Often, only a comparison of the investments is required, in which case the line having the greatest slope indicates the best investment. (Sect. IV, Chap. III.)

EXAMPLE (v)—

Given that 1 litre = 1·76 pints, draw a graph connecting these units, and hence express 3·2 litres in pints and $4\frac{1}{4}$ pints in litres.

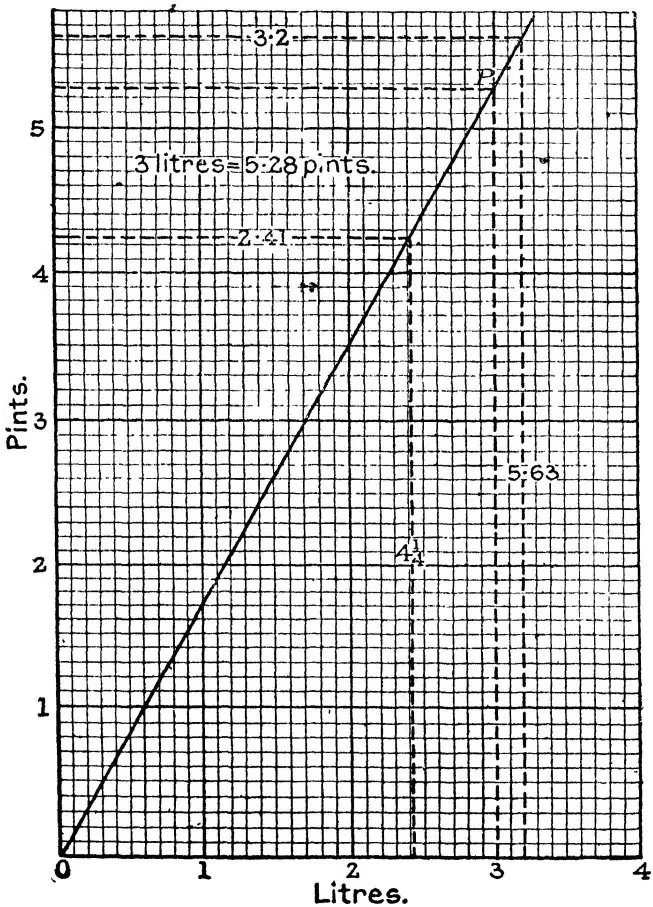


FIG. 83.

$\begin{cases} 3{\cdot}2 \text{ litres} = 5{\cdot}63 \text{ pints} \\ 4\frac{1}{4} \text{ pints} = 2{\cdot}41 \text{ litres} \end{cases}$

NOTE 6.—If a number of quantities of a liquid be, first, measured in litres and, secondly, in pints, the number of litres will be proportional to the number of pints, for in the case of each pair of corresponding numbers of litres and pints the ratio will be 1 : 1·76. The graph, therefore, will be a straight line through O.

To draw the line as accurately as possible, the point to which O is joined should be as far from O as possible. From the relation given, 3 litres = 5·28 pints, and from this the point P is obtained; but if space had permitted, it would have been better to obtain the point by the relation 5 litres = 8·8 pints.

EXAMPLE (vi)—

The distances between stations A and B, B and C, and C and D are 6, 10, and 8 miles respectively. A train leaves A at 9 a.m. and stops 3 min. at B, 4 min. at C, and 5 min. at D, the average speeds between A and B, B and C, and C and D being 12, 15, and 15 miles per hour respectively, while after leaving D its average speed is 20 miles per hour. A second train leaves A at 10 27 a.m. and travels in the same direction at a uniform speed of 40 miles per hour. When and where does it overtake the first? If a third train leaves D at 10.30 a m. and travels in the opposite direction at a uniform speed of 25 miles per hour, when and where will it pass the first two trains?

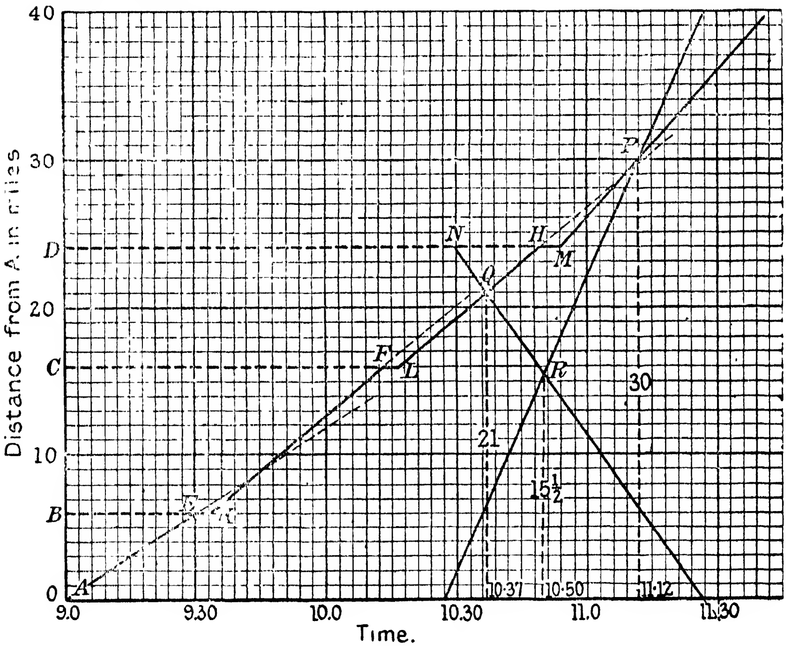


FIG. 84.

2nd train overtakes the 1st at 11 12 a.m., 30 miles from A
3rd passes 1st at 10.37 a.m. approx., 21 miles approx. from A
3rd ,, 2nd ,, 10.50 a.m. ,, 15½ ,, ,,

NOTE 7—If a train travels at uniform speed from a certain point, the distance from that point is proportional to the time taken, so that the graph connecting distance with time will be a straight line passing through that point. The continued dotted lines show how the slopes of the lines AE, KF, and LH are

obtained. While the train is stationary, the graph will be a straight line parallel to the time axis, for the distance from A remains the same as the time increases.

The point P where the graphs, representing the motions of the 1st and 2nd trains, intersect determines the time and distance from A required.

At 10.30 a.m., the 3rd train is at D, so that the graph representing its motion is a straight line starting from N. Now, as the distance from A decreases as the time increases, the slope of the line is $\frac{25}{1}$ downwards. The points of intersection Q and R determine the other results required.

If the graph be drawn on a large scale, accurate results are obtained. Graphs of this kind are employed by railway companies. Large frames are used; the positions A, E, K, etc., obtained; and pegs are placed in holes in the frame: then threads (of different colour to represent motions of different trains) are drawn round the pegs, so that on being drawn tight they at once determine the graphs. The points where the threads cross determine the times when and positions where the trains overtake or pass one another.

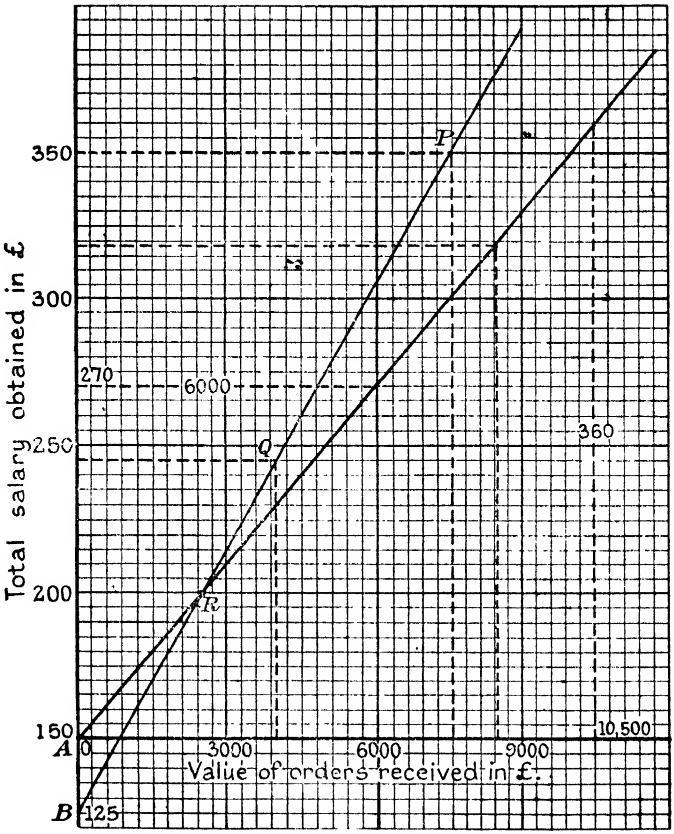


FIG. 85

EXAMPLE (vii)—

A commercial traveller received a fixed salary of £150 and a certain commission on the value of orders obtained. The first year he obtained orders to the value of £8,450, and received £319. What should he receive

if he obtained orders to the value of £10,500 ? What would be the value of the orders if the total salary received had been £270 ?

Ansr. (Fig. 85.) If orders amounted to £10,500, salary would be £360.
,, salary had been £270, orders would have amounted to £6,000.

NOTE 8.—As the amount of salary above £150 is proportional to the value of orders received, the graph will be a straight line passing through the point *A*.

EXAMPLE (viii)—

Another traveller whose total salary consists of a fixed amount plus commission on the value of orders, received £245 one year when he obtained £4,000 worth of orders, and £350 when the orders were valued at £7,500. What was the fixed portion of his total salary ?

The fixed portion of salary is £125.

NOTE 9.—The points *P* and *Q* (Fig. 85) are obtained from the data, and from consideration of Example (vii) it is seen that the graph is a straight line through *P* and *Q*. The line *PQ* cuts the salary axis at *B*, which determines the fixed portion of salary.

The position of *R*, the point of intersection of the graphs, shows that if the value of orders be £2,500, the total salary for each traveller would be the same, namely, £200

91. Fluctuations can readily be seen by graphical representation.

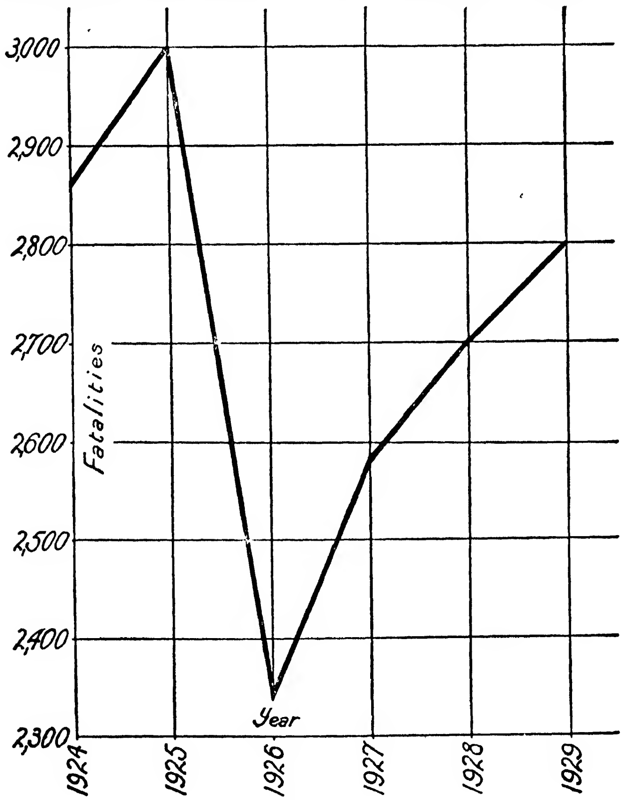


FIG. 86.

EXAMPLE (ix)—

The numbers of fatal accidents to workmen where compensation was paid during the period 1924–29 inclusive were 2,865, 3,019, 2,332, 2,581, 2,705, and 2,799. Draw a graph illustrating the fluctuation (Fig. 86).

92. INTERPOLATION

It has been seen that if one quantity varies directly as another, the graph connecting these quantities is a straight line passing through the origin. If now a constant quantity be added to the second, the graph will be a straight line parallel to the former. Thus if one quantity varies as the difference of another quantity from a certain fixed amount, the graph will be a straight line, not, however, passing through the origin.

If one quantity varies inversely, or as the square, etc., of another quantity, the graph is a curve which can be drawn when sets of corresponding values are known. The graph can then be used for finding the approximating value of one quantity for a given value of the other quantity. This process is known as Interpolation.

EXAMPLE (x)—

Find the cube root of 14·6.

Cube Root	Number.
2	8
2·2	10·648
2·4	13·824
2·6	17·576

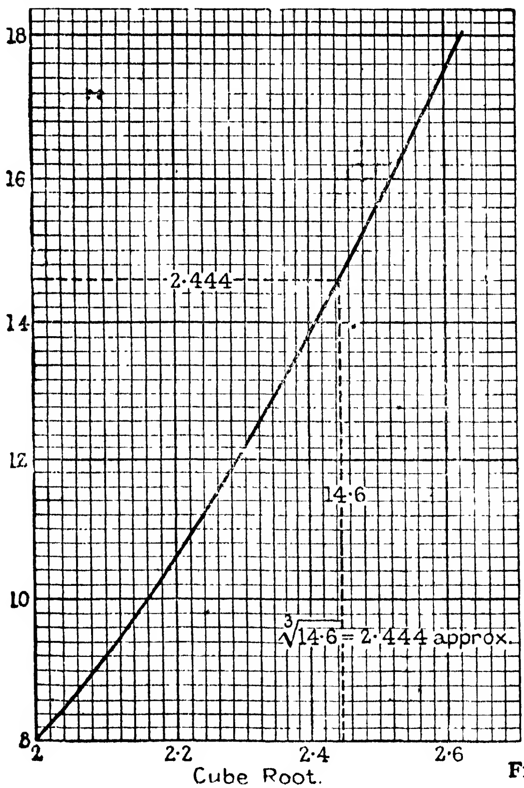


FIG. 87

Example (xi)—

A furniture retailer offers to sell articles whose cash values are 10, 50, and 100 guineas respectively for 8s. 4d., £1 16s., and £3 7s. 6d. per month during a period of 3 years. What should be paid per month for furniture valued at 70 guineas? What value of furniture could be bought for 25s. per month?

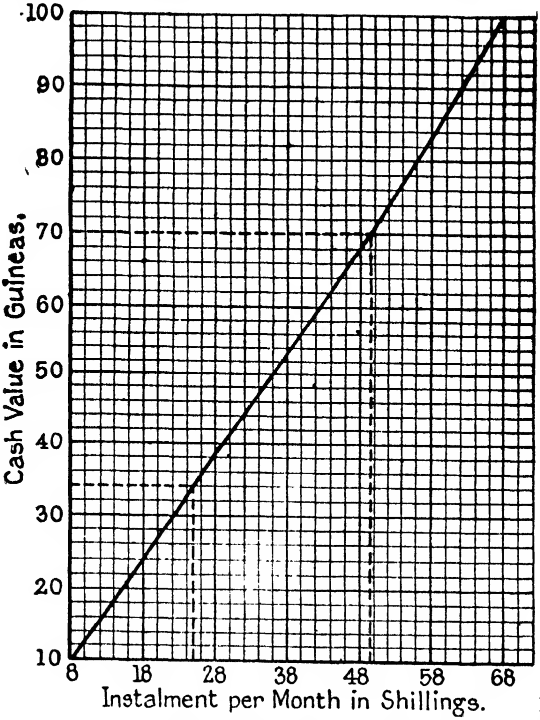


Fig. 88.

Cash value, 70 guineas; instalment, £2 9s. 4d. per month.
Instalment, 25s. per month; cash value, 34 guineas.

Example (xii)—

The distance between two stations is 120 miles. Draw a graph connecting the average speed of a train with the time taken to perform the journey. Find the speed, in order that the train should take 3 hours 10 min. over the journey.

Speed in m.p.h.	Time in hours.
60	2
50	$2\frac{2}{5}$
40	3
30	4
24	5

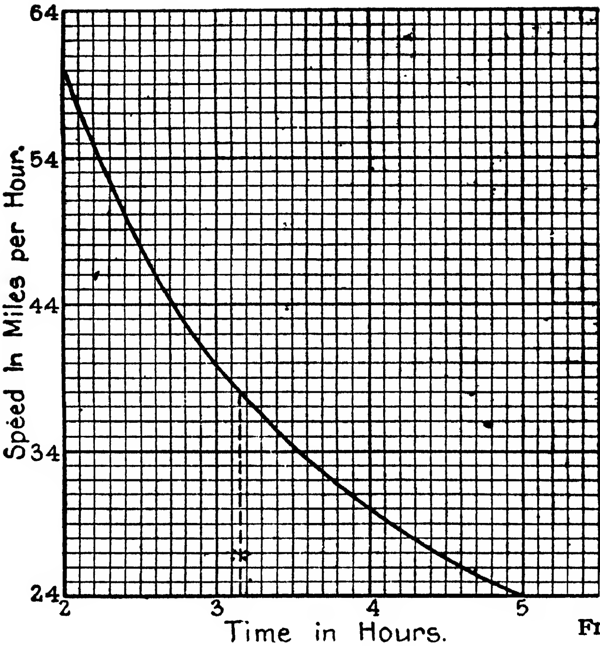


FIG. 89.

Ansr—38 miles per hour (approx.).

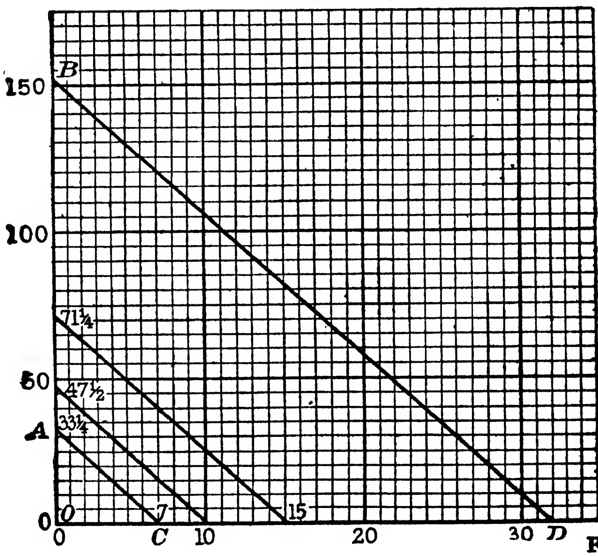


FIG. 90.

93. PROPORTIONAL PARTS.

The fact that the sides of equiangular triangles are proportional enables problems on proportional division to be solved graphically.

Example (xiii)—

Divide £152 between A, B, and C in the ratios 7 : 10 : 15.

Ansr. (Fig. 91) $\begin{cases} A \text{ receives } £33 \;\; 5s. \\ B \quad ,, \quad £47 \;\; 10s. \\ C \quad ,, \quad £71 \;\; 5s. \end{cases}$

Note 10.—As OAC and OBD are similar triangles, AC being parallel to BD

$$\frac{OA}{OB} = \frac{OC}{OD} \; i.e., \; OA = \frac{OC}{OD} \times OB = \frac{7}{32} \times 152 \text{ units.}$$

Note 11.—The student must not overestimate the utility of graphical methods, for, unless the sheet of paper be sufficiently large, results giving greater errors than are required are likely to arise.

94. INTEREST.

It has been seen that the formula giving the Simple Interest (I) in terms of the Principal (P), Rate per cent. (r), and Number of Years (n) is $I = \frac{Prn}{100}$, from which it is seen that if any two of the quantities P, r, and n be constant, I varies as the remaining quantity, so that the graph connecting I with this quantity will be a straight line.

The formula can be put in the form $r = \frac{100I}{Pn}$, from which it is clear that if I and P are constant, r varies inversely as n, while if I and n are constant, r varies inversely as P. Each of the graphs connecting r and n, and r and P, will be of the type illustrated in Ex. xii of this Chapter. If I be constant, and any one of the quantities P, r, and n also be constant, the graph connecting the remaining two will be of this type; and to construct the graph, a set of simple corresponding values must be at first obtained.

Example (xiv)—

At what rate per cent. would £80 amount to £99 16s. in $4\frac{1}{2}$ years, simple interest ?

Interest on £80 for $4\frac{1}{2}$ years at 0% = £0
,, ,, ,, 8% = £28$\frac{4}{5}$.

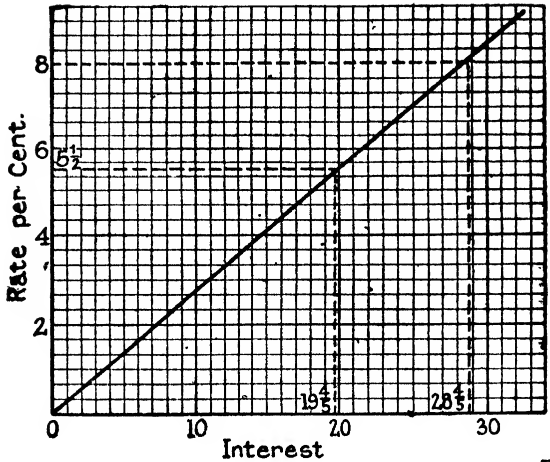


FIG. 91.

Ansr.—$5\frac{1}{2}$%.

EXAMPLE (xv)—

Tabulate corresponding values of rate per cent. ranging from 2 per cent. to 10 per cent. and number of years such that £450 will produce as interest £36.

$$n = \frac{100 \times 36}{450 \times r} = \frac{8}{r}$$

Rate per Cent.	Number of Years.	Rate per Cent.	Number of Years.
2	4	7	$1\frac{1}{7}$
3	$2\frac{2}{3}$	8	1
4	2	9	$\frac{8}{9}$
5	$1\frac{3}{5}$	10	$\frac{4}{5}$
6	$1\frac{1}{3}$		

NOTE 12.—From these sets of corresponding values, a graph can be drawn which should be as large as space permits. The student is advised to construct the graph and use it to find the rate per cent. when the time is 2 years 3 months, etc., verifying each result by calculation.

For a given rate per cent., compound interest, a graph showing the relationship between the amount of a given sum with the corresponding time can be drawn.

EXAMPLE (xvi)—

£100 is deposited, and the interest is allowed to accumulate at 5 per cent. per annum, payable yearly. Draw a graph connecting the amounts with the corresponding times.

Use the graph to find the compound interest on £160 for $9\frac{1}{2}$ years; and to find the time a sum of money doubles itself, at 5 per cent. compound interest, payable yearly.

GRAPH, SHOWING THE AMOUNT OF £100 FOR ANY PERIOD OF TIME UP TO 15 YEARS, COMPOUND INTEREST 5% PER ANNUM, PAYABLE YEARLY.

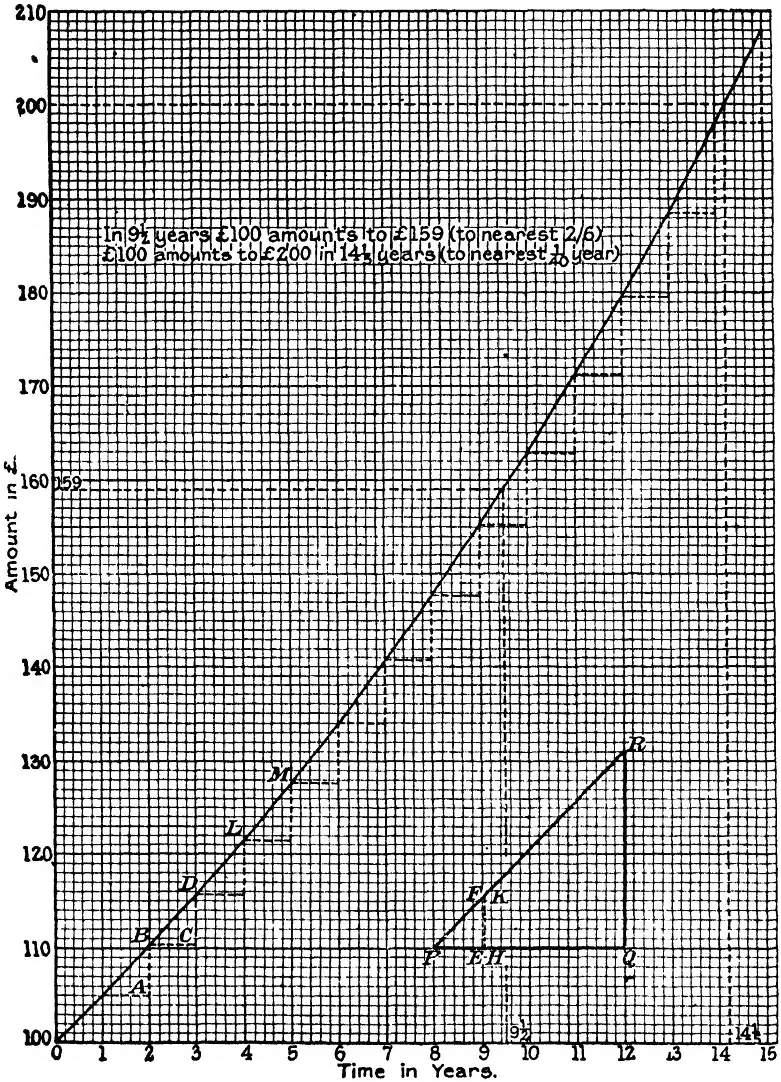


FIG. 92.

Ansr. $\left\{\begin{array}{l}\text{In } 9\frac{1}{2} \text{ years, £100 amts. to £159 (to nearest 2/6).}\\ \text{£100 amounts to £200 in } 14\frac{1}{5} \text{ years (to nearest } \frac{1}{10} \text{ year).}\end{array}\right.$

From the graph on preceding page, it is seen that £100 amounts to £159 approx. in $9\frac{1}{2}$ years.

$$\therefore \text{ Compound Int. on } £160 = £\frac{59 \times 160}{100} \text{ approx.}$$

$$= £94 \text{ 10s. correct to nearest } 5/-.$$

The time taken for a sum of money to be increased two-fold is independent of the magnitude of the sum of money, for the ratio of amount to principal each year is 105 : 100, so that the ratio of original sum to the amount at the end of n years is $105^n : 100^n$, which is the same, whatever magnitude the original sum may be.

From the graph it is seen that any sum of money is doubled at 5 per cent. compound interest, payable yearly in 14 years $2\frac{1}{2}$ months to the nearest half-month.

NOTE 13.—The ratio of the amount at the beginning of any year to that at the beginning of the previous year is 105 : 100, *i.e.*, 21 : 20. As the interest each year is proportional to the amount at the beginning of the year, it is clear that the ratio of the interest for any year to that for the previous year is also 21 : 20. Thus the yearly interests are £5, £5 $\times \frac{21}{20}$, £5 $\times \frac{21}{20} \times \frac{21}{20}$, etc., respectively

Now, by similar triangles, any quantity can be increased in a given ratio by a simple geometric construction. Referring to the triangle PQR, $\frac{QR}{PQ} = \frac{21}{20}$, so that $\frac{EF}{PE} = \frac{21}{20}$. PE represents the interest 1st year, therefore EF represents the interest 2nd year, and this distance is marked from A to B, the latter determining the amount at the end of two years. If H is marked so that $PH = EF$, then as $\frac{HK}{PH} = \frac{21}{20}$ $\therefore$ $\frac{HK}{EF} = \frac{21}{20}$, so that HK represents the interest for the 3rd year, and this distance is marked from C to D, so that D determines the amount at the end of 3 years. Proceeding in the same way, the position of the points L, M, etc., is obtained by careful use of a pair of dividers.

NOTE 14.—If the most regular curve be drawn through the points $BDLM$, etc., the resulting graph is that of the relation $A = 100 \times (1{\cdot}05)^n$, where $A \equiv$ amount of £100 at the end of n years. This is in accordance with the theory of finance; but, in practice, if the period of time involves a fraction of a year, the interest for the fraction of a year is calculated by the simple interest formula: hence the graph is obtained by joining the points B, D, L, M, etc., by straight lines.

95. MAXIMA AND MINIMA.

There is no exact mathematical law showing the relationship between the quantity of goods sold and the price at which they are sold. By plotting results obtained by actual practice, however, it is sometimes possible to discover the approximate price for which the goods should be sold, in order to gain the maximum profit which might be obtained.

EXAMPLE (xvii)—

A manufacturer can produce an article for £5 12s. For equal periods

of time he sells the goods at £7 7s., £7, £6 15s., and £6 6s., the numbers sold being 248, 320, 420, and 645 respectively. At what price per article should he sell in order to gain as large a profit as possible?

Selling Price.	Number Sold.	Total Profit.
147 shillings	248	£434
140 „	320	£448
135 „	420	£483
126 „	645	£451 10s.

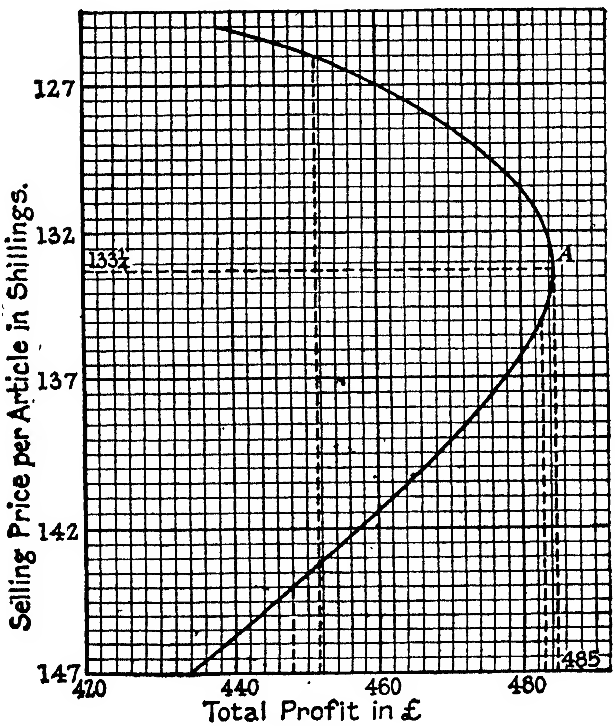


FIG. 93

Selling Price to give Maximum Profit is approximately £6 13s. 3d.

Approx. number sold $= \dfrac{485}{1\frac{1}{16}} = 456$

TEST EXERCISES I, 9.

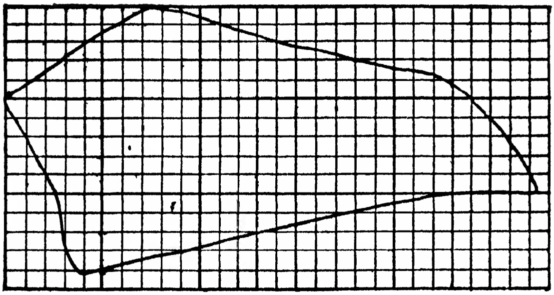


Fig. 94.

(1) The figure represents the plan of a park drawn on a scale of 6 in. to a mile. What is the area of the park in acres ?

(2) On squared paper, draw a circle and a square whose side is equal to the radius of the circle. Find the ratio of the area of the circle to that of the square.

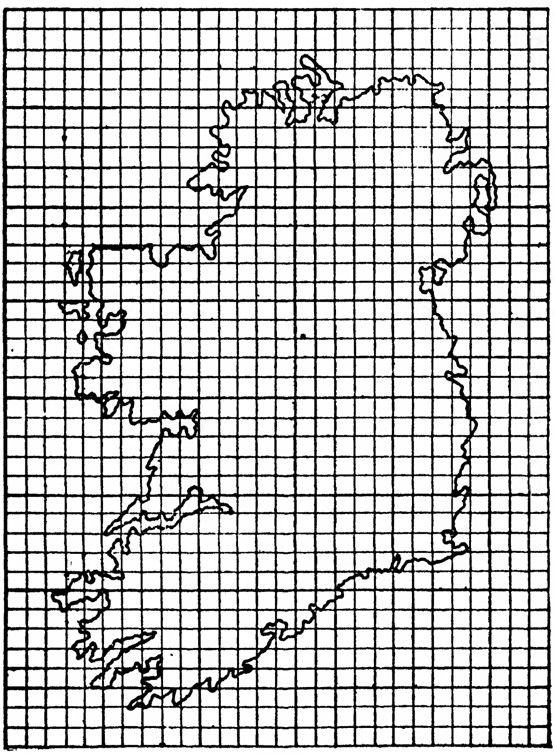
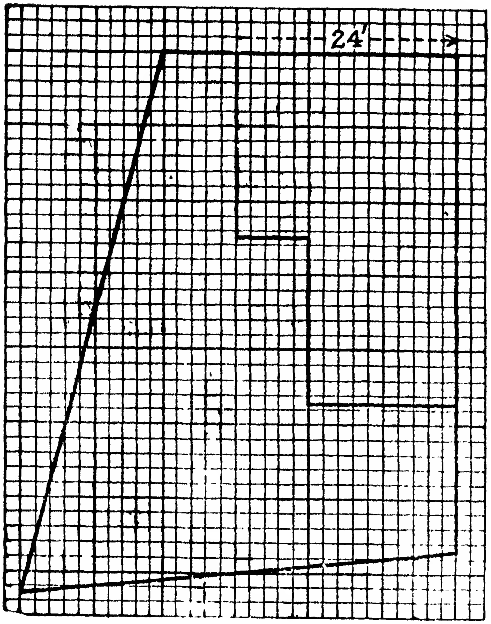


Fig. 95.

(3) From the outline Map of Ireland drawn on a scale such that $1\frac{1}{8}$ in. represents 100 miles, determine the Area of Ireland to the nearest 100 square miles.



(4) The figure is the plan of a house and garden drawn to scale, the frontage of the house being 24 ft.

Find the total area of the land. If a man bought the land at the rate of £4,050 an acre and received a yearly rental of 10 gns., what percentage interest would he obtain on his outlay ?

FIG. 96.

(5) The dutiable imports (Food, Drink, and Tobacco) into the United Kingdom during a certain year were—

Cocoa	£8,449,932	Refined Sugar .	£12,719,677
Coffee	£4,936,543	Unrefined ,, . .	£19,092,483
Dried Fruit . .	£1,034,542	Tea	£19,578,952
Condensed Milk .	£2,097,867	Wine	£ 2,917,276
Spirits	£2,719,516	Tobacco . . .	£ 8,549,629

Draw a diagram illustrating the comparative values of the above imports.

(6) The following table gives Great Britain's receipts from Customs due to the importation of tobacco and snuff.

Year ended 31 March.	*Tobacco and Snuff.*
	£
1930	62,909,202
1931	64,187,910
1932	63,400,948
1933	67,341,021
1934	67,638,814

Show a comparison of the yearly receipts by means of a diagram.

(7) The greatest seaports of the World are as follows—

Seaport.	*Entered Tons.*	*Cleared Tons.*
Antwerp	23,604,000	24,152,000
Colombo	8,943,786	8,885,964
Genoa	9,167,000	9,011,057
Hamburg . . .	19,652,000	19,783,000
Hong-Kong . . .	14,910,026	14,922,232
Liverpool . . .	13,944,000	12,773,000
London	21,417,036	11,723,156
Marseilles . . .	11,575,000	11,299,000
New York . . .	20,558,000	21,327,000
Rotterdam . . .	17,783,000	14,689,000
Southampton . . .	10,606,000	10,583,000

Draw diagrams illustrating the comparative totals of entered and cleared tons in the case of (1) New York, Antwerp, London, Hamburg, Rotterdam, and Liverpool; (2) Colombo, Genoa, Hong-Kong, Marseilles, Southampton.

(8) By the aid of a graph, write down the following fractions in order of magnitude: $\frac{13}{17}$, $\frac{31}{43}$, $\frac{23}{40}$, $\frac{51}{73}$.

(9) A man invested £575, £420, and £335 cash respectively in three businesses. The interests received at the end of the year were £25 5s., £17 10s., and £13 12s. respectively. Which was the best and which was the worst investment?

(10) A man deposited £750 in a bank, and the interest at the end of the year was £22 15s. He also deposited 1,200 francs in a French bank, which paid him 38·75 francs as interest for one year. Which bank paid the higher rate of interest?

(11) Given that 1 mile = 1·61 Km., by means of a graph express 53¼ miles in Km. and 100 Km. in miles.

(12) Draw a graph by means of which weights expressed in Kg. may be at once expressed in lb. and *vice versa*, given that 1 Kg. = 2·205 lb. Use the graph to express 1 cwt. in Kg.

(13) Using the relationship 1 ac. = ·405 hect., find by means of a graph the number of hectares equivalent to 1 ac. 2 rd. 15 sq. pl. and the number of acres, etc., equivalent to 1 hect. 35 ares.

(14) If 88·26 francs be equivalent to £1, find the numbers of francs equivalent to £18 10s. and £10 16s. Also find the numbers of £ and shillings equivalent to 1,200 francs and 925 francs.

(15) Find graphically the approx. cost of 15 cwt. 3 qr. 14 lb. of tea at £170 per ton.

(16) The rates paid by a man whose house is assessed at £47 10s. was £18 4s. 2d. Find graphically the amount that should be paid by a man living in the same borough whose house is assessed at £26.

(17) On the Fahrenheit scale the boiling and freezing points of water are 212° and 32° respectively, whereas on the Centigrade scale they are 100° and 0° respectively. By means of a graph, find the reading on the Fahrenheit scale for temperatures of 18° C. and −7° C., and find the readings on the Centigrade scale for temperatures 0° F. and 88° F.

(*Note.*—The number of degrees Fahrenheit above the freezing point is proportional to the number of degrees Centigrade.)

(18) A retailer decided to reduce the price of all his articles by 15 per cent. Draw a graph that will enable him to find the reduced prices for

original prices up to £2. Find the reduced prices of articles previously marked 10s. 9d., 18s. 6d., £1 13s.

(19) A printer charged 7s. 7d. for printing 100 booklets, and offered to supply 250 more for a total cost of 12s. 9½d. Assuming that he makes a fixed charge for setting up the type, together with an amount proportional to the number of booklets printed, find graphically the cost of 500 booklets.

(20) A and B are commercial travellers. A received a fixed salary of £175, together with 1½ per cent. commission on the value of orders obtained. B receives no fixed salary, but has 2½ per cent. commission. Draw graphs connecting the values of orders obtained with the corresponding total salaries received in each case. Hence find the values of orders for their total salaries (i) to be equal, (ii) for A's to be twice B's, (iii) for B's to be £100 more than A's.

(21) A, B, C and D are four stations, the distances of B, C, and D from A being 1½, 4, and 7 miles respectively. A train leaves A at 2.15 p.m., reaches B at 2.19 p.m., where it stops 1 min.; reaches C at 2.26 p.m., where it stops 2 min. and reaches D at 2.35 p.m. A non-stop train passes through A at 2.25 p.m., and it is required to overtake the slow train at a point 6¼ miles from A. Find approximately what the speed of the non-stop train should be.

(22)

Miles from Paddington			
—	Paddington . . .	dep. 9.0 a.m.	dep. 11.20 a.m.
36	Reading	arr. 9.46 a.m. dep. 9.48 a.m.	—
77¼	Swindon	arr. 10.57 a.m. dep. 11.0 a.m.	arr. 12.49 p.m. dep. 12.52 p.m.
133½	Newport	arr. 12.10 p.m. dep. 12.13 p.m.	arr. 2.8 p.m. dep. 2.13 p.m.
145¼	Cardiff	arr. 12.30 p.m.	arr. 2.30 p.m.
145¼	Cardiff	dep. 8.55 a.m.	dep. 10.0 a.m.
133½	Newport	arr. 9.11 a.m. dep. 9.13 a.m.	arr. 10.17 a.m. dep. 10.22 a.m.
77¼	Swindon	—	—
36	Reading	arr. 11.12 a.m. dep. 11.14 a.m.	—
—	Paddington . . .	arr. 12.0 noon	arr. 1.0 p.m.

From the above time-table, find approximately when and where the two down trains pass each of the two up trains, assuming that between stations the trains travel at uniform speeds.

(23) Three cars travel to and fro between two stations, X and Y, 1½ miles apart. They each take 6 min. over the single journey, which is performed without stopping, but wait 3 min. at X and Y before commencing the return journey, and they leave a station every 6 min. The track is a single one, except where the cars pass one another. Where should the loop lines be placed?

(24) METEOROLOGICAL SUMMARY (LONDON), 1930–1931

Month.	*Temperature:*	*Rainfall:*		*Pressure:*	*Sunshine:*
	Mean.	*Days.*	*Amount.*	*Mean.*	*Percentage.*
1930—October . .	54·2° F.	16	1·11 in.	29·87 in.	34
„ —November . .	46·1° F.	17	3·94 in.	29·87 in.	19
„ —December . .	41·7° F.	16	1·61 in.	29·91 in.	5
1931—January . .	39·9° F.	16	1·14 in.	29·91 in.	12
„ —February . .	40·3° F.	19	1·60 in.	29·89 in.	16
„ —March . . .	42·1° F.	7	0·23 in.	29·95 in.	29
„ —April . . .	48·5° F.	18	3·76 in.	29·88 in.	26
„ —May . . .	55·5° F.	16	2·77 in.	29·88 in.	36
„ —June . . .	61·5° F.	11	1·71 in.	30·03 in.	38
„ —July . . .	62·5° F.	15	2·56 in.	29·83 in.	31
„ —August . .	61·5° F.	18	3·92 in.	29·90 in.	30
„ —September .	55·7° F.	14	2·28 in.	30·17 in.	30

WHITAKER'S ALMANACK, 1932

Draw graphs showing the variation of Temperature, Rainfall, Pressure, and Amount of Sunshine for the period October, 1930, to September, 1931.

(25) POPULATION OF CANADA.

Census Year.	*Population.*	*Census Year.*	*Population.*
1871 . .	3,689,257	1901 . .	5,371,315
1881 . .	4,324,810	1911 . .	7,206,643
1891 . .	4,833,239	1921 . .	8,788,483

Draw a graph showing the fluctuations of the population of Canada. Also, assuming that throughout the period 1891–1911 the population increases in conformity to a certain law, what was the approximate population in 1896 and 1906?

(26) A certain commodity is sold in 2 oz., 4 oz., and 8 oz packets, the prices being 1s. 3d., 2s. 3d., and 4s. respectively. If it be decided to sell in 6 oz. packets, what should the price be?

(27) According to the mortality tables, during the three years 1920–22, of 1,000 babies born, the numbers surviving after different periods of time are as shown below.

Age.	*Male.*	*Female.*	*Age.*	*Male.*	*Female.*
1 . .	910	931	11 . .	855	878
2 . .	889	911	15 . .	849	871
4 . .	874	896	20 . .	837	859
7 . .	863	885			

By means of a graph, find the approximate numbers of survivors at the ages of 3 and 13 years.

(28) The following table gives the Expectation of Life at certain ages.

Age.	*Male.*	*Female.*	*Age.*	*Male.*	*Female.*
30 . .	37·4 years	40·3 years	42 . .	27·6 years	30·2 years
34 . .	34·1 ,,	36·9 ,,	46 . .	24·4 ,,	26·9 ,,
38 . .	30·8 ,,	33·5 ,,	50 . .	21·4 ,,	23·7 ,,

By plotting a graph, find the Expectation of Life at the ages of 32, 36, 40, 44, and 48 years.

(29) The following table shows the approximate sum to which an Annuity of £1 accumulating at compound interest will amount in certain times at different rates per cent.

Years.	3%.	4%.	5%.	Years.	3%.	4%.	5%.
1 .	£ 1	£ 1	£ 1	20 .	£26·87	£29·78	£33·07
5 .	£ 5·31	£ 5·42	£ 5·53	25 .	£36·46	£41·65	£47·73
10 .	£11·46	£12·01	£12·58	30 .	£47·58	£56·09	£66·44
15 .	£18·60	£20·02	£21·58	35 .	£60·46	£73·65	£90·32

By means of graphs, find the amount of an annuity of £1 at the end of 12, 18, and 28 years at 3 per cent., at 4 per cent., and at 5 per cent., to the nearest shilling.

Also find the amount at the end of 15 years at $3\frac{1}{2}$ per cent. and at $4\frac{1}{2}$ per cent.

(30) Draw a graph representing the relation between the radius and area of a circle from radius 2 in to radius 6 in. Hence find the area of a circle whose radius is 3·6 in., and find the radius of a circle whose area is 75 sq. in. ($\pi = \frac{22}{7}$).

(31) Find, correct to two places of decimals, the cube root of 42·8.

(32) Draw a graph representing the relation between the speed of a steamer and the time it takes to perform a journey of 84 miles. Find the time taken when the speed is 15·7 miles per hour, and find the speed in order that the journey should be completed in 4 hr. 40 min.

(33) 15 cwt. of meat is available for feeding a number of men for one week. Draw a graph representing the relation between the number of men and the quantity received per week by each man. What should be received by each man if the number of men be 720; also how many men could be supplied with 2 lb. 10 oz. per week?

(34) A man's assets are £510, and he has three creditors, to whom he owes £450, £250, and £150. Find graphically the amount each creditor should receive.

(35) At what rate per cent. per annum would £75 amount to £84 15s. in $3\frac{1}{4}$ years' simple interest?

(36) Find graphically the time taken for £420 to amount to £504 at 5 per cent. per annum, simple interest.

(37) Tabulate corresponding values of Principal and Time from 1 year to 5 years, such that £100 will be produced as interest at 4 per cent. per annum, simple interest. Hence find the time taken for £750 to amount to £850 at 4 per cent. per annum, and also find what sum will produce £100 interest in $4\frac{1}{2}$ years.

(38) From the graph shown in Figure 92, find the time taken for a sum of money to treble itself at 5 per cent. per annum, compound interest.

(*Note.*—£200 will amount to £300 in the same time as £100 amounts to £150; find this time and add it on to $14\frac{1}{5}$ years.)

(39) The approximate times necessary for a sum of money to double itself at 3, 4, and 5 per cent. per annum compound interest, payable yearly, are $23\frac{2}{5}$, $17\frac{2}{3}$, and $14\frac{1}{5}$ years. Find the approximate times in the case when the rates are $3\frac{1}{2}$ and $4\frac{1}{2}$ per cent. respectively.

(40) Construct a graph giving the amount of £100 at any time up to 15 years at $4\frac{1}{2}$ per cent. per annum interest, payable yearly, using the table given in Chapter II, Section IV. Use the graph to find the amount in $7\frac{1}{2}$ years and also to find in what time £100 will amount to £160.

(41) Using the table of amounts of £1 at 3 per cent. per annum, compound interest, payable yearly, given in Section IV, Chapter II, draw a graph giving the amount of £100 at 6 per cent. per annum, compound interest, payable half-yearly. Find the time taken for £62 10s. to amount to £100 under the latter conditions.

(*Note.*—This is the same time as that taken by £100 to amount to £160.)

(42) The largest parcel that can be sent by parcel post is cylindrical in shape and is such that its length added to its girth is 6 ft. Tabulate the volumes in the case when the lengths are 1, 2, 3, 4, and 5 ft. respectively. Draw a graph representing the relation between the lengths and the corresponding volumes, and hence find the length when the volume is the greatest possible.

(43) A man wishes to make a tank, in the shape of a square prism, to hold 1,000 gall. Find the total areas of the bottom and the four sides in the case when the lengths of the side of the base are 5, 6, 7, and 8 ft. respectively. By plotting a graph, find the dimensions of the tank that requires the least amount of material to construct.

(44) An article which costs a retailer 3s. is sold at different times at 3s. 6d., 3s. 11d., 4s. 6d., and 4s. 9d. each, the sales being 970, 615, 354, and 252 per week respectively. Find the price at which the articles should be sold for the retailer to obtain the most profit.

CHAPTER X.

LOGARITHMS AND THEIR APPLICATIONS.

96. LAWS OF INDICES.

THE quantity a^x is known as a **Power** of the quantity a, and x is called the **Index**. Thus the first, second, third, and fourth powers of a are a, a^2, a^3, and a^4 respectively, the indices being 1, 2, 3, and 4 respectively.

$$\text{Now } a^2 \times a^3 = a \times a \times a \times a \times a = a^5$$
$$\text{also } a \times a^2 \times a^4 = a \times a \times a \times a \times a \times a \times a = a^7$$

so that it is clear that the index of the product of two or more powers of the same quantity is the sum of the indices.

$$\text{Also } a^3 \div a \quad i.e., \ \frac{a^3}{a} = \frac{\cancel{a} \times a \times a}{\cancel{a}} = a \times a = a^2$$

$$\text{and } a^6 \div a^3, \ i.e., \ \frac{a^6}{a^3} = \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a \times a}{\cancel{a} \times \cancel{a} \times \cancel{a}} = a \times a \times a = a^3$$

so that it is obvious that the index of the quotient of two powers of the same quantity is the difference of the indices of the powers.

Thus, if x, y, and z be positive integers, and a be any numerical quantity, $a^x \times a^y \times a^x = a^{x+y+z}$ and $a^x \div a^y = a^{x-y}$

It remains to see what interpretation can be given to a^x, when (1) x is a positive fraction and when (2) x is a negative quantity, integer or fraction.

Let the interpretation given to powers in both cases be that which makes the identities $a^x \times a^y \times a^z = a^{x+y+x}$ and $a^x \div a^y = a^{x-y}$ true even when x, y, and z are fractions or negative quantities.

Consider the quantity $a^{\frac{1}{2}}$. By the above assumption, $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a$, so that $a^{\frac{1}{2}}$ denotes $\sqrt{a}$. Again, $a^{\frac{2}{3}} \times a^{\frac{2}{3}} \times a^{\frac{2}{3}} = a^{\frac{2}{3}+\frac{2}{3}+\frac{2}{3}} = a^2$, so that $a^{\frac{2}{3}}$ denotes $\sqrt[3]{a^2}$. Also as $a^{1\frac{1}{4}} \times a^{1\frac{1}{4}} \times a^{1\frac{1}{4}} \times a^{1\frac{1}{4}} = a^{1\frac{1}{4}+1\frac{1}{4}+1\frac{1}{4}+1\frac{1}{4}} = a^5$, it is clear that $a^{\frac{5}{4}} \equiv \sqrt[4]{a^5}$.

Proceeding thus for any fractional index, the following generalization, is obtained: If m and n are positive integers,

$$a^{\frac{m}{n}} \equiv \sqrt[n]{a^m}.$$

Consider the quantity a^{-1}. By the assumption, $a^2 \div a^3 = a^{2-3} = a^{-1}$.

$$\text{Therefore } a^{-1} \text{ denotes } \frac{a^2}{a^3}, \text{ i.e., } \frac{\cancel{a}\times\cancel{a}}{\cancel{a}\times\cancel{a}\times a}, \text{ i.e., } \frac{1}{a}$$

$$\text{Also as } a^2 \div a^5 = a^{2-5} = a^{-3},\ a^{-3} = \frac{\cancel{a}\times\cancel{a}}{\cancel{a}\times\cancel{a}\times a\times a\times a} = \frac{1}{a^3}$$

$$\text{Again } a^{-\frac{1}{2}} \times a^{-\frac{1}{2}} = a^{-\frac{1}{2}-\frac{1}{2}} = a^{-1} = \frac{1}{a}. \quad \text{Therefore } a^{-\frac{1}{2}} = \frac{1}{\sqrt{a}} = \frac{1}{a^{\frac{1}{2}}}$$

$$\text{Further } a^{-\frac{3}{4}} \times a^{-\frac{3}{4}} \times a^{-\frac{3}{4}} \times a^{-\frac{3}{4}} = a^{-\frac{3}{4}-\frac{3}{4}-\frac{3}{4}-\frac{3}{4}} = a^{-3} = \frac{1}{a^3}$$

$$\text{Therefore } a^{-\frac{3}{4}} = \frac{1}{\sqrt[4]{a^3}}.$$

Proceeding thus for any negative index, the following generalization is obtained—

If m and n are positive integers,

$$a^{-\frac{m}{n}} \equiv \frac{1}{a^{\frac{m}{n}}} \equiv \frac{1}{\sqrt[n]{a^m}}$$

As $a \div a = a^{1-1} = a^0$ by the assumption, and as $a \div a = \frac{a}{a} = 1$, it is clear that $a^0 \equiv 1$.

EXAMPLE (1)—

$$\frac{\sqrt[3]{32} \times \sqrt{8}}{4\sqrt[6]{2}} = \frac{\sqrt[3]{2^5} \times \sqrt{2^3}}{2^2 \times \sqrt[6]{2}} = \frac{2^{\frac{5}{3}} \times 2^{\frac{3}{2}}}{2^2 \times 2^{\frac{1}{6}}}$$
$$= 2^{\frac{5}{3}+\frac{3}{2}-2-\frac{1}{6}} = 2^1 = \underline{2}$$

97. DEFINITION OF A LOGARITHM.

If a numerical quantity N be expressed in the form a^x, x is said to be the **Logarithm** of N to base a. This statement can more briefly be made as follows—

If $N = a^x$, then $x = \log_a N$, the abbreviation "log." standing for "the logarithm of."

As $16 = 2^4$, $4 = \log_2 16$; and as $5 = 25^{\frac{1}{2}}$, $\frac{1}{2} = \log_{25} 5$.

The logarithms in common use are those whose base is 10, and so for the remaining part of the chapter it is understood that 10

is always the base. Thus, as $100 = 10^2$, $2 = \log 100$; as $10 = 10^1$, $1 = \log 10$; as $1 = 10^0$, $0 = \log 1$; as $\cdot 1 = \frac{1}{10} = 10^{-1}$, $-1 = \log \cdot 1$; and as $\cdot 01 = \frac{1}{10^2} = 10^{-2}$, $-2 = \log \cdot 01$.

In simple language, if a number be expressed in the form of a power of 10, the index of this power of 10 is the logarithm of the number.

98. EXPRESSION OF A NUMBER IN THE FORM 10^x.

It requires now to show that it is possible to express any positive numerical quantity in the form 10^x.

It has already been shown that 100, 10, 1, ·1, ·01 can be expressed in this form. In a similar way, $1{,}000 = 10^3$, $10{,}000 = 10^4$, etc., and $\cdot 001 = 10^{-3}$, $\cdot 0001 = 10^{-4}$, etc.

As $\sqrt{10}$	$= 3\cdot 1623$ approx.,	then	$3\cdot 1623 = 10^{\frac{1}{2}}$	$= 10^{\cdot 5}$
,, $\sqrt{3\cdot 1623}$	$= 1\cdot 7783$	,, ,,	$1\cdot 7783 = 10^{\frac{1}{4}}$	$= 10^{\cdot 25}$
,, $3\cdot 1623 \times 1\cdot 7783$	$= 5\cdot 6235$	,, ,,	$5\cdot 6235 = 10^{\frac{3}{4}}$	$= 10^{\cdot 75}$
,, $10 \times 3\cdot 1623$	$= 31\cdot 623$	,,	$31\cdot 623 = 10^{1\frac{1}{2}}$	$= 10^{1\cdot 5}$
,, $10 \times 1\cdot 7783$	$= 17\cdot 783$	,,	$17\cdot 783 = 10^{1\frac{1}{4}}$	$= 10^{1\cdot 25}$
,, $10 \times 5\cdot 6235$	$= 56\cdot 235$	,,	$56\cdot 235 = 10^{1\frac{3}{4}}$	$= 10^{1\cdot 75}$
,, $100 \times 3\cdot 1623$	$= 316\cdot 23$	,,	$316\cdot 23 = 10^{2\frac{1}{2}}$	$= 10^{2\cdot 5}$
. . . .			. . . .	

Also as $10^{\frac{1}{2}} \div 10^1$	$= 10^{-\frac{1}{2}}$	then	$\cdot 31623 = 10^{-\frac{1}{2}}$	$= 10^{-\cdot 5}$
,, ,, $10^{\frac{3}{4}} \div 10^1$	$= 10^{-\frac{1}{4}}$	,,	$\cdot 56235 = 10^{-\frac{1}{4}}$	$= 10^{-\cdot 25}$
,, ,, $10^{\frac{1}{4}} \div 10^1$	$= 10^{-\frac{3}{4}}$	,,	$\cdot 17783 = 10^{-\frac{3}{4}}$	$= 10^{-\cdot 75}$
,, ,, $10^{\frac{1}{2}} \div 10^2$	$= 10^{-1\frac{1}{2}}$	,,	$\cdot 031623 = 10^{-1\frac{1}{2}}$	$= 10^{-1\cdot 5}$
. . . .			. . . .	

By marking the numbers 1, 1·7783, 3·1623, as 5·6235 and 10 along one axis, and the corresponding indices of 10, namely, 0, ·25, ·5, ·75, and 1, a graph can be drawn and by the principle of interpolation any number between 1 and 10 can approximately be expressed in the form 10^x.

Suppose from the graph it is seen that $7 = 10^{\cdot 8451}$, then as $70 = 7 \times 10^1$ and $\cdot 007 = 7 \times 10^{-3}$, etc., $70 = 10^{1\cdot 8451}$ and $\cdot 007 = 10^{-3+\cdot 8451}$, etc. Thus having noticed that 10^x is always positive, whether x is positive or negative, it is seen that any positive number can approximately be expressed in the form 10^x, x being the logarithm of the number.

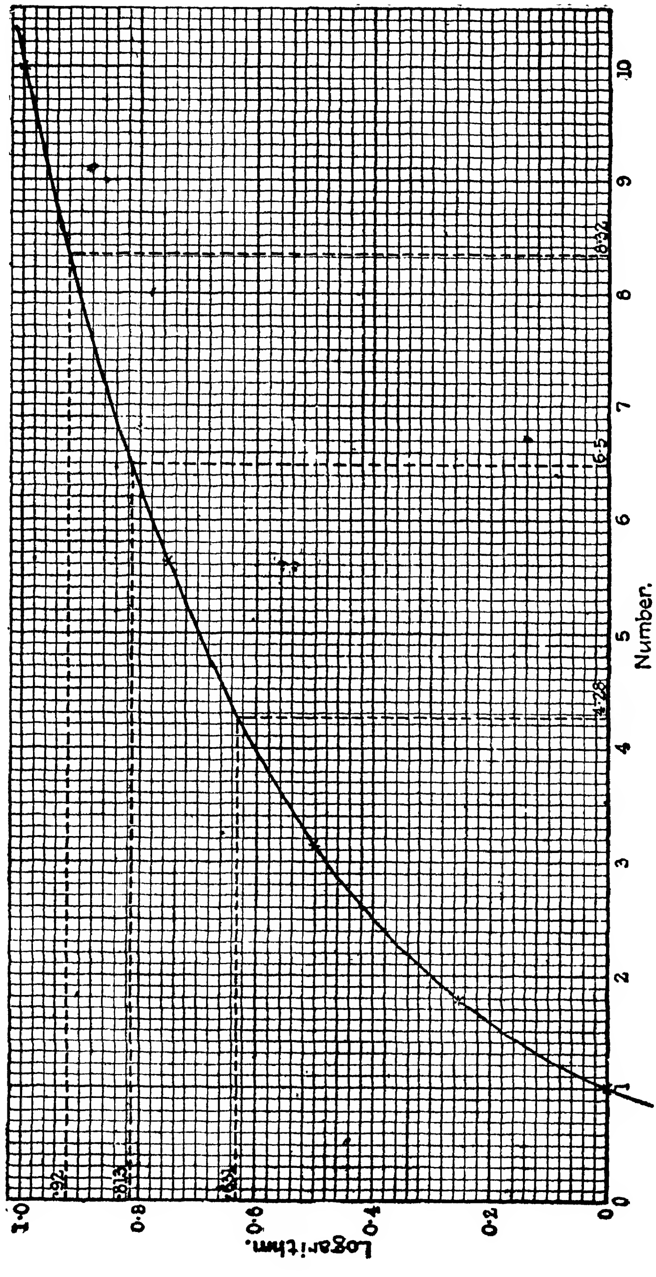


FIG. 97.

GRAPH. SHOWING THAT ANY POSITIVE NUMBER CAN BE EXPRESSED IN THE FORM 10^x.

Example (ii)—

Using the following table, draw a graph by means of which the logarithms of any numbers can approximately be obtained. Find approximately the logarithms of 6·5 and 42·8, and find the number whose logarithm is ·92.

Number.	Logarithm.	Number.	Logarithm.
1	0	5·624	0·75
1·778	0·25	10	1
3·162	0·5	—	—

From the graph, log 6·5 = ·813 and log 4·28 = ·631. From the latter, by definition $4{\cdot}28 = 10^{\cdot 631}$, therefore $42{\cdot}8 = 10^{\cdot 631} \times 10^{1} = 10^{1{\cdot}631}$: thus log 42·8 = 1·631. Also ·92 = log 8·32.

Note 1.—The larger the graph, the closer the approximation; but, on the other hand, the further apart will be the points plotted, with the result that there would be an error in the shape of the curve. More points could be plotted, however, by evaluating $\sqrt{1{\cdot}7783}$, *i.e.*, $10^{\frac{1}{8}}$. By multiplying this number by 1·7783, 3·1623, and 5·6235, *i.e.*, by $10^{\frac{1}{4}}$, $10^{\frac{1}{2}}$, and $10^{\frac{3}{4}}$, the numbers whose logarithms are $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$ (*i.e.*, ·375, ·625, and ·875) are obtained. This process could be extended by evaluating $10^{\frac{1}{16}}$, $10^{\frac{1}{32}}$, $10^{\frac{1}{64}}$, etc., by finding the square root of $10^{\frac{1}{8}}$, $10^{\frac{1}{16}}$, $10^{\frac{1}{32}}$ etc., respectively, so that by multiplying by the results already obtained, the values of 10 to the powers of any number of sixteenths, thirty-seconds, sixty-fourths, etc., can be obtained.

99. MANTISSA AND CHARACTERISTIC.

With the exception of integral powers of 10, *i.e.*, 10^1, 10^2, 10^{-3}, etc., the logarithms of commensurable numbers are incommensurable. Although by means of a large scale graph, the logarithms of numbers can be obtained to a fairly close degree of approximation; yet in commercial problems it is often necessary to know the values correct to seven or even nine places of decimals. By means of an algebraic series, the proof of which is beyond the scope of this book, the logarithms of numbers can be calculated to any degree of accuracy required. These calculations have been performed and the results tabulated: from *Chambers' Mathematical Tables* the logarithms of quantities from 1 to 108000 correct to seven places of decimals can be readily obtained.

As log 1 = 0 and log 10 = 1, any number between 1 and 10 has a logarithm which lies between 0 and 1. In other words, **the logarithm of a number which has one figure before the decimal point is a quantity which has no figures before the decimal point. In tables of logarithms, only the logarithms of quantities between 1 and 10 are given,** but from these the logarithms of numbers less than 1 and greater than 10 can be at once determined.

If it be required to find the logarithms of 309·17, 309170, ·30917,

and ·0030917, the logarithm of 3·0917 is obtained from a book of tables and found to be ·4901973.

$$\text{Now } 309{\cdot}17 = 3{\cdot}0917 \times 10^2 = 10^{\cdot 4901973} \times 10^2 = 10^{2\cdot 4901973}$$
$$\text{Also } 309170 = 3.0917 \times 10^5 = 10^{\cdot 4901973} \times 10^5 = 10^{5\cdot 4901973}$$

Thus log 309·17 = 2·4901973 and log 309170 = 5·4901973.

$$\text{Again, } \cdot 30917 = 3{\cdot}0917 \times \tfrac{1}{10} = 10^{\cdot 4901973} \times 10^{-1} = 10^{-1+\cdot 4901973}$$
$$\text{Also } \cdot 0030917 = 3{\cdot}0917 \times \tfrac{1}{1000} = 10^{\cdot 4901973} \times 10^{-3} = 10^{-3+\cdot 4901973}$$

Thus, log ·30917 = $\bar{1}$·4901973 and log ·0030917 = $\bar{3}$·490173,

it being understood that the minus sign refers only to the figure before the decimal point, so that the decimal portion is always positive.

It is seen that a logarithm consists of two parts, namely, the decimal portion called the **mantissa**, which is obtained from the tables and is independent of the position of the decimal point of the number; and the integral part called the **characteristic**, which may be positive or negative, and depends entirely on the position of the decimal point of the number. By a study of the results immediately above, it is clearly seen that **if starting from the position after the first figure of the number, the number of places be counted until the decimal point is reached, this will give the magnitude of the characteristic : moreover, the sign of the latter will be + or – according as to whether the counting is made to the right or to the left.**

The above rule applied inversely will enable the position of the decimal point of a number to be fixed when its logarithm is known. Suppose it be required to find the numbers whose logarithms are $\bar{2}$·9385798, 4·9385798, and 1·9385798. From the tables it can be found that ·9385798 = log 8·6812. Moving the decimal point two places to the left, four places to the right, and one place to the right respectively, it is clear that $\bar{2}$·9385798 = log ·086812 4·9385798 = log 86812, and 1·9385798 = log 86·812.

100. READING OF THE TABLE OF LOGARITHMS.

From a study of the following examples in conjunction with the specimen page taken from a book of Mathematical Tables, no difficulty will be found in learning how to read a table of Logarithms.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
3050	484 2998	3141	3283	3426	3568	3710	3853	3995	4137	4280	
51	4422	4564	4707	4849	4991	5134	5276	5418	5561	5703	
52	5845	5988	6130	6272	6414	6557	6699	6841	6984	7126	
53	7268	7410	7553	7695	7837	7979	8121	8264	8406	8548	
54	8690	8833	8975	9117	9259	9401	9543	9686	9828	9970	
55	485 0112	0254	0396	0539	0681	0823	0965	1107	1249	1391	
56	1533	1676	1818	1960	2102	2244	2386	2528	2670	2812	
57	2954	3096	3239	3381	3523	3665	3807	3949	4091	4233	142
58	4375	4517	4659	4801	4943	5085	5227	5369	5511	5653	1 14
59	5795	5937	6079	6221	6363	6505	6647	6788	6930	7072	2 28
60	7214	7356	7498	7640	7782	7924	8066	8208	8350	8491	3 43
3061	8633	8775	8917	9059	9201	9343	9484	9626	9768	9910	4 57
62	486 0052	0194	0336	0477	0619	0761	0903	1045	1186	1328	5 71
63	1470	1612	1754	1895	2037	2179	2321	2462	2604	2746	6 85
64	2888	3029	3171	3313	3455	3596	3738	3880	4021	4163	7 99
65	4305	4446	4588	4730	4872	5013	5155	5297	5438	5580	8 114
66	5722	5863	6005	6146	6288	6430	6571	6713	6855	6996	9 128
67	7138	7279	7421	7563	7704	7846	7987	8129	8270	8412	
68	8554	8695	8837	8978	9120	9261	9403	9544	9686	9827	
69	9969	0110	0252	0393	0535	0676	0818	0959	1101	1242	
70	487 1384	1525	1667	1808	1950	2091	2232	2374	2515	2657	
3071	2798	2940	3081	3222	3364	3505	3647	3788	3929	4071	
72	4212	4353	4495	4636	4778	4919	5060	5202	5343	5484	
73	5626	5767	5908	6050	6191	6332	6473	6615	6756	6897	
74	7039	7180	7321	7462	7604	7745	7886	8027	8169	8310	
75	8451	8592	8734	8875	9016	9157	9299	9440	9581	9722	
76	9863	0004	0146	0287	0428	0569	0710	0852	0993	1134	
77	488 1275	1416	1557	1698	1839	1981	2122	2263	2404	2545	
78	2686	2827	2968	3109	3251	3392	3533	3674	3815	3956	
79	4097	4238	4379	4520	4661	4802	4943	5084	5225	5366	141
80	5507	5648	5789	5930	6071	6212	6353	6494	6635	6776	1 14
3081	6917	7058	7199	7340	7481	7622	7763	7904	8045	8185	2 28
82	8326	8467	8608	8749	8890	9031	9172	9313	9454	9594	3 42
83	9735	9876	0017	0158	0299	0440	0580	0721	0862	1003	4 56
84	489 1144	1285	1425	1566	1707	1848	1989	2129	2270	2411	5 71
85	2552	2692	2833	2974	3115	3256	3396	3537	3678	3818	6 85
86	3959	4100	4241	4381	4522	4663	4804	4944	5085	5226	7 99
87	5366	5507	5648	5788	5929	6070	6210	6351	6492	6632	8 113
88	6773	6914	7054	7195	7335	7476	7617	7757	7898	8038	9 127
89	8179	8320	8460	8601	8741	8882	9023	9163	9304	9444	
90	9585	9725	9866	0006	0147	0287	0428	0569	0709	0850	
3091	490 0990	1131	1271	1412	1552	1693	1833	1973	2114	2254	
92	2395	2535	2676	2816	2957	3097	3238	3378	3518	3659	
93	3799	3940	4080	4220	4361	4501	4642	4782	4922	5063	
94	5203	5343	5484	5624	5765	5905	6045	6186	6326	6466	
95	6607	6747	6887	7027	7168	7308	7448	7589	7729	7869	
96	8010	8150	8290	8430	8571	8711	8851	8991	9132	9272	
97	9412	9552	9693	9833	9973	0113	0253	0394	0534	0674	
98	491 0814	0954	1094	1235	1375	1515	1655	1795	1935	2076	
99	2216	2356	2496	2636	2776	2916	3057	3197	3337	3477	
3100	3617	3757	3897	4037	4177	4317	4457	4597	4738	4878	

EXAMPLE (iii)—

Write down the logarithms of 306, ·030831, 3083·2, 3·07486, 3·074853, ·000031.

	Number.	Logarithm.		Number.	Logarithm.
(1)	306	2·4857214	(4)	3·07486	·4878254
(2)	·030831	$\bar{2}$·4889876	(5)	3·074853	·4878244
(3)	3083·2	3·4890017	(6)	·000031	$\bar{5}$·4913617

NOTE 2.—In all cases, no notice is taken of the position of the decimal point until after the mantissa is obtained. In the case where a line occurs above the last four figures of a logarithm, the first three figures are those on the left, a line below. The tables of differences on the right give the amounts to be added on in the cases where a sixth significant figure occurs in the number. If a seventh significant figure occur, one-tenth of the amount given in the table of differences should be added. For example, log 3·074853 = ·4878169 + ·0000071 + ·0000004 = ·4878244.

These results should be compared with those obtained from the tables in the Appendix. In most problems on Finance four-figure logarithms do not give answers to the required degree of accuracy.

EXAMPLE (iv)—

Write down the numbers whose logarithms are 3·4857782, $\bar{1}$·4870818, ·4902142, $\bar{3}$·4892349.

	Logarithm.	Number.		Logarithm.	Number.
(1)	3·4857782	3060·4	(3)	·4902142	3·09182
(2)	$\bar{1}$·4870818	·30696	(4)	$\bar{3}$·4892349	·003084856

NOTE 3.—A number which is such that its logarithm is a given number is sometimes called the anti-logarithm of the given number. For example, 3060·4 is the anti-logarithm of 3·4857782.

NOTE 4.—In all cases, no notice is taken of the characteristic until the digits composing the anti-logarithms are obtained.

Referring to part (3), ·4902142 = ·4902114 + ·0000028, so that ·4902142 is the logarithm of 3·09182, the sixth significant figure being obtained from the difference table.

Referring to part (4), ·4892349 = ·4892270 + ·0000071 + ·0000008, so that ·4892349 is the logarithm of 3·084856, for as 85 added on corresponds to a 6 as the sixth significant figure, one-tenth of this (*i.e.*, 8 approx.) corresponds to a 6 as the seventh significant figure.

101. FUNDAMENTAL LAWS OF LOGARITHMS.

There are three laws in connection with logarithms which are of vital importance in the application of logarithms to arithmetical problems.

LAW 1.—N, M, and P are three positive quantities whose logarithms are x, y, and z respectively. Then it follows from the definition of a logarithm that $N = 10^x$, $M = 10^y$, and $P = 10^z$.

Now from Paragraph 96 it is seen that $N \times M \times P = 10^x \times 10^y \times 10^z = 10^{x+y+z}$ so that by the definition $\log (N \times M \times P) = x + y + z$, but as $x = \log N$, $y = \log M$ and $z = \log P$, it follows that

$$\log (N \times M \times P) = \log N + \log M + \log N.$$

Law 2.— $$\frac{N}{M} = \frac{10^x}{10^y} = 10^{x-y}$$

so that from the definition $\log\left(\frac{N}{M}\right) = x - y$

$$\text{i.e. } \log\left(\frac{\mathbf{N}}{\mathbf{M}}\right) = \log \mathbf{N} - \log \mathbf{M}.$$

Note 5.—Consider the following examples of powers raised to a power—

$$(a^2)^3 = a^2 \times a^2 \times a^2 = a^6$$

$$(a^{\frac{2}{3}})^4 = a^{\frac{2}{3}} \times a^{\frac{2}{3}} \times a^{\frac{2}{3}} \times a^{\frac{2}{3}} = a^{\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}} = a^{\frac{8}{3}}$$

$$(a^3)^{\frac{1}{2}} = \sqrt{a^3} = a^{\frac{3}{2}}$$

$$(a^{\frac{2}{3}})^{\frac{1}{4}} = \sqrt[4]{a^{\frac{2}{3}}} = a^{\frac{\frac{2}{3}}{4}} = a^{\frac{2}{12}} = a^{\frac{1}{6}}$$

$$\left(a^{-3}\right)^{\frac{1}{2}} = \left(\frac{1}{a^3}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{a^3}} = \frac{1}{a^{\frac{3}{2}}} = a^{-\frac{3}{2}}$$

$$\left(a^{-\frac{1}{2}}\right)^{-\frac{1}{3}} = \left(\frac{1}{a^{\frac{1}{2}}}\right)^{-\frac{1}{3}} = \frac{1}{(a^{\frac{1}{2}})^{-\frac{1}{3}}} = \left(a^{\frac{1}{2}}\right)^{\frac{1}{3}} = \sqrt[3]{a^{\frac{1}{2}}} = a^{\frac{\frac{1}{2}}{3}} = a^{\frac{1}{6}}$$

In each example it is seen that in the case of a power raised to a power, the equivalent power has an index which is the product of the two indices. In general, if r and x be any $+ve$ or $-ve$ integers or fractions, $(a^x)^r = a^{rx}$.

Law 3—As $N^r = (10^x)^r = 10^{rx}$, where r is any $+ve$ or $-ve$ integer or fraction, then, by the definition rx is the logarithm of N^r.

$$\therefore \log(\mathbf{N}^r) = \mathbf{r} \times \log \mathbf{N}.$$

It is clear that as regards Law 1, a similar result is obtained, however many numbers are multiplied together, so that the three laws may be stated as follows—

1. **The logarithm of a product equals the sum of the logarithms of the factors of the product.**
2. **The logarithm of a quotient equals the logarithm of the dividend minus the logarithm of the divisor.**
3. **The logarithm of a power of a number equals the product obtained by multiplying the index by the logarithm of the number.**

From Laws 1 and 3, it is seen that the logarithm of a composite number can easily be obtained if the logarithms of its prime factors are known.

Example (v)—

Given that log 3 = ·4771213, log 5 = ·6989700, and log 7 = ·8450980, find the logarithms of 105, 1·35, and ·0245.

$$105 = 3 \times 5 \times 7 \quad \therefore \log 105 = \log 3 + \log 5 + \log 7$$
$$= \underline{\underline{2 \cdot 0211893}}$$

$135 = 3^3 \times 5 \quad \therefore \log 135 = 3 \log 3 + \log 5$
$= 1{\cdot}4313639 + {\cdot}6989700$
$= 2{\cdot}1303339$
$\therefore \log 1{\cdot}35 = {\cdot}1303339$

$245 = 5 \times 7^2 \quad \therefore \log 245 = \log 5 + 2 \log 7$
$= {\cdot}6989700 + 1{\cdot}6901960$
$= 2{\cdot}3891660$
$\therefore \log {\cdot}0245 = \bar{2}{\cdot}3891660$

Example (vi)—

Express the logarithms of the following expressions in terms of the logarithms of a, b, and c.

$$\frac{a^2\sqrt{b}}{c}, \quad \sqrt{\frac{ab^3}{c^5}}, \quad \frac{a^2}{\sqrt[3]{b^2c}}$$

$\log \frac{a^2\sqrt{b}}{c} = \log (a^2\sqrt{b}) - \log c$ by Law 2

$= \log a^2 + \log \sqrt{b} - \log c$ by Law 1

$= 2 \log a + \frac{1}{2} \log b - \log c$ by Law 3 as $\sqrt{b} = b^{\frac{1}{2}}$

$\log \sqrt{\frac{ab^3}{c^5}} = \frac{1}{2} \log \frac{ab^3}{c^5}$ by Law 3

$= \frac{1}{2} [\log a + 3 \log b - 5 \log c]$ by Laws 1, 2, and 3.

$\log \frac{a^2}{\sqrt[3]{b^2c}} = 2 \log a - \frac{1}{3} \log b^2c$ by Laws 2 and 3.

$= 2 \log a - \frac{1}{3} [2 \log b + \log c]$ by Laws 1 and 3.

102. APPLICATIONS OF LOGARITHMS.

Note 6.—Before dealing with the applications of logarithms, it is necessary for the student to make sure that he is able quickly and accurately to manipulate with negative integers.

(*a*) Add the following—

$3{\cdot}1416273$
$\bar{1}{\cdot}9047368$
$\bar{5}{\cdot}8360914$
${\cdot}2131004$
$\bar{1}{\cdot}0955559$

The decimal parts being all positive, these are added in the ordinary way and $+2$ is carried. Now $+2$ and $-5 = -3$, -3 and $-1 = -4$ and -4 and $+3 = -1$.

(*b*) Perform the following subtractions—

${\cdot}4973$
${\cdot}8176$
$\bar{1}{\cdot}6797$

After subtracting the figures in the first decimal place, in accordance with paragraph 13, part (*b*), 1 is added to the units figure (which is 0) of the lower quantity. Then $0 - 1 = -1$.

$\bar{1}{\cdot}0983$
$\bar{2}{\cdot}1694$
${\cdot}9289$

On adding 1 to -2, the result is -1. Then $-1-(-1) = -1+1 = 0$.

$$\begin{array}{r} \cdot 4731 \\ \bar{3}\cdot 8649 \\ \hline \bar{4}\cdot 6082 \end{array}$$

On adding 1 to 3, the result is 4. Then $0 - 4 = -4$.

(*c*) Perform the following multiplication—

$$\begin{array}{r} \bar{1}\cdot 3286 \\ 4 \\ \hline \bar{3}\cdot 3144 \end{array}$$

There is 1 to carry after multiplying the figure in the first decimal place. Then $4 \times (-1) = -4, \quad -4 + 1 = -3$.

(*d*) Perform the following divisions—

$$\begin{array}{r} 2|\bar{1}\cdot 372947 \\ \hline \bar{1}\cdot 6864735 \end{array}$$

As the characteristic must always be an integer, the negative characteristic of dividend must be divisible by 2. The dividend is therefore regarded as being $\bar{2} + 1\cdot 372947$.

$$\begin{array}{r} 5|\bar{7}\cdot 2983622 \\ \hline \bar{2}\cdot 6596724 \end{array}$$

The dividend is regarded as being $\overline{10} + 3\cdot 2983622$.

When the characteristics are $+ve$, the four rules are performed in accordance with the elementary rules.

By means of logarithms, calculations, which, if performed by the elementary rules, would involve laborious work, can be speedily made. Cube root and higher roots of numbers can be quickly evaluated if logarithms are employed. The following examples illustrate the advantage of logarithms, a table giving the latter correct to seven decimal places being employed.

EXAMPLE (vii)—

Evaluate $£\dfrac{41\cdot 3657 \times 35\cdot 249 \times \cdot 7429}{7695\cdot 3 \times \cdot 00917}$, to the nearest penny.

$$£\frac{41\cdot 3657 \times 35\cdot 249 \times \cdot 7429}{7695\cdot 3 \times \cdot 00917}$$

$= £15\cdot 35051$

$= £15 \text{ 7s. 0d.}$

Number.	Logarithm.
41·3657	1·6166404
35·249	1·5471468
·7429	$\bar{1}$·8709304
N	3·0347176
7695·3	3·8862256
·00917	$\bar{3}$·9623693
D	1·8485949
15·35051	1·1861227

NOTE 7.—3·0347176 is the logarithm of the numerator of the fraction and 1·8485949 is that of the denominator. 1·1861227 is the logarithm of the value of the fraction, so that its antilogarithm gives the required result.

EXAMPLE (viii)—

Find the radius of a sphere which has the same volume as a rectangular prism 17·473″ by 14·941″ by 7·367″.

Let r in. ≡ rad. of sphere,

then $\dfrac{4 \times 3{\cdot}14159\, r^3}{3} = 17{\cdot}473 \times 14{\cdot}941 \times 7{\cdot}367$

$\therefore\ r = \sqrt[3]{\dfrac{17{\cdot}473 \times 14{\cdot}941 \times 22{\cdot}101}{12{\cdot}56636}}$

Radius of sphere = 7·7147 inches correct to 4 places of decimals

Number.	Logarithm.
17·473	1·2423675
14·941	1·1743797
22·101	1·3444119
N	3·7611591
12·56636	1·0992095
F	2·6619496
7·714655	·8873165

NOTE 8.—2·6619496 is the logarithm of the fraction, and the value of this to the power $\frac{1}{3}$ is required. In accordance with Law 3, 2·6619496 is multiplied by $\frac{1}{3}$, *i.e.*, divided by 3 in order to obtain the logarithm of the cube root of the fraction.

EXAMPLE (ix)—

Find the amount of £100 in 20 years' time at $3\frac{3}{4}$ per cent. per annum, compound interest payable half-yearly.

Amount = £100 × $(1{\cdot}01875)^{40}$

= £210·2364

= £210 4s. 9d. to nearest 1d.

Number.	Logarithm.
100	2·0000000
1·01875	·0080677
$(1{\cdot}01875)^{40}$	·3227080
210·2364	2·3227080

NOTE 9.—The formula of paragraph 74, $A = P\left(1 + \frac{i}{2}\right)^{2n}$, is applied.

EXAMPLE (x)—

Evaluate $\dfrac{1}{\sqrt[5]{{\cdot}36947}}$

$\dfrac{1}{({\cdot}36947)^{\frac{1}{5}}}$

= 1·22035 correct to 5 dec. places

Number.	Logarithm.
1	0·0000000
·36947	$\bar{1}$·5675792
$({\cdot}36947)^{\frac{1}{5}}$	$\bar{1}$·9135158
1·220349	·0864842

Example (xi)—

What sum will amount to £1,000 in 12 years' time at 4½ per cent. per annum, compound interest?

Let £x ≡ the sum

then $x \times (1{\cdot}045)^{12} = 1000$

$$\therefore\ x = \frac{1000}{(1{\cdot}045)^{12}}$$

$$= 589{\cdot}6636$$

Number.	Logarithm.
1000	3·0000000
1·045	·0191163
$(1{\cdot}045)^{12}$	·2293956
589·6636	2·7706044

Ansr.—£589 13s. 3d. to nearest 1d.

Example (xii)—

How long would a sum of money take to double itself if allowed to accumulate at 3½ per cent. compound interest, payable quarterly?

Let £P ≡ sum of money
,, n ≡ no. of years
then $2P = P \times (1{\cdot}00875)^{4n}$

$$\therefore\ (1{\cdot}00875)^{4n} = \frac{2P}{P} = 2$$

Now, taking logarithms of both sides,
$4n \times \log 1{\cdot}00875 = \log 2$

$$\therefore\ 4n = \frac{\log 2}{\log 1{\cdot}00875}$$

$$= \frac{{\cdot}3010300}{{\cdot}0037836}$$

$$\therefore\ n = \frac{{\cdot}3010300}{{\cdot}0151344}$$

$$= 19{\cdot}89045$$

Number.	Logarithm.
·3010300	$\bar{1}{\cdot}4786098$
·0151344	$\bar{2}{\cdot}1799652$
19·89045	1·2986446

Ansr.—19 years 9 months 52 days.

Example (xiii)—

£475 12s. 7d. is deposited at a bank and left to accumulate at 3 per cent compound interest. How long must it be left in order to amount to £725?

Let n ≡ no. of years
then $725 = 475{\cdot}6292 \times (1{\cdot}03)^n$

$$\therefore\ (1{\cdot}03)^n = \frac{725}{475{\cdot}6292}$$

$$\therefore\ n \log 1{\cdot}03 = \log 725 - \log 475{\cdot}6292$$

$$\therefore\ n = \frac{2\cdot8603380 - 2\cdot6772685}{\cdot0128372}$$

$$= \frac{\cdot1830695}{\cdot0128352}$$

$$= 14\cdot26086$$

Number.	Logarithm.
·1830695	$\bar{1}$·2626161
·0128372	$\bar{2}$·1084703
14·26086	1·1541458

Ansr.—14 years 96 days.

NOTE 10.—The answers to Examples (xii) and (xiii) are obtained on the assumption that the formulae $A = P\left(1 + \frac{i}{4}\right)^{4n}$ and $A = P(1 + i)^n$ hold good in calculating the interest for fractions of a period. It has already been pointed out that for fractions of a period, the interest is that fraction of the interest for a complete period. It is necessary, then, to see if a discrepancy arises.

Referring to Example (xii), £1 amounts to £2 in a time which lies between 79 and 80 quarter-years. In 79 quarter-years, £1 amounts to £$(1\cdot00875)^{79}$, *i.e.*, £1·990235. The question now becomes—in how many days will £1·990235 produce £·009765 as interest? The answer to this is $\frac{100 \times \cdot009765 \times 365}{1\cdot990235 \times 3\cdot5}$ days, *i.e.*, 51·17 days, whereas the number of days by the former method was 51·26. As fractions of days are never dealt with; and as in 19 years 9 months 51 days £1 has not quite amounted to £2, the answer by both methods is 19 years 9 months 52 days. Thus, no discrepancy arises in this case, and it is clear that at most the error, resulting by the above assumption in problems like this, is one day.

Referring to Example (xiii), the student is advised to calculate the amount of £475·6292 in 14 years, then to find the number of days this sum takes to amount to £725, using the simple interest formula and employing logarithms whenever it is of advantage to do so, and to compare results.

EXAMPLE (xiv)—

A savings certificate bought for 15s. 6d. amounted to £1 in 5 years. At what rate per cent. compound interest would 15s. 6d. amount to £1 in the same time?

Let £i ≡ interest on £1 for 1 year,
then $1 = \cdot775 \times (1 + i)^5$

$$\therefore\ (1 + i)^5 = \frac{1}{\cdot775}$$

$$\therefore\ 1 + i = \left(\frac{1}{\cdot775}\right)^{\frac{1}{5}} \text{ by taking the 5th root of each}$$

$$= 1\cdot0523$$

$$\therefore\ i = \cdot0523$$

$\therefore$ interest on £100 = £5·23.

Number.	Logarithm.
1	0·0000000
·775	$\bar{1}$·8893017
f	·1106983
1·0523	·0221396

Ansr.—5·23%.

EXAMPLE (xv)—

£100 amounted to £117 3s. 6d. in 3 years, compound interest payable half-yearly. What would the amount be after another 4 years at the same rate of interest?

Let $£i \equiv$ interest on £1 for 1 year, if payable yearly,

$$\text{then } 117{\cdot}175 = 100 \times \left(1 + \frac{i}{2}\right)^6$$

$$\therefore\ 1 + \frac{i}{2} = \left(\frac{117{\cdot}175}{100}\right)^{\frac{1}{6}}$$

$$\text{Now, amount after 7 years} = £100 \times \left(1 + \frac{i}{2}\right)^{14}$$

$$= £100 \times \left(\frac{117{\cdot}175}{100}\right)^{\frac{14}{6}}$$

$$= £100 \times (1{\cdot}17175)^{\frac{7}{3}}$$
$$= £144{\cdot}7487$$
$= £144$ 15s. to nearest 1d.

Number.	Logarithm.
1·17175	·0688349
1·447487	·1606148

EXAMPLE (xvi)—

The population of England and Wales was 37,885,000 in 1921 and 39,948,000 in 1931. Find the percentage increase per year, assuming it to be constant throughout the period. What was the probable population in 1927?

Let $r \equiv$ rate per cent. of increase per year,

$$\text{then } 37{,}885{,}000 \times \left(1 + \frac{r}{100}\right)^{10} = 39{,}948{,}000$$

$$\therefore\ 1 + \frac{r}{100} = \left(\frac{39{,}948}{37{,}885}\right)^{\frac{1}{10}}$$
$$= 1{\cdot}005316$$
$$\therefore\ r = {\cdot}5316$$

$$\text{Pop. in 1907} = 37{,}885{,}000 \times (1{\cdot}005316)^6$$
$$= 39{,}110{,}000$$

Number.	Logarithm.
39948	4·6014950
37885	4·5784673
f	·0230277
1·005316	·0023028
$(1{\cdot}005316)^6$	·0138166
37885000	7·5784673
39109645	7·5922839

***103. GEOMETRICAL PROGRESSION** is the name given to series of quantities which have the property that the ratio of any one quantity to the next is constant. For example, 3, 6, 12, 24,

48 . . . is a series in Geometrical Progression, the constant ratio being 2.

Let $a \equiv$ first term of a G.P.
,, $r \equiv$ constant ratio
,, $S \equiv$ sum of n terms of the series
then $ar =$ 2nd term of the G.P.
,, $ar^2 =$ 3rd ,, ,,
,, $ar^3 =$ 4th ,, ,,
and $ar^{n-1} =$ nth ,, ,,

so that $S = a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-2} + ar^{n-1}$

Multiplying each side by r, it is clear that,

$$rS = ar + ar^2 + ar^3 + ar^4 + \ldots\ldots + ar^{n-1} + ar^n$$

By subtracting, it follows that,

$rS - S = ar^n - a$, as all the intermediate terms cancel one another,

i.e., $S(r-1) = a(r^n - 1)$

$\therefore\ S = \dfrac{a(r^n - 1)}{r-1}$ or $\dfrac{a(1 - r^n)}{1-r}$ by multiplying numerator and denominator by -1.

Now suppose that r is less than one, then the greater the number of times r is multiplied by itself, the smaller will be the result. If n is an infinitely large number, then r^n is infinitesimally small and may be considered as being zero.

Thus, $a + ar + ar^2 + \ldots\ldots$ ad. inf. $= \dfrac{a}{1-r}$, if r is less than 1.

*EXAMPLE (xvii)—

At the beginning of a year a man decided to deposit £30 at the end of each year in a bank which paid $3\frac{1}{2}$ per cent. interest. If the instalments were allowed to accumulate, what was the total amount standing to his credit at the end of 15 years?

Amount of 1st instalment $= £30 \times (1{\cdot}035)^{14}$
,, 2nd ,, $= £30 \times (1{\cdot}035)^{13}$
. . . .
,, 14th ,, $= £30 \times 1{\cdot}035$
,, 15th ,, $= £30$.

$\therefore$ Total amount $= £30 + 30 \times 1{\cdot}035 + \ldots + 30 \times (1{\cdot}035)^{13} + 30 \times (1{\cdot}035)^{14}$

$$= \frac{£30\ [(1{\cdot}035)^{15} - 1]}{1{\cdot}035 - 1}$$

$$= \frac{£30 \times {\cdot}675346}{{\cdot}035}$$

$= £578{\cdot}868$
$= £578$ 17s. 4d. to nearest 1d.

Number.	Logarithm.
1·035	·0149403
1·675346	·2241045

*EXAMPLE (xviii)—

A certain property produces a return of £350 annually, and the lease expires in 48 years' time. What should be paid for the property, assuming

the purchaser will receive his first instalment of £350 one year after the date of purchase and reckoning compound interest at 6 per cent. per annum? What would be the value if the production of the £350 per annum continued perpetually?

$$\text{Present value of 1st year's return} = \pounds\frac{350}{1{\cdot}06}$$

$$\text{,, 2nd ,,} = \pounds\frac{350}{(1{\cdot}06)^2}$$

$$\cdots\cdots\cdots\cdots$$

$$\text{,, 48th ,,} = \pounds\frac{350}{(1{\cdot}06)^{48}}$$

$$\therefore \text{Total present value} = \pounds\frac{350}{1{\cdot}06} + \frac{350}{(1{\cdot}06)^2} + \ldots + \frac{350}{(1{\cdot}06)^{48}}$$

$$= \frac{\pounds\frac{350}{1{\cdot}06}\left[1 - \left(\frac{1}{1{\cdot}06}\right)^{48}\right]}{1 - \frac{1}{1{\cdot}06}}$$

$$= \frac{\pounds\frac{350}{1{\cdot}06} \times {\cdot}93900183}{1 - \frac{1}{1{\cdot}06}}$$

Number.	Logarithm.
1	0·0000000
1·06	·0253059
	$\bar{1}$·9746941
·06099817	$\bar{2}$·7853168

$$= \pounds\frac{350 \times {\cdot}93900183}{{\cdot}06} \quad \text{by multiplying } N \text{ and } D \text{ by } 1{\cdot}06$$

$$= \pounds5{,}477 \ 10\text{s. } 3\text{d. to nearest 1d.}$$

$$\text{If perpetual, total present value} = \pounds\frac{350}{1{\cdot}06} + \frac{350}{(1{\cdot}06)^2} + \ldots\ldots\ ad.\ inf.$$

$$= \pounds\frac{\frac{350}{1{\cdot}06}}{1 - \frac{1}{1{\cdot}06}} = \pounds\frac{350}{{\cdot}06}$$

$$= \pounds5{,}833 \ 6\text{s. } 8\text{d.}$$

NOTE 11.—The second part of the question is the same as "What principal produces £350 a year at 6% per annum?" From the simple interest formula, $P = \pounds\frac{100 \times 350}{6 \times 1} = \pounds5{,}833\frac{1}{3}$, so that the results tally.

*EXAMPLE (xix)—

A man borrows £10,000 at 5 per cent. He clears it off in 12 years by

equal annual payments, beginning at the end of the first year. Find the annual payment necessary.

Let £a ≡ annual payment

$$\text{then } £\frac{a}{1\cdot05} = \text{present value of 1st payment}$$

$$£\frac{a}{(1\cdot05)^2} = \quad ,, \quad ,, \quad \text{2nd} \quad ,,$$

. . . .

$$£\frac{a}{(1\cdot05)^{12}} = \quad ,, \quad ,, \quad \text{12th} \quad ,,$$

$$\therefore \frac{a}{1\cdot05} + \frac{a}{(1\cdot05)^2} + \dots + \frac{a}{(1\cdot05)^{12}} = 10{,}000$$

$$\therefore \frac{\frac{a}{1\cdot05}\left[1 - \left(\frac{1}{1\cdot05}\right)^{12}\right]}{1 - \frac{1}{1\cdot05}} = 10{,}000$$

$$\therefore \frac{a\left[1 - \left(\frac{1}{1\cdot05}\right)^{12}\right]}{\cdot05} = 10{,}000$$

$$\therefore a = \frac{500}{1 - \left(\frac{1}{1\cdot05}\right)^{12}}$$

$$= \frac{500}{\cdot4431626}$$

$$= 1128\cdot254$$

Number.	Logarithm.
1	0·0000000
1·05	·0211893
f	$\bar{1}\cdot9788107$
·5568374	$\bar{1}\cdot7457284$
500	2·6989700
·4431626	$\bar{1}\cdot6465632$
1128·254	3·0524068

∴ Each payment is £1,128 5s. 1d.

*Example (xx)—

The weight of a frustum of a cone is given by the formula

$$W = \tfrac{1}{3}\pi hp(R^2 + Rr + r^2),$$

where R and r are the radii, h is the perp. height, and p is the weight of unit volume of the material. Find the weight of a stone column 45 ft. high, whose lower and upper diameters are 3′ 4½″ and 2′ 9¾″ respectively, if 1 cub. ft. of the material weighs 353·47 lb.

$$\text{Weight of column} = \frac{3\cdot14159 \times 45 \times 353\cdot47}{3} \times (1\cdot6875^2 + 1\cdot6875 \times 1\cdot40625 + 1\cdot40625^2)\text{ lb.}$$

$= 3{\cdot}14159 \times 15 \times 353{\cdot}47 \times 7{\cdot}198242$ lb.

$= 119900{\cdot}2$ lb.

Ans.—53 tons 1180 lb.

Number.	Logarithm.
1·6875	·2272438
2·847656	·4544876
1·40625	·1480625
2·373047	·3753063
1·977539	·2961250
3·14159	·4971496
15	1·1760913
353·47	2·5483526
7·198242	·8572264
119900·2	5·0788199

TEST EXERCISES I, 10.

(For the solutions of the following questions, a copy of "Chambers' Mathematical Tables" should be obtained.)

(1) Evaluate—(*a*) $\bar{1}{\cdot}4387 - \bar{2}{\cdot}3974 + 1{\cdot}9634 + {\cdot}0117$,
(*b*) $\cdot 3948 - (1{\cdot}4098 + {\cdot}3729 - 2{\cdot}1887)$,
(*c*) $(\bar{1}{\cdot}3186749 - \bar{1}{\cdot}7930482) \times 14$;
(*d*) $\frac{1}{4}(\bar{2}{\cdot}1386) - \frac{1}{3}(\bar{5}{\cdot}2487) + \frac{2}{5}(1{\cdot}1417 - 1{\cdot}2308)$

(2) (*a*) Tabulate the logarithms of 14, 2075·4, ·390487, 214076900, 000499362, and 8·700438.

(*b*) Tabulate the anti-logarithms of $\bar{1}{\cdot}8850557$, 3·6532897, $\bar{2}{\cdot}7038116$, ·048629, $\bar{1}{\cdot}202718$, and 5·4700000.

(3) By means of logarithms, evaluate the following—

(*a*) $\dfrac{7{\cdot}406 \times 8{\cdot}9309 \times 212{\cdot}38}{{\cdot}006891 \times {\cdot}737 \times 83{\cdot}746}$

(*b*) $\sqrt{\dfrac{{\cdot}1}{2{\cdot}983 \times {\cdot}87764}}$

(*c*) $(1{\cdot}185)^{17}$

(*d*) $\sqrt{\dfrac{1{\cdot}17294 \times (3{\cdot}172)^{5}}{(4{\cdot}7263)^{4}}}$

(*e*) $\dfrac{1}{{\cdot}079368 \times 1{\cdot}21173 \times 1{\cdot}989}$

(4) Show by expressing in terms of log 2, log 3, and log 5 that $\log \frac{15}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243} = \log 2$.

(5) Given that log 2 = ·3010300 and log 3 = ·4771213, calculate the values of log 5, log ·006, log 4·5, log ·648, log 112·5.
(*Note.*—Log 10 − log 2 = log 5.)

(6) Given that log 12 = 1·0791812 and log 18 = 1·2552725, find log 2, log 3, log ·192, and log 14·58.
(*Note.*—Log 18 − log 12 = log 1·5, and log 12 − log 1·5 = log 8 = 3 log 2.)

(7) Find the value of a rectangular prism of metal, the dimensions being 4·117″ × 3·253″ × 1·047″, the specific gravity 11·3, and the cost 3s. 4½d. per oz. Tr. (1 cub. ft. of water weighs 1,000 oz. Av.)

(8) How many spheres of lead of diameter ·585 in. could be made from a block of lead in the shape of a square prism whose dimensions are 3·67″ × 3·67″ × 13·97″? ($\pi = 3\frac{1}{7}$)

(9) The volume (V) of the frustum of a pyramid is given by the formula $V = \frac{h}{3}(A_1 + \sqrt{A_1A_2} + A_2)$, where A_1 and A_2 are the areas of the parallel sections and h is the perpendicular distance between them. Find the volume of a frustum of a square pyramid of height 7·57″, the length of the sides of the section being 4·132″ and 2·497″ respectively.

(10) Find the ratio of the area of surface of a cube to that of a sphere having an equal volume.

(11) 1 peck of a certain powder weighs 19 lb. 5 oz. What are the internal dimensions of a cylindrical tin whose height equals its diameter, which will just hold ½ lb. of the powder? (1 gall. = 277·274 cub. in. π = 3·1416).

(12) Find the volume of a hollow sphere whose external and internal diameters are 5·731 in. and 3·946 in. respectively.

(13) From the relationships given in paragraph 61, and taking £1 as the equivalent of 89·17 francs, express—

(*a*) 1s. 1¾d. per ton per mile in francs per metric ton per kilometre.
(*b*) 7s. 11d. per yard in francs per metre.
(*c*) ·65 francs per litre in pence per pint.
(*d*) 346·20 francs per hectare in £ *s.* *d.* per acre.

(14) At a certain time, £437 15s. 9d. was equivalent to 12719·70 francs. Later on, when the number of francs equivalent to £1 had increased by 2·92 per cent., what was the equivalent in francs of £295 7s. 10d.?

(15) Find the weight of ⅛ mile of metal wire ·0057 in. in diameter, given that 1 cub. ft. of the metal weighs 513 lb. 11 oz.

(16) A statuette whose height is 4·7 in., weighs 3 lb. 7·4 oz. What would be the weight of a statue of similar shape and same material, if its height were 8 ft. 6 in.?

What would be the height of a statue of similar shape and same material if its weight were 3 ton 11 cwt. 15 lb.?

(17) The population of a town increased in 5 consecutive years by 5·1 per cent., 3·73 per cent., 2·94 per cent., 4·13 per cent., and 5·78 per cent. per year respectively. If the population at the beginning of the period was 17,494, what was it 5 years later?

(18) A sold an article to B, thereby gaining 13·7 per cent. on his outlay. B sold it to C and gained 7·4 per cent. on his outlay. C sold it to D, who paid a sum which was 28·3 per cent. greater than its cost to A. What percentage profit on his outlay did C gain?

(19) Find the interest on £711 14s. 6d. for 68 days at 4¼ per cent. per annum.

(20) In how many days would £965 amount to £1,000 at 4⅝ per cent. per annum?

(21) Find the compound interest on £713 6s. 5d. for 3 years at 5¼ per cent. per annum. Also find the total interest, if the period of time was 3 years 114 days.

(22) A company pays interest at 7½ per cent. per annum payable quarterly. To what rate per cent. per annum, payable yearly, is this equivalent?

(23) What sum, if allowed to accumulate at 3½ per cent. per annum, payable half-yearly, would amount to £750 in 7½ years?

(24) What sum, if allowed to accumulate at 4¾ per cent. per annum, would amount to £1,000 in 12 years 219 days' time?

(*Note.*—Find what £1 amounts to in 12 years 219 days, then divide this amount into £1,000.)

(25) How long would it take £693 17s. 6d. to amount to £1,050 at 4¼ per cent. per annum, if interest is payable (i) yearly, (ii) quarterly, (iii) monthly?

(26) At what rate per cent. per annum would a sum of money allowed to accumulate, treble itself in 20 years?

(27) A man borrowed £400 from a money-lender, who, 4 years later, demanded £500. What rate per cent. per annum compound interest did the latter require for lending his money?

*(28) A and B each start working at £150 per annum. A is promised a yearly increase of £10 to a maximum of £300 and B an increase of 5 per cent. each year for 15 years. Who has the higher maximum?

What are the total amounts they have earned during the period of 16 years?

(*Note.*—As A's increments are constant, his average salary for the period of 16 years is £$\frac{150 + 300}{2}$ = £225.)

(29) The population of Oxford in 1921 was 57,052 and in 1931 it was 80,540. What was the annual increase per thousand, assuming it remained constant throughout the period?

*(30) Find the amount of 21 annual payments of £25 9s. 6d. at 4 per cent. per annum: interest payable yearly and first payment made 1 year from now.

*(31) A man borrows £1,000 at $5\frac{1}{4}$ per cent. and promises to pay off the debt by 14 annual payments beginning at the end of the first year. What is the annual payment necessary?

(32) The interest on £507 for 3 years was £73 11s. 10d., the interest being paid yearly. What would be the additional interest if the money were allowed to accumulate for another two years at the same rate of interest?

*(33) What is the value of an estate which produces a net income of £975 per annum perpetually. What would be its present value if the lease were to expire in 30 years' time, assuming the first instalment of £975 to be immediately due? (Reckon interest at 6 per cent. per annum.)

(34) A man borrowed £15,750 for two months at 5 per cent. per annum. At the end of two months, the interest was added on and the debt renewed for another two months. This was continually repeated until the end of 22 months, when the debt and interest were repaid. Find the total interest.

*(35) A man at the beginning of 1918 subscribed an amount which, put out at interest at 4 per cent. per annum, would be sufficient to provide a hospital with £500 per annum for the period 1922–1932, the money to be paid at the beginning of each year. What was the amount he subscribed?

SECTION II.

THE BUSINESS UNDERTAKING.

INTRODUCTION.

THE object of establishing a business concern is to supply commodities or services to a country, town, or district where they are desired. The production and the distribution of these commodities give rise to business activity on the part of persons who, in return for the use of their capital and the carrying on of their work, expect some reward in the form of profits. According to the disposal of the profits, a business belongs to one of two groups; namely, the purely Capitalistic Concern, the profits of which go to the owner, owners, or shareholders; and the Communistic Concern, which is so organized as to enable the profits to be shared among the consumers and the employees, or else used for the benefit of the community. The subdivisions of these groups are shown in the table on the opposite page.

As regards losses, the law of bankruptcy, full details of which are beyond the scope of this book, affords relief to insolvent debtors, who, by surrendering their property for the benefit of their creditors, are relieved from liability as regards their debts. This law enables the creditors to receive payments in proportion to the amounts owing to them, and thus prevents some creditors from taking an undue advantage over others. Proceedings are commenced by a creditor (who must, either alone or with others, have at least an unsecured sum of £50 owing), or the debtor himself, presenting a petition to the Court. If the petition is fully in order and certain fees are paid, the Court issues a receiving order, which must be advertised in the *London Gazette* and in a local paper, and notifies the Board of Trade. An **official receiver**, appointed by the latter, becomes the protector of the debtor's business until a trustee (sometimes the official receiver himself) has been appointed by the creditors. The debtor must furnish a complete statement of his affairs, and if he be found guilty of fraud he is liable to be punished. The Court also appoints a day for the public examination of the debtor. If the debtor be declared a

				Profits.
Business Undertakings	Capitalistic Concerns	The Sole Trader		Disposes of all
		Partnership		Divided among partners
		Limited Partnership		
		Public Limited Company		Divided among shareholders
		Private Limited Company		
		Combinations	Trusts	
			Cartels	
	Communal Concerns	Co-operative Societies		Divided among employees and customers
		Municipal Undertakings (*e.g.*, Supply of water, gas, and electric power, running of tramways)		Used for the benefit of the community
		Government Undertakings (*e.g.*, Postal services, savings bank, national insurance.)		Used for the benefit of the Nation

bankrupt, the trustee winds up the business by realizing the property and other assets; and after deducting sufficient to pay in full the **preferential claims** (for example, employees' wages, rent, rates, taxes, and legal expenses of the bankruptcy proceedings), he declares a dividend of so much in the £ to be paid to the ordinary creditors. A man adjudicated a bankrupt is not allowed to be a Member of Parliament or to hold municipal office; and should he obtain a sum exceeding £10 on credit without stating at the time he is a bankrupt, he is guilty of a criminal offence.

To avoid the expense, trouble, and disgrace of bankruptcy proceedings, the creditors and the debtor can agree to a *deed of arrangement*, which takes various forms, the most common being: (1) Deed of Assignment, by which a trustee can realize the estate and distribute the proceeds to the creditors in proportion to the amounts of their debts; (2) Deed of Composition, whereby the creditors can release the debtor from his liabilities on condition that he agrees to pay them so much in the £ in a lump sum or by instalments; (3) Deed of Inspectorship, which allows the debtor to carry on his business, but under the supervision of a trustee appointed by the creditors.

This Section is mainly concerned with profits and losses. In the case of a business belonging to a sole trader, the net profits being entirely at the disposal of the owner himself, no arithmetical problems involving the division of profits arise.

In the first chapter, therefore, an opportunity is taken of dealing with several problems which confront traders engaged in various kinds of businesses.

The remaining chapters, in the main, deal respectively with the divisions of profits and losses among (1) the partners of firms; (2) the shareholders of companies; and (3) the shareholders, members, and employees of co-operative associations and other communal concerns.

As in Section I, the examples and test exercises marked with an asterisk are comparatively difficult, and might with advantage be left by the student until a final revision.

CHAPTER XI.

THE SOLE TRADER.

104. A Sole Trader is a person who has exclusive ownership of a business. The law holds him responsible for all the debts and losses of the business, even if incurred by neglect or carelessness on the part of his employees. He is not compelled to have his accounts audited, and he need not issue a balance sheet. The profits can be disposed of entirely in accordance with his wishes, and the law does not prevent him from winding up or selling his concern at any time. The capital is introduced by him, but he has the right to make use of money obtained by loans.

In most cases, businesses organized in this way are not extensive in character owing to the fact that the owner is frequently his own organizer and manager, and thus has little time, even supposing he be not short of capital, to consider the question as to how his business could be increased. It has frequently happened that a one-man business has so prospered that partners have been called in and, later on, the concern has been transformed into a company.

The proprietor, whether his business be that of a hawker, retailer, merchant, manufacturer, etc., has, in order to ascertain his financial condition, to keep accounts. These, combined with other problems of a kind which vary according to the nature of his business, render it imperative that he should possess a thorough knowledge of the elements of Arithmetic. Some examples involving profit and loss are worked as follows—

Example (i)—

A silk importer buys silk in France at so many francs per metre. The cost of transport, etc., increases this price by 4 per cent. Taking 89·30 francs to be equivalent to £1 and 1·09 yd. to 1 metre, find a multiplier to 3 places of decimals, which he can use to make out a price list for silks in shillings per yard, which would give him a profit of 12½ per cent. on his total outlay. Use the multiplier in the cases where he bought silk for (1) 15·75 fc. per metre and (2) 22·20 fc. per metre.

If cost of 1 metre = 1 franc

then selling price of 1 ,, $= 1 \times \frac{104}{100} \times \frac{112 \cdot 5}{100}$ franc.

,, ,, 1 yard $= \frac{104 \times 112 \cdot 5}{100 \times 100 \times 1 \cdot 09}$ franc

$= \frac{104 \times 112 \cdot 5 \times 20}{100 \times 100 \times 1 \cdot 09 \times 89 \cdot 3}$ shilling

$= \cdot 240$ shilling.

Buying Price.	Selling Price.
15·75 fc. per metre	15·75 × ·24 shillings per yard = 3/9½ per yard
22·20 fc. ,,	22·2 × ·24 ,, ,, = 5/4 per yard

Example (ii)—

The liabilities of a bankrupt were £13,600 and his assets were estimated at £9,750. The creditors received a first dividend of 9s. 6d. in the £. On being wound up, the assets only realized 85 per cent. of the estimated amount; and the legal expenses, the salary of the trustee, and the payments of preferential claims together absorbed 12½ per cent. of the sum realized. What additional amount in the £ should the creditors receive? What total sum should a creditor for £2,750 receive?

Sum to be divided among creditors $= £9{,}750 \times \frac{85}{100} \times \frac{87\frac{1}{2}}{100}$

∴ Total dividend in the £ $= \frac{9750 \times 85 \times 175 \times 20}{100 \times 200 \times 13600}$ shillings

$= 10 \cdot 664$ shillings

= 10s. 8d. to nearest ½d.

∴ Additional dividend in the £ = 1s 2d ,, ,,

Total sum received by creditor for £2,750 $= £\frac{2750 \times 10\frac{2}{3}}{20}$

= £1,466 13s. 4d.

Example (iii)—

Thomas Ross owed James Cowan £915, payable on 24th October, 1930. The former went bankrupt, with the result that Cowan received 8s. 3d. in the £ on 12th January, 1931, and a final dividend of 3s. 9½d. in the £ on 25th August, 1931. What was Cowan's loss, reckoning that he could always put out his capital to gain 5 per cent. per annum?

£100 from 24th October, 1930, to 12th January, 1931, at 5% per annum amounts to $£101\frac{7}{73}$.

∴ Value on Oct 24th, 1930, of first dividend $= £\frac{915 \times 8\frac{1}{4} \times 100}{101\frac{7}{73} \times 20}$

Again, £100 from 24th October, 1930, to 25th August, 1931, at 5% per annum amounts to £$104\frac{13}{73}$.

∴ Value on 24th Oct., 1930, of first dividend $= £\dfrac{915 \times 3\frac{49}{60} \times 100}{104\frac{13}{73} \times 20}$

∴ Value on 24th Oct., 1930, of complete dividend $= £4575 \times \left(\dfrac{33 \times 73}{4 \times 7380} + \dfrac{229 \times 73}{60 \times 7605}\right)$

$$= £\frac{4575 \times 73}{4} \times \left(\frac{33}{7380} + \frac{229}{15 \times 7605}\right)$$

$$= £\frac{4575 \times 73 \times 5454495}{4 \times 7380 \times 15 \times 7605}$$

$= £540$ 19s. 1d. to nearest 1d.

∴ Extent of loss on 24*th October*, 1930, was £374 0s. 11d.

NOTE 1.—If Cowan wished to know the total extent of his loss, say, on 31st December, 1931, he would find the amount of £374 0s. 11d. for the period 24th October, 1930, to 31st December, 1931, at 5 per cent. per annum.

105. GOODWILL.

When a commercial concern changes hands it is likely that the former customers will continue to make purchases from the business under its new proprietorship. This results in the new owner making greater profit than would have been the case had he started an entirely new business, for in the latter case, besides having to wait a considerable time, he would have to apply ingenuity and advertisement before the amount of business carried on would result in profits that would justify his initial outlay. Thus a business having already a good connection and reputation is worth a considerable amount in addition to the price of premises, appliances, stock, etc. This asset of a business due to its connection is known as **Goodwill**, the value of which is usually reckoned as being worth from two to five years' purchase, that is, the amount of the profits during the past two to five years.

EXAMPLE (iv)—

A man bought and managed a business, giving £975 for property, £720 for stock, and 4 years' purchase for goodwill, the profits of the business for the previous 4 years being £525, £490, £570, and £584 respectively. During the first complete year the expenses incurred amounted to £375, and the money paid for new stock was £1,345, all of which, including expenses, was paid out of the money received from the sale of the goods. His gross receipts were £2,635, and he had stock to the value of £625 at the end of the year. Assuming that he could have obtained 5 per cent. on his capital and that he gave up the management of a similar business at a salary

of £275 per year, what was his net profit for the year, allowing 5 per cent. depreciation in the value of the property bought?

$$\text{Gross profit} = £2{,}635 - £1{,}345 - £375 - £95 - £\tfrac{5}{100} \times 975$$
$$= £771\ 5\text{s.}$$
$$\text{Interest on initial outlay at } 5\% = £\tfrac{5}{100} \times 3864$$
$$= £193\ 4\text{s.}$$
$$\therefore \text{ Net profit} = £771\ 5\text{s.} - £193\ 4\text{s.} - £275$$
$$= £303\ 1\text{s.}$$

TEST EXERCISES II, 1.

(1) A manufacturer employed 35 men in a workshop at 11½d. per hour. They worked 9 hr. per day and 5 hr. on Saturdays, and overtime counted as time and a quarter; in addition, work on Sundays counted as time and a half. A bonus of 10 per cent. on the amounts earned was allowed. If 17 men altogether worked overtime each to the extent of 11 hr. on week-days and 8 of these also worked 7 hr. on Sundays, what was the total amount necessary to pay the wages?

(2) An agent received each year from his employer a certain sum for travelling expenses and a commission of a certain percentage of the amount of his sales. In two successive years he earned £325 and £412 10s., the amounts of his sales being £11,520 and £16,560 respectively. What must be the amount of his sales during the third year, in order that his total salary should be £500?

(3) A retailer bought 238 lb. of tea at £8 15s. per cwt. He sold it at such a price per lb. as to realize a profit of 16 per cent. on his outlay. When the tea had been sold, he found that his takings were 10s. 10½d. less than that estimated; and put it down to the fact that some tea had been pilfered. What amount had been pilfered and what was his actual percentage profit?

(4) What is the total rent paid by a farmer for a farm of 112 ac. 2 rd. 15 sq. pl. at £4 15s. per acre?

(5) A man was offered £115 for five horses, but refused to sell. He then employed an auctioneer, who sold two for £24 5s. each, two for £22 15s. each, and one for £27 10s. If the auctioneer charged a commission of 2½ per cent., what did the man gain or lose by employing him?

(6) A wine merchant bought in Italy 35 hectolitres of wine at 7 lire 35 centesimi per litre. What, in English money, must he pay, if £1 is equivalent to 68 lire 74 centesimi?

(7) A manufacturer supplies a certain commodity which is such that a cylindrical tin whose internal height and diameter are 6 in. and 4¼ in. respectively, as near as possible holds 1 lb. when full. He decides to sell tins containing 2 lb. and 3 lb. respectively. If the internal heights are 8 in. and 10 in. respectively, what should the diameters be if they are to be an exact number of eighths of an inch?

(8) What is the interest due on a loan of £500 from 29th September 1931, to 5th March, 1932, at 5½ per cent. per annum?

(9) A retailer marks his goods 33⅓ per cent. above the cost price, but he allows 5 per cent. discount for cash. What is the actual percentage profit?

(10) A merchant is in the habit of borrowing sums of money not exceeding £1,000 and not for a greater time than 4 months. Construct a table giving the interest on £1 for 1 day at 4½, 4¾, 5, 5¼, and 5½ per cent. per annum by means of which he can calculate to the nearest penny the interests on his loans. Use the table to find the interest on (i) £450 for

75 days at 4¾ per cent. per annum and (ii) £500 for 64 days at 5½ per cent. per annum.

(11) A man bought oil imported from the United States at an inclusive cost of a number of cents per U.S. gallon. Given that £1 = $3·40. find a multiplier which he could use to make out a price list of oils in pence per Imperial gallon which would give him a gross profit of 6 per cent. Use the multiplier to determine selling prices when he bought oil at (i) 55 cents per U.S. gallon, (ii) 75 cents per U.S. gallon. (1 U.S. gallon = 231 cub. in.)

(12) 1 cwt. of a mixture is made by mixing a quantity of inferior tea at £10 5s. 4d. per cwt. with a quantity of better tea at £11 13s. 4d. per cwt., so that by being sold at 2s. 4d. per lb. a profit of 20 per cent. on cost price could be made. How many pounds of each kind of tea should be mixed?

(13) What amount, in kroner and ore, should an importer of provisions pay per centner of butter, in order that by selling at £4 15s. per firkin he would gain a profit of 10 per cent., if £1 = 17·55 kroner? (1 kroner = 100 ore; 1 centner = 110·231 lb.; 1 firkin (of butter) = 56 lb.)

(14) A square enclosure is required to contain 350 sheep, so that on an average each shall have not less than 9 sq. ft. of land surface. If hurdles 6 ft. long are used, what will be the least number of hurdles required?

(15) What is the value of gold in a chain made of standard gold (22 carat) which weighs 2 oz. Tr. 17 dwt. 6 gr., if pure gold is valued at £5 12s. 6d. per ounce Troy?

(16) During a sale an article previously marked at £8 6s. was reduced to £6 13s. by selling at which price the shopkeeper lost 5 per cent. on what the article cost him. What percentage profit on cost price would he have gained if he had sold the article before the sale?

(17) A man bought three houses of the same kind leasehold for £1,025; and spent £130 in making improvements. The total ground rent was £22 1s., and he reckoned repairs could be effected at £24 per annum. The houses are each assessed at £35 per annum, and the rates are 7s. 11d. in the £ on the assessed rentals. What weekly rent, in shillings and sixpences (if necessary) must he charge in order to obtain 10 per cent. on the money he has sunk? If one house is empty for 9 months in the year, what is his percentage profit or loss?

(18) What is the cost of 3 ℥ 1 ʒ 2 ℈ of a drug at 5s. 4d. per ounce?

(19) An importer bought flour at $105 per short ton. Transport, etc., increased this price by 7½ per cent. At what price in English money should he sell the flour per barrel in order to gain 12 per cent. on his total outlay, if £1 = 3·36 dollars? [1 short ton = 2,000 lb., 1 barrel (of flour) = 196 lb.]

(20) A farmer employed two men, X and Y, to fence in a field, and paid them 2s. 3d. and 2s. 9d. per day respectively. The first day, X put up 35 yd. and Y 40 yd.: on each subsequent day, X and Y accomplished 30 yd. and 47½ yd. respectively. If, after 12 days, the work was finished, find who was the most profitable man to employ, and estimate (i) the time this man would have taken if working alone (taking a fraction of a day as a whole day or a half-day, according as to whether it is more or less than a half-day) and (ii) the amount which would have been saved by employing him only.

(21) A merchant had two kinds of silk, which cost him £7 5s. and £8 10s. 6d. per piece. A customer requires 15 pieces of the first and 6 pieces of the second, and offers £8 5s. per piece for the former kind. If the merchant wishes to make on the cost price 12 per cent. profit, what price per piece should he ask for the second kind of silk?

(22) The debts of a bankrupt amount to £2,158 3s. 9d., and his assets consist of property estimated at £915 and two bills, the first for £275 due 4 months hence, and the second £168 10s. due 3 months hence, interest in each case being reckoned at 5 per cent. The property was sold at a figure 5 per cent. higher than that estimated, but the expense of winding up the business was 8 per cent. of the gross assets. How much in the £ could he pay his creditors?

(23) A builder agrees to build a house for £750, and to wait a period not exceeding 4 years for his money on the understanding that $\frac{1}{40}$ of the amount owing shall be charged as interest every half-year and added to the principal. What should be the amounts if payment be made (i) at the end of $1\frac{1}{2}$ years, (ii) at the end of 3 years?

(24) A packing case is required to hold 10 gross of packets each 8″ long, 5″ wide, and 3″ deep. Assuming that the packets are to be arranged to lie all the same way, find the dimensions of the packing case that would require the least wood.

(*N.B.*—To satisfy last condition, the case should be as near a cube as possible.)

(25) A man borrowed £150 from a money-lender and agreed to pay four instalments of £45 each at the end of 3, 6, 9, and 12 months. What rate of interest was the money-lender charging?

(26) A retailer gets $33\frac{1}{3}$ per cent. trade discount off the prices in a manufacturer's list, and he wished to price his articles in such a way that, after allowing his customers $7\frac{1}{2}$ per cent. discount for cash, he will still make a profit of 15 per cent. on what the goods cost him. Find by how much per cent. the prices he marks the articles are greater or less than the manufacturer's list prices.

(27) A tobacco importer gained 15 per cent. gross profit on his cost price. Owing to increase of duty, the tobacco costs him 11d. per lb. more. By how much per lb. should he increase his selling price so as to gain the same percentage profit?

(28) A bankrupt owes £15,475. His assets are £75 10s. 6d. cash; buildings valued at £4,750; plant valued at £1,325; stock valued at £825 10s.; and outstanding debts of total value £2,746 5s. The buildings, plant, and stock are sold at 5 per cent. above, 10 per cent. below, and $7\frac{1}{2}$ per cent. below the estimated amounts respectively. Of the debts, one for £725 realizes 13s. 9d. in the £, one for £1,017 10s. realizes 14s. 10d. in the £, and the remainder realizes the full amount. If the legal expenses, together with the preferential claims, amounted to £1,137 15s., what dividend could be declared?

(29) A bankrupt paid 11s. 7d. to his creditors, but later a debt of £2,214 10s., which had been written off as bad by the trustee, realized 6s. $8\frac{1}{2}$d. in the £. If the trustee's additional expenses were 45 guineas, and the original sum owed by the bankrupt to the ordinary creditors was £3,225, what additional amount in the £ should the latter receive?

(30) A merchant who became insolvent was released from his liabilities on 5th May by the creditors under a deed of inspectorship. Under the supervision of the trustee, a dividend of 7s. 6d. in the £ was made on 5th May of the next year, another 7s. 6d. in the £ on 5th May of the following year, and 3s. 6d. in the £ on 5th November of this year. The creditors then completely released the merchant from his liabilities. Reckoning interest at 5 per cent., what was the loss on 5th May of the first year of insolvency to a creditor to whom originally the merchant owed £10,000?

(31) A retailer started a new business after making an outlay of £755, of which £245 was for appliances, apart from stock. His profits for the first four years of working were £55, £275, £290, and £315. He then sold the business for £1,450, which included payment for goodwill at three years' purchase, payment for appliances at 65 per cent. of their original cost, and the rest, payment for stock. For what amount did he sell the stock?

(32) A and B are two merchants, each being the sole owner of a business, and their capitals are £2,250 and £5,420 respectively. The profits for the past three years were £475 and £512, £448 and £436, £452 and £485 respectively. If each values the goodwill of his business at three years' purchase less 5 per cent. per annum on capital, how much was the goodwill of A's business greater or less than that of B's?

(*N.B.*—Regard goodwill as being worth the sum of the past three years' net profits, *i.e.*, the profits obtained by deducting interest on capital.)

(33) A traveller was appointed by a merchant at a rate of £150 per annum plus 5 per cent. on value of orders obtained. In the first four years he obtained orders to the value of £500, £740, £976, and £1,086, the goods being sold at $33\frac{1}{3}$ per cent. above cost price. To what salary was the traveller entitled each year? How much profit each year did the merchant make out of the traveller's efforts, if additional expenses attached to the sale of the above goods was $1\frac{1}{4}$ per cent. of selling price? Also if goodwill be reckoned at $3\frac{1}{2}$ years' purchase, by how much was its value increased by the traveller?

CHAPTER XII.

PARTNERSHIP.

106. **PARTNERSHIP**, as defined by the Partnership Act, 1890, is "the relation which subsists between persons carrying on business in common with a view to profit." The number of partners in a firm (the name given collectively to persons in partnership) may consist of from two to a maximum of twenty members, except in the case of a banking partnership, where the number must not exceed ten. Each of the partners must be competent to contract: suppose an infant entered into partnership, he could not by law be held responsible for the debts of the firm.

The contract of a partnership can be entered into orally; but, in general, an agreement in writing, called the **Articles of Partnership**, is made, and this, besides other clauses, usually contains conditions relating to: (1) Duration of partnership; (2) amounts of capital subscribed; (3) interest on capital; (4) salaries; (5) sharing of profits and losses; (6) powers of the partners to make contracts binding to the firm; (7) dissolution of partnership and retirement of one or more partners; (8) goodwill. Every partner, whether active or dormant, is responsible for the debts and liabilities of the firm. The act of any partner, such as the engaging of a servant; the buying and selling of goods; the giving of a receipt for payment, or any transaction, provided it comes within the ordinary course of business and within his sphere according to the articles of agreement, binds the other partners. Should debts be incurred or wrongful acts committed by a member of a firm in the ordinary course of business, the ensuing liabilities of the firm must be borne jointly by all the partners. A former partner of a firm who has had his retirement notified in the *London Gazette* is legally free from the liabilities of transactions of the firm made after his retirement.

In cases where no articles of agreement exist and the partners are not in mutual agreement, the *Partnership Act*, 1890, rules that—

1. Partners are entitled to share equally in the capital and profits, and must contribute equally to the losses, whether capital or otherwise.

2. Partners are entitled to receive interest at 5 per cent. per annum on any loans, apart from capital, made to the firm.

3. Partners are not entitled to be credited with interest on their capital prior to the ascertainment of profits.

4. Partners are not entitled to any salary for acting in the partnership business.

In settling accounts between partners after a dissolution of partnership, the above Act also provides that—

1. Losses, including losses and deficiencies of capital, are to be paid first, out of profits; next, out of capital; and, lastly, by the partners individually in the ratios in which they were entitled to share profits.

2. Assets are to be applied in the following order: (I) In paying debts and liabilities of the firm to persons other than partners; (II) in paying to each partner rateably what is due from the firm to him for losses as distinct from capital; (III) in paying to each partner rateably what is due from the firm to him in respect of capital; and (IV) the ultimate residue, if any, shall be divided among the partners in the ratio in which profits are divisible.

107. By the **Limited Partnership Act, 1907**, a **Limited Partnership** must consist of one or more **general partners**, who shall be liable for all debts and obligations of the firm; and one or more persons called **limited partners**, who have contributed capital, but who are not liable for the debts and obligations of the firm beyond the amounts contributed. The conditions as regards the maximum numbers of members are the same as in ordinary partnership. Under this form of partnership a limited partner can invest money in a firm and receive a return varying with the profits of the business, at the same time being free from loss to a greater extent than the amount of capital subscribed; whereas a general partner is liable, if necessary, to make good the losses of the firm from his own private wealth. Limited partners do not take an active part in the management of a firm, but can be looked to for advice when required. They cannot during the continuation of the partnership withdraw any part of their capital, but with the consent of the general partners they may transfer their shares.

The following examples illustrate the applications of the clauses of the Partnership Act, 1890, and the terms agreed upon in the various articles of partnership.

EXAMPLE (i)—

The firm of A, B, and C altogether has a capital of £25,000. The total assets realized £11,750. What should each receive or pay out on dissolving partnership if (1) no record or agreement as to the amount of capital each has subscribed exist; (2) the capitals of A, B, and C are £20,000, £4,000, and £1,000 respectively; but no agreement as to division of profits or losses exists; and (3) their capital is same as in (2); but by agreement they are to share $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{4}$ respectively of the profits and losses?

Total loss = £25,000 − £11,750 = £13,250.

(1) By the Act of 1890, the loss to be borne by each = £4,416 13s. 4d.
Also the share of capital of each = £8,333 6s. 8d.
∴ Each partner should receive £8,333 6s. 8d. − £4,416 13s. 4d.
= £3,916 13s. 4d.

(*i.e.*, same as $\frac{1}{3}$ of £11,750.)

(2) By the Act of 1890, the loss to be borne by each = £4,416 13s 4d.
∴ A should receive £20,000 − £4,416 13s. 4d. = £15,583 6s. 8d.
Also B „ pay out £4,416 13s. 4d. − £4,000 = £ 416 13s. 4d.
and C „ „ £4,416 13s. 4d. − £1,000 = £ 3,416 13s. 4d.

Thus A takes £11,750 and, in addition, B pays him £416 13s. 4d., and C pays him £3,416 13s. 4d.

(3) Loss to be borne by A = $\frac{1}{2}$ of £13,250 = £6,625.
„ „ B = $\frac{1}{4}$ of £13,250 = £3,312 10s.
„ „ C = $\frac{1}{4}$ of £13,250 = £3,312 10s.
∴ A should receive £20,000 − £6,625 £13,375;
Also B „ „ £4,000 − £3,312 10s. = £ 687 10s.;
and C „ pay out £3,312 10s. − £1,000 = £ 2,312 10s.

Thus C pays out £2,312 10s., which, added to the net assets, gives a total of £14,062 10s, of which A's share is £13,375 and B's share is £687 10s.

EXAMPLE (ii)—

Brown, Smith & Robinson are in partnership with capitals of £10,000, £8,000, and £4,500 respectively at the beginning of a year. It is agreed that Robinson should receive £500 at the end of the year for his services as manager and that the profits should be divided strictly in proportion to the capital subscribed. After 2 months, Brown withdrew £2,000 capital; after 5 months, Smith withdrew £1,000 capital; and after 6 months Robinson added £500 capital. What amounts are due to each at the end of the year, if the net profits amounted to £4,284?

Brown's contribution of capital is equivalent to
£20,000 + £80,000 for 1 month, *i.e.*, £100,000 **for a month.**

Smith's contribution of capital is equivalent to
£40,000 + £49,000 for 1 month, *i.e.*, £89,000 **for 1 month.**

Robinson's contribution of capital is equivalent to
£27,000 + £30,000 for 1 month, *i.e.*, £57,000 **for 1 month.**

Now, as £4,284 is net profit, Robinson's salary has already been deducted being included in the salaries account.

∴ Brown should receive £$\frac{100}{246}$ × £4,284 = £1,741 9s. 3d.
Also Smith „ „ £$\frac{89}{246}$ × £4,284 = £1,549 18s. 1d.
and Robinson „ „ £$\frac{57}{246}$ × £4,284 + £500 = £1,492 12s. 8d.

Example (iii)—

X and Y have decided that their capital in the firm should bear interest at 5 per cent. per annum, that withdrawals during the year should count as withdrawals of capital, and that profits should be equally divided. At the beginning of the year 1931, X's capital was £15,000, but he withdrew £400 on 23rd May and £400 on 15th August. Y's capital was £12,000, but he withdrew £250 on 7th April and £300 on 18th September. If the net profit at end of the year (after paying interest on capital) was £1,824, and X and Y kept two-thirds of their shares of profits and the whole of their interest as capital in the business, find (1) their total incomes for the year's working and (2) their capitals at the beginning of the next year.

Interest on £400 from 23rd May to 31st Dec. $= £\dfrac{400 \times 5 \times 222}{36500}$

" £400 " 15th Aug. " $= £\dfrac{400 \times 5 \times 138}{36500}$

∴ X's income from interest on capital $= £750 - £\dfrac{400 \times 50 \times 36}{36500}$

= £750 - £19 14s. 6d.

= £730 5s. 6d.

∴ X's total income = £730 5s. 6d. + £912 = **£1,642 5s 6d.**

Interest on £250 from 18th April to 31st Dec $= £\dfrac{250 \times 5 \times 257}{36500}$

" £300 " 18th Sept. " $= £\dfrac{300 \times 5 \times 104}{36500}$

∴ Y's income from interest on capital $= £600 - £\dfrac{5 \times 50 \times 1909}{36500}$

= £600 - £13 1s. 6d.

= £586 18s. 6d.

∴ Y's total income = £586 18s. 6d. + £912 = **£1,498 18s. 6d.**

X's capital at beginning of next year = £15,000 - (£800 + £19 14s. 6d.) + £750 + £608

= **£15,538 5s. 6d.**

Y's " " " = £12,000 - (£550 + £13 1s. 6d.) + £600 + £608

= **£12,644 18s. 6d.**

Note 1.—£400 on 23rd May + £400 on 15th August is equivalent to £819 14s. 6d. on 31st December. The withdrawal of these amounts has the same effect, as regards interest, as keeping the £15,000 intact until 31st December and then withdrawing £819 14s. 6d.

Note 2.—In a large number of firms, each partner has a *drawings account* on which interest on capital and share of profits are entered on *Cr.* side and withdrawals on *Dr.* side. The capital of each partner is kept at its original figure, so

that any balance on the drawings account is not transferred to the capital account. Interest would be allowed on a surplus in this account or charged against a deficit at a rate agreed upon in the articles of agreement. If X and Y had kept their capitals fixed and had had drawings accounts, X's account would be as follows—

Dr. X's DRAWINGS ACCOUNT. *Cr.*

1931.		£	s.	d.	1931.		£	s.	d.
May 23	To Cash	400	–	–	Dec. 31	By Interest on Capital	750	–	–
Aug. 15	,, ,,	400	–	–	,, 31	,, Share of Profit	912	–	–
Dec. 31	,, Interest	19	14	6					
,, 31	,, Cash	304	–	–					
,, 31	,, Balance c/d	538	5	6					
		£1,662	–	–		£	1,662	–	–
					1932. Jan. 1	To Balance b/d	538	5	6

In the event of winding up a firm owing to loss, the credit balance of a partner's drawings account would usually be considered as a loan made by the partner to the firm and, as such, it should be repaid before any return of the fixed capital is made.

EXAMPLE (iv)

Swain & Todd have fixed capitals of £4,000 and £2,000 in their firm, and by agreement they are entitled to (1) interest at 4 per cent. per annum on capital payable at the end of the year; (2) interest at 5 per cent. per annum on sums standing to their credit in their respective drawings account; (3) salaries of £300 and £200 per annum respectively paid by equal instalments at the end of each month out of the working expenses of the firm; (4) equal share of net profits. On 1st January, 1930, the sums standing to their credit in the drawings account were £342 10s. and £415 12s. respectively. The net profits for 1930 were £562. What were the respective sums standing to their credit in their drawings accounts on 1st January, 1931, given that they withdrew £25 and £23 6s. 8d. each month respectively in addition to salary ?

Interest charged on Swain's excess of monthly drawings $= £\frac{25 \times 5}{100 \times 12}\left[11 + 10 + \ldots + 1\right]$

$= £\frac{25 \times 5}{100 \times 12} \times 6 \times 11$

$= £6$ 17s. 6d.

∴ Swain's balance on drawings account on 1st January, 1931 $= £342$ 10s. + £160 + (£17 2s. 6d. – £6 17s. 6d.) + £281

= £793 15s. 0d.

Interest charged on Todd's excess of monthly drawings $= £\frac{23\frac{1}{3} \times 5}{100 \times 12}\left[11 + 10 + \ldots + 1\right]$

$= £\frac{70 \times 5}{3 \times 100 \times 12} \times 6 \times 11$

$= £6$ 8s. 4d.

$\therefore$ Todd's balance on drawings account on 1st January, 1931 $=$ £415 12s. + £80 + (£20 15s. 7d. − £6 8s. 4d.) + £281

$= \underline{\text{£790 19s. 3d.}}$

NOTE 3.—Swain drew £25 per month in addition to his salary, which does not appear in the drawings account. Interest on £25 for 11, 10 . . . months is $£\frac{25 \times 4 \times 11}{100 \times 12}$, $£\frac{25 \times 4 \times 10}{100 \times 12}$ . . . respectively. The total interest is $£\frac{25 \times 5}{100 \times 12}$ [11 + 10 + . . . + 1]. To rapidly evaluate the sum of a series of numbers which have a common difference, the average of the numbers, which is obtained by halving the sum of the first and last, should be multiplied by the number of numbers in the series. A series of numbers such that the difference between any two consecutive numbers is constant is known as an arithmetic progression.

EXAMPLE (v)—

It is agreed by the firm of White, James & Ford that capital subscribed shall bear interest at 4 per cent. per annum, and that profits and losses shall be shared in proportion to capital at the end of the year. At the beginning of the year their capitals were £7,500, £4,500, and £2,000 respectively. After 3 months, White withdrew £1,500 and James made a *loan* of £1,000 to the firm. After 6 months, Ford increased his capital by £750. After 9 months, White was allowed by his partners to increase his capital to the original figure. If the net profit was £2,840 7s. 6d., find the total incomes at the end of the year. Find also what their incomes would be if, before paying interest on capital, their profit had been only £287 10s.

$$\text{White's interest on capital} = £300 - £\frac{1500 \times 4 \times 1}{2 \times 100}$$

$$= £300 - £30 = £270$$

$$\text{James's} \quad ,, \quad ,, \quad = £180 + £\frac{1000 \times 5 \times 3}{4 \times 100} \quad (\textit{see}\ \text{Note 4})$$

$$= £180 + £37\ 10\text{s.} = £217\ 10\text{s.}$$

$$\text{Ford's} \quad ,, \quad ,, \quad = £80 + £\frac{750 \times 4 \times 1}{2 \times 100}$$

$$= £80 + £15 = £95$$

Their capitals at the end of the year are £7,500, £4,500, and £2,750 respectively. (*See* Note 4.)

$\therefore$ White's share of profit $= \frac{750}{1475} \times$ £2,840 7s. 6d. $=$ £1,444 5s. 2d.

Also James's ,, ,, $= \frac{450}{1475} \times$ £2,840 7s. 6d.

$= \frac{3}{5} \times$ £1,444 5s. 2d. $=$ £ 866 11s. 1d.

and Ford's ,, ,, $=$ £2,840 7s. 6d. − £2,310 16s. 3d.

$=$ £ 529 11s. 3d.

$\therefore$ White's total income $=$ £270 + £1,444 5s. 2d. $= \underline{\text{£1,714 5s. 2d.}}$

also James's total income = £217 10s. + £866 11s. 1d.
= **£1,084 1s. 1d.**

and Ford's „ „ = £95 + £529 11s. 3d. = £ 624 11s. 3d.

Total interest on capital = £582 10s.

∴ Net loss = £582 10s. – £287 10s.

= £295

White's share of loss = $\frac{450}{1475}$ × £295 = £150

James's „ = $\frac{3}{5}$ × £150 = £ 90

Ford's „ = £295 – £240 = £ 55

∴ White's net income = £270 – £150 = £120

Also James's „ „ = £217 10s. – £90 = £127 10s.

and Ford's „ „ = £95 – £55 = £40

NOTE 4.—According to the Act of 1890, the interest on a *loan* must be at the rate of 5 per cent. per annum, unless another rate has been previously agreed upon The loan could have been withdrawn at any time, and does not count as being *capital.*

EXAMPLE (vi)—

Lawson & Gedd dissolved partnership on 31st December, 1930. The accounts showed the assets at £15,000 and the liabilities other than capital at £2,000. Lawson's capital was £8,000 and Gedd s £5,000: Lawson was entitled to three-fifths of the profits and Gedd to two-fifths. Gedd, who undertook to realize the assets, was entitled to 5 per cent. on the gross proceeds realized to completely cover expenses and remuneration. Gedd's gross receipts were: January, 1931, £1,500; February, £1,200; March, £3,600; April, £7,750; and in May the remaining assets were completely realized, the amount being £2,400. If distribution of the realization of the assets was made at the end of each month, find out how much Lawson & Gedd were entitled to receive at the end of each month ?

January's realization of assets, viz., £1,500, should be completely used in paying outside creditors at rate of 15s. in the £.

February's amount, viz., £1,200, should be used to pay outside creditors the final 5s. in the £. Gedd should receive 5 per cent. of £2,700, *i.e.*, £135; and the remaining sum (*i.e.*, £1,200 – £500 – £135, *i.e.*, £565) should be divided among Lawson and Gedd, Lawson's share being £$\frac{8}{13}$ × 565 and Gedd's share £$\frac{5}{13}$ × 565.

Of March's amount, viz., £3,600, Gedd should receive 5 per cent. of £3,600 (*i.e.*, £180), and the remainder, £3,420, should be divided such that Lawson's and Gedd's shares should be £$\frac{8}{13}$ × 3420 and £$\frac{5}{13}$ × 3420 respectively.

Of April's amount, viz., £7,750, Gedd should receive 5 per cent. of £7,750 (*i.e.*, £387 10s.), and the remaining £7,362 10s. should be divided between Lawson and Gedd, their shares being £$\frac{8}{13}$ × 7362½ and £$\frac{5}{13}$ × 7362½ respectively.

The amount of capital not yet paid is £13,000 – £565 – £3,420 – £7,362 10s. (*i.e.*, £1,652 10s.) Of this, Lawson is entitled to £$\frac{8}{13}$ × 1652½ and Gedd to £$\frac{5}{13}$ × 1652½. The remaining part of May's realization, after Gedd has received 5 per cent. of £2,400 (*i.e.*, £120) is £627 10s.; and as this is profit, Lawson and Gedd should receive £$\frac{3}{5}$ × 627½ and £$\frac{2}{5}$ × 627½ respectively.

Thus in—	*Lawson is entitled to*	*Gedd is entitled to*
January .	*nil*	*nil**
February .	$£\frac{8}{13} \times 565$ $= £347$ 13s. 10d.	$£\frac{5}{13} \times 565 + £135$ $= £352$ 6s. 2d.
March . .	$£\frac{8}{13} \times 3420$ $= £2{,}104$ 12s. 4d.	$£\frac{5}{13} \times 3420 + £180$ $= £1{,}495$ 7s. 8d.
April . .	$£\frac{8}{13} \times 7362\frac{1}{2}$ $= £4{,}530$ 15s 5d.	$£\frac{5}{13} \times 7362\frac{1}{2} + £387$ 10s. $= £3{,}219$ 4s 7d.
May . .	$£\frac{8}{13} \times 1652\frac{1}{2} + £\frac{3}{5} \times 627\frac{1}{2}$ $= £1{,}393$ 8s 5d.	$£\frac{5}{13} \times 1652\frac{1}{2} + £\frac{2}{5} \times 627\frac{1}{2} + £120$ $= £1{,}006$ 11s. 7d.

* As the assets are sufficient for the repayment to the creditors to be made in full, it is immaterial whether Gedd deducts 5 per cent. from January's realization or waits until the creditors have been paid in full. In the case where there is a deficiency, the expenses would be deducted from each month's realization, but Gedd would not be entitled to remuneration. Lawson, however, would probably allow a consideration to Gedd for his services when the time came for the partners to share the loss.

NOTE 5.—This example illustrates the ruling that, on assets being realized, payments are made in the following order: (1) Amounts due to outside creditors; (2) amounts due to partners other than capital; (3) capital of partners paid rateably; (4) profits (if any) divided according to agreement. If, say, the final realization in May had been only £1,400, the net loss would be £322½: the partners would share this loss either equally or in a ratio agreed upon, so that the partners would each receive the remainder of their capitals less their respective contributions to the loss.

NOTE 6.—If the net division of realization of assets had been required, the following only would have been necessary—

Gross realization of assets $= £16{,}450$.

Total liabilities, including capital and commission to Gedd $= £2{,}000 + £13{,}000 + £822$ 10s.
$= £15{,}822$ 10s.

Net profit $= £627$ 10s.

∴ Amount due to Lawson $= £8{,}000 + £\frac{3}{5} \times 627\frac{1}{2}$
$= £8{,}376$ 10s.

and ,, ,, Gedd $= £5{,}000 + £\frac{2}{5} \times 627\frac{1}{2} + £822\frac{1}{2}$.
$= £6{,}073$ 10s.

108. **GOODWILL** is the name given to the asset of a business undertaking due to the probability that customers will continue to carry on business with the organization in question. As, in general, the more customers there are, the more likelihood of greater profits there will be, it follows that goodwill has a monetary value which, however, is difficult to estimate.

In the case of a new partner entering a firm, as he will share in the goodwill of the business, he will, besides subscribing capital, make a payment to the old partners for allowing him to share with them the goodwill of the concern. This payment may be

made directly in two ways: first, by the old partners accepting cash, which they use for purposes independent of the firm; and, secondly, by the money being divided among the old partners in any agreed ratios and the respective amounts added to their capitals. As an example of this latter method, suppose X paid £5,000 capital and £1,000 for goodwill to the firm of A and B, and that A and B divide the latter between them in the ratio 3 : 2 and add the amounts to their respective capitals. On the liabilities side of the balance sheet the total increase of capital is: A, £600; B, £400; and X, £5,000; while on the assets side the cash item is increased by £6,000, the amount of the goodwill remaining the same as before. Another and indirect way of paying for goodwill is sometimes made: referring to the above example, suppose X paid £5,000 capital, but instead of paying cash to A and B it was agreed that their capitals should be increased by £600 and £400 respectively, then on the liabilities side of the balance sheet the increase of capital would be £6,000; while on the assets side the cash item would be increased by £5,000 and the goodwill item by £1,000. By this last method, on dissolving partnership the realized assets would be no greater than they would have been had A's and B's capital account not been nominally increased; but, having been increased, A and B would take a larger share of the realized assets, and thus X would be left with a smaller share. In this way (apart from the increased interest on the capitals of A and B causing the net profits of the firm to decrease), X indirectly makes payment for his share of the goodwill of the firm.

EXAMPLE (vii)—

A and B have capitals of £10,000 and £8,000 in a firm, and their custom is to allow interest of 5 per cent. on capital and to share profits equally. At the end of 1929 the firm's net profit was 15 per cent. of the total capital. At the beginning of 1930, C entered the firm, paying £5,000 capital and £1,170 for goodwill, the latter sum being added on to the capitals of A and B in proportion to the amounts of their original capital. A, B, and C agreed that C should have $\frac{1}{5}$ of the net profit, and that the remainder should be equally divided between A and B. At the end of 1930 the net profit was again 15 per cent. of the total capital.

Find (1) by how much A's and B's incomes for 1930 were greater or less than those of 1929 and (2) the amount of C's income in 1930.

$$\text{(1) A's income in 1929} = £500 + £\frac{15 \times 18000}{100 \times 2} = £1,850$$

$$\text{B's} \quad ,, \quad 1929 = £400 + £1,350 \qquad = £1,750$$

A's, B's, and C's capitals during 1930 were £10,650, £8,520, and £5,000 respectively.

$\therefore$ A's income in 1930 $= £532\ 10s. + £\frac{2}{5} \times \frac{15 \times 24170}{100}$

$= £1,982\ 14s.$

and B's income in 1930 $= £426 + £1,450\ 4s. = £1,876\ 4s.$

$\therefore$ A's and B's incomes increased by $\underline{\underline{£132\ 14s.}}$ and $\underline{\underline{£126\ 4s.}}$ respectively.

(2) C's income in 1930 $= £250 + £725\ 2s. = \underline{\underline{£975\ 2s.}}$

Example (viii)—

In 1931, Neale & Hope with capitals of £2,400 and £1,600 respectively made a total profit of £662 10s., no interest being allowed on capital. At the beginning of 1932, Dixon became a partner by paying £2,000 capital, and it was agreed that the capitals of Neale & Hope should jointly be increased by £600 and individually in proportion to the amounts, and the assets of the business by goodwill be increased by £600. If the profit in 1932 was £1,015, find the increase of Neale & Hope's incomes, and find what percentage profit Dixon made, assuming that profits were divided in proportion to capitals.

Also find what the profits of each partner would have been in 1932 if the goodwill of the business had not been considered, resulting in the capitals of Neale & Hope remaining the same as in 1931.

Neale's income in 1931 $= £\frac{24}{40} \times 662\frac{1}{2} = £397\ 10s.$

Hope's " " $= £265.$

Neale's " 1932 $= £\frac{276}{660} \times 1015 = £424\ 9s.\ 1d.$

Hope's " " $= £\frac{184}{660} \times 1015 = £282\ 19s.\ 5d.$

Dixon's " " $= £307\ 11s.\ 6d.$

$\therefore$ Neale's and Hope's incomes increased by $\underline{\underline{£26\ 19s.\ 1d.}}$ and $\underline{\underline{£17\ 19s.\ 5d.}}$ respectively.

Dixon's percentage profit $= \frac{307\frac{23}{40} \times 100}{2000} = \underline{\underline{15 \cdot 4\%}}.$

Neglecting goodwill, Neale's profit would have been $£\frac{24}{60} \times 1015$

$= \underline{\underline{£406}}$

" " Hope's " " $£\frac{16}{60} \times 1015$

$= \underline{\underline{£270\ 13s.\ 4d.}}$

" " Dixon's " " $£\frac{20}{60} \times 1015$

$= \underline{\underline{£338\ 6s.\ 8d.}}$

109. DISSOLUTION OF PARTNERSHIP.

On dissolving partnership, if a loss is incurred, it has been seen that the partners individually must bear this loss either equally or in accordance with a mutual agreement. It is best to regard these contributions as being paid into the firm's account, and when this has been done there will be sufficient to repay the capital in full to each partner. This, of course, amounts to the

same thing as deducting the losses to be borne by each partner from their respective capitals and paying out the remainders.

Now suppose it happens that a partner's contribution to a loss is greater than his capital in the firm, then it follows that out of his own private means he must pay out the difference. If, however, owing to personal insolvency he is unable to make this payment, an additional loss will have to be borne by the remaining partners. This loss must not be regarded as an additional loss to the firm, but as a personal loss to the remaining partners, and, as such, must be borne by them in proportion to their capitals. The following example will make this clear—

Example (ix)—

At the time of dissolving partnership, the partners A, B, and C have £5,000, £3,000, and £1,000 capital respectively in the firm, and by agreement they are to bear losses in the ratios 3 : 3 : 2 The net loss is £5,600, but C is entirely unable to pay his contribution. How much do A and B receive? Suppose C, instead of having a capital of £1,000, had overdrawn his capital to the extent of £400, and was able to pay neither this nor his contribution to the loss: how much should A and B receive, £5,600 being the loss excluding capital irrecoverable from C?

If contributions to cover losses are paid to the firm, the assets would then amount to £9,000. As C does not contribute, the capital to be shared is £9,000 − £1,400 (*i.e.*, £7,600), which will be divided among A and B only as C's share in the loss is greater than his capital in the firm.

$\therefore$ A and B have £$\frac{5}{8}$ × 7600 and £$\frac{3}{8}$ × 7600 respectively.

Thus A receives £$\frac{5}{8}$ × 7600 − £2100 = £2,650

and B „ £$\frac{3}{8}$ × 7600 − £2100 = £750

In the second case, C should pay into the firm £1,800, which, together with the contributions of £2,100 each from A and B plus the assets available for repayment of capital, would make a total of £8,000, which, in their case, is the total capital of the firm considering C's capital as zero.

As C does not contribute, the capital to be shared by A and B is £8,000 − £1,800 (*i.e.*, £6,200).

$\therefore$ A and B have £$\frac{5}{8}$ × 6200 and £$\frac{3}{8}$ × 6200 respectively.

Thus A receives £$\frac{5}{8}$ × 6200 − £2100 = £1,775

and B „ £$\frac{3}{8}$ × 6200 − £2100 = £225.

Note 7.—The above method is in accordance with that in Book-keeping. A much shorter method is as follows—

1st case. Net loss to firm by C's insolvency = £400

$\therefore$ Total loss borne by A = £2,100 + £$\frac{5}{8}$ × 400
= £2,350

$\therefore$ A should receive £5,000 − £2,350 = £2,650

2nd case. Net loss to firm by C's insolvency = £1,800

$\therefore$ Total loss borne by A = £2,100 + £$\frac{5}{8}$ × 1,800
= £3,225

$\therefore$ A should receive £5,000 − £3,225 = £1,775

NOTE 8.—In the 1st case, the realization of assets for repayment of capital is £9,000 − £5,600 = £3,400 = £2,650 + £750. In the 2nd case, the total loss is £6,000, as the £400 owing to the firm by C realized nothing: the sum available for repayment of capital is £8,000 − £6,000 = £2,000 = £1,775 + £225. Thus in both cases the answers are verified.

110. BANKRUPTCY.

In the case when all the partners constituting a firm become bankrupt, the creditors of the firm, except those who hold security of certain assets, will receive rateably the realization of the assets after expenses and preferential claims have been paid. If there be a surplus of the separate estate of any one partner beyond that which will satisfy the separate creditors, it will be applied towards the deficiency remaining to the joint creditors of the firm. In other words, he must meet the losses of the firm as far as the realization of his separate estate will allow.

If a person be an ordinary or general partner in two or more firms, all of which become bankrupt, the surplus of his separate estate would be distributed among the firms rateably in proportion to his liabilities.

EXAMPLE (x)—

X is a partner in four firms as follows, and they are dissolved as X becomes a bankrupt.

Firm.	Kind of Partner.	Total Capital	X's Capital.	Deficiency or Loss.	X's Share of Losses.
A & Co.	Ordinary	£10,000	£4,500	Deficiency: £ 2,000	one-half
B & Co.	,,	£ 5,000	£3,000	,, £ 1,400	one-half
C & Co.	,,	£12,000	£5,000	,, £ 1,800	one-third
D & Co.	Limited	£15,000	£2,000	Loss: £12,000	one-quarter

If the surplus of X's private estate be £790, what amounts are due to or from the above firms?

Amount owing to A & Co. = $\pounds\frac{1}{2} \times 12{,}000 - \pounds 4{,}500 = \pounds 1{,}500$

,, ,, B & Co. = $\pounds\frac{1}{2} \times 6{,}400 - \pounds 3{,}000 = \pounds 200$

,, due from C & Co. = $\pounds 5{,}000 - \pounds\frac{1}{3} \times 13{,}800 = \pounds 400$

As $\frac{1}{4}$ of £12,000 exceeds X's capital in D & Co., X is not entitled to any amount from this firm; but, on the other hand, being a limited partner he is not liable for losses beyond the loss of his capital.

Thus, £1,190 is available for payment to A & Co. and B & Co.

∴ Amount to be paid to A & Co. = $\pounds\frac{15}{17} \times 1190 = \underline{\pounds 1{,}050.}$

and ,, ,, B & Co. = $\pounds\frac{2}{17} \times 1190 = \underline{\pounds 140.}$

EXAMPLE (xi)—

On 1st January, 1929, three partners bought a business for £250,000. On 31st December, 1931, they were compelled to stop payment. Their assets realized: Property and Plant, £180,000; Sundry Investments, £23,500; Cash and Bills Receivable, £2,950; Stock, £128,000; Debtors, £4,830. The liabilities were: Salaries unpaid, £5,220; Bills Discounted, £11,200; Sundry Creditors, including Bank, £354,000. The expenses of winding up the business amounted to £6,125; and securities which realized £57,000 and £24,500 were held by fully-secured creditors, to whom the firm owed £50,000 and £22,250 respectively. What was the deficiency and how much in the £ could be given to the remaining creditors, if £4,450, £5,130, and £2,734 were obtained after realizing the assets and paying the liabilities of the separate estates of the partners respectively?

Total realization of assets = £339,280
,, liabilities = £370,420
∴ Deficiency = £ 31,140

Expenses + Preferential Claims
+ full payments to fully-secured creditors = £ 83,595
Amount of remaining liabilities = £370,420 − £77,470 = £292,950
,, for final distribution = £339,280 − £83,595 + £12,314
= £267,999

$$\therefore \text{ Amount in £ for remaining creditors} = \frac{267{,}999}{292{,}950} \times 20 \text{ shillings}$$

= 18s. 3½d. to nearest ½d.

NOTE 9.—A creditor who holds mortgage, charge, or lien upon the property of a debtor is said to be a secured creditor. If he hold security sufficient to cover the whole of the debt, he is said to be fully secured; but should the security realize more than the amount of the debt, the debtor is entitled to the balance: if, however, it realize less than the amount of the debt, the creditor, who is thus only partly secured, is entitled to receive repayment on the adverse balance at the same rate as that received by the unsecured creditors. A banker who holds securities in his possession (not merely deposited for safety) is entitled to use them as a security for the unpaid balance of the debtor's account; while a solicitor is also a secured creditor in respect of title deeds or other securities in his possession.

NOTE 10.—The expense item, £6,125, is not considered as a liability, but is deducted from the amount available for distribution. The securities held by the secured creditors are included in the total realization of assets; while the sums owing to these creditors are included in the item, " full payments to fully-secured creditors."

TEST EXERCISES II, 2.

(1) In the firm of A & Co., A, B, and C having capitals of £7,500, £5,500, and £3,000 respectively. They agree to divide net profits, so that A, B, and C shall have ½, ⅓, and ⅙ respectively. If interest at 4 per cent. per annum be paid on capital, and B and C receive £350 and £250 per annum for their services to the firm, what would each receive by the end of the year's working, if the net profit be £2,240?

(2) In Question (1), what would each receive if £2,240 represented the profit before (i) interest on capital was paid; (ii) B's and C's salaries were paid; and (iii) interest on capital and salaries of B and C were paid?

(3) Taylor, Fisher, and Brown had agreed to divide the profits of the firm in proportion to their capitals, which were £2,500, £2,200, and £1,300 respectively at the beginning of the year. After 3 months, Taylor added

£500, and Fisher withdrew £500; while after 7 months, Brown added £300 capital. If the profit was £1,176 14s. 2d., what amount should each receive?

(4) A, B, C, and D having capitals of £12,000, £5,000, £2,000, and £1,000 respectively in their firm, agree to pay interest at 5 per cent. on their capital, and to share profits and losses in the ratios 4 : 3 : 2 : 1. At the end of the year it is found the firm has incurred a net loss of £550. If the partners desire to start the next year without an adverse balance, find what sums should be paid by or to the firm in the case of each partner?

(5) X, Y, and Z had capitals of £10,500, £4,500, and £3,000 in their firm. On dissolving partnership, their liabilities, including repayment of capital, were £20,500, and their assets realized £14,750. What should each receive as repayment of capital (i) if no agreement existed as to division of losses? (ii) if they had agreed to bear $\frac{1}{2}$, $\frac{3}{10}$, and $\frac{1}{5}$ of the losses respectively? and (iii) if they had agreed to share losses in proportion to their capital?

(6) Parke & Bowyer decided to keep their capitals, namely, £25,000 and £20,000, at their original figures, and to pay interest at 5 per cent. per annum on capital and also on balances in their drawings accounts. They also agreed to share equally the profits of the firm. On 1st January, 1932, they had favourable balances in their drawings accounts of £1,750 and £840 respectively. After three months, Parke withdrew £500 and Bowyer £400; after six months they withdrew £500 and £450 respectively; after nine months, £400 and £420 respectively; and at the end of the year, £600 and £480 respectively. The net profit (after deducting interest) of the firm for 1932 was £2,375. What were the amounts standing to their credit in their respective drawings accounts at the beginning of 1933? Make out their drawings accounts.

(7) Young, Short & Hemming were in partnership, their capitals at the beginning of 1931 being £6,240, £4,720, and £4,280 respectively. They agreed to allow interest at 5 per cent. on capital, and decided to divide net profits equally. At the end of each year the balances in their drawings account were transferred to their capital accounts. During the years 1931 and 1932 Young drew £40 per month, Short drew £60 every two months, and Hemming £80 every three months. The firm's net profits for 1931 and 1932 were £680 and £832 respectively. What were their respective capitals at the beginning of 1933?

(8) Referring to Question (7), what would have been the capitals of Young, Short & Hemming respectively at the beginning of 1933 had the firm incurred a net loss of £500 in 1931 and a net profit of £1,000 in 1932. Make out the respective drawings accounts in this case.

(9) Lampton & Cross, on forming partnership with £5,000 each, agreed to allow interest at 4 per cent. per annum as capital, and to divide profits in proportion to what their capitals would be at the end of the year. Lampton withdrew £500 capital on 10th May and £300 on 24th September. Cross withdrew £300 capital on 4th June and £250 on 6th October. The net profit for the year was £873 10s. 9d., and each took £100 of his share and allowed the remainder, together with the interest on capital, to be added to his capital. What were the respective capitals at the beginning of the next year?

(10) Pindar & Cook were in partnership, but no Partnership Deed had been entered into; and no agreement respecting interest on capital and loans, and as to how profits should be shared, had been made. Pindar contributed all the capital, £4,500, and after six months Cook advanced £2,000 to the firm as a loan. Prior to charging any interest which may be due to either partner, the profits of the firm for the complete year amounted to £960. To what share of this amount is Pindar entitled?

(11) Stanton, North, Cook & Wood are partners, and share profits and losses: Stanton, one-third; North, one-quarter; Cook, one-quarter; and Wood,

one-sixth. On 31st December, 1932, the firm was dissolved, the Balance Sheet on that date being as follows—

BALANCE SHEET.

	£		£
Stanton, Capital Account .	14,372	Property	21,200
North, " "	11,749	Stock	15,385
Cook, " "	7,137	Sundry Debtors . .	4,146
Sundry Creditors . .	8,273	Wood, Capital Account .	800
	£41,531		£41,531

The assets were realized as follows: Property, £22,500; Stock, £11,537; Sundry Debtors, £3,935 The expenses of the realization amounted to £428 Calculate the amount Wood must pay and the amounts due to each of the other partners.

(12) On 1st January, 1932, the capital of Messrs Girling & Co. was as follows: Robert Girling, £4,000; William Stone, £4,536; Henry Leach, £1,327; Thomas Low, £738. On 31st March, 30th June, 30th September, and 31st December, Girling and Stone withdrew £105 each; Leach, £90; Low, £70—these drawings being subject to interest at 5 per cent. per annum. Interest on capital was credited at 5 per cent. per annum, and net profits divided as follows: Girling, three-eighths; Stone, one-quarter; Leach, five-twenty-fourths; Low, one-sixth. The profit of the firm, prior to paying interest on capital, during the year amounted to £3,375 13s. 8d. What were the partners' respective capitals on 1st January, 1933? and what were their incomes for 1932 by the end of the year? (Having found interest on capital in each case from £3,375 13s. 8d., deduct the total interest on capital, and the net profit is obtained)

(13) Bilton, Scott & Turner traded in partnership, sharing profits and losses, $\frac{5}{12}$, $\frac{1}{3}$, and $\frac{1}{4}$ respectively. They decided to dissolve partnership on 31st December, 1932, when their Balance Sheet showed as follows—

	£		£
Bilton, Capital Account .	1,720	Sundry Debtors . .	1,750
Scott, " " .	1,140	Stock	1,410
Sundry Creditors . .	943	Cash	185
		Turner, Capital Account .	458
	£3,803		£3,803

Jason & Co. bought the Book Debts for £1,500 and the Stock for £996. What amounts should Bilton and Scott receive, and Turner pay?

(14) Suppose that Turner, instead of paying his deficiency of Capital and share of loss in full, was only able to pay £300 all told. What would Bilton and Scott then receive?

(15) Referring to Question (11), if Wood were unable to make any payment at all, what would Stanton, North, and Cook be entitled to receive respectively?

(16) After realizing assets and meeting all liabilities except repayment of capital, the firm of A & Co., consisting of A, capital £5,700; B, capital £4,200; C, capital £700, have £2,845. How should this be divided among A and B, the agreement being that losses should be borne in the ratios 2 : 2 : 1, if (i) C cannot pay any part of his share of the loss and (ii) C can only pay £500?

(17) At the time of dissolving partnership, the partners, Watson, Brand & Murray, had capital accounts of £825, £980, and £720 respectively, and Brand also had made a loan of £500 to the firm. An outside trustee was

employed to wind up the business, and he was allowed 4 per cent. of the gross realization of the assets for remuneration and to cover expenses. The liabilities amounted to £1,750 apart from capital and the loan from Brand and monthly repayments were to be made according to the extent of the realization of the assets. The gross realization was as follows: March, £972; April, £1,372; May, £1,954; June, £983; and July, £495. If profits and losses were shared equally, draw up a table showing how the above realizations were disposed of each month.

(18) Referring to the above question, if the liabilities to outside creditors amounted to £3,750, and the partners knowing there would be a loss decided to wait until the realization was completed, what amounts would Brand be entitled to receive each month and what should each partner receive when repayment of capital was made?

(19) A partner in a firm had a capital account of £1,120, on which interest at 4 per cent. per annum was paid; and he had also made a loan of £750, which bore interest at 6 per cent. per annum. He was entitled to one-third of the total net profits, which during 1927, 1928, and 1929 were £450, £745, and £996 respectively. What percentage each year is his total income from the firm of the total amount invested, assuming he withdraws interest and profits each year?

(20) A is a general partner, and B and C are limited partners in a firm whose liabilities (including capital) amounted to £11,468 14s. 6d., and whose assets realized £7,173 7s. 4d. The capitals of A, B, and C are £3,000, £2,000, and £500; and by agreement, B and C bear $\frac{1}{3}$ and $\frac{1}{8}$ respectively of the losses; but, being limited partners, their contributions to the loss must not exceed the amounts of their respective capitals. What are A and B entitled to receive? If the firm were an ordinary partnership, what would A and B receive?

(21) Rudd, Wilson & Henley were the partners in a firm, having capitals of £2,250, £1,750, and £1,200 respectively. It was agreed that the business should be wound up; and Wilson, who undertook to realize the assets, was allowed 5 per cent. of the gross receipts to cover expenses and remuneration. The liabilities of the firm, apart from capital, were £3,745, which included a loan of £500 from Rudd. Wilson and Henley were entitled to receive $\frac{1}{3}$ and $\frac{1}{4}$ of the net profits, and Rudd, the remainder. Wilson's receipts were. 1st quarter, £3,950 4s.; 2nd quarter, £4,735 11s 6d.; 3rd quarter, £3,136 3s. 4d.; and, finally, £2,076 during the 4th quarter. If repayments were made at the end of each quarter, what amount was each partner entitled to receive at the end of each quarter?

(22) X and Y were partners in a firm and had capitals of £7,500 and £6,000 respectively. Interest at 5 per cent. per annum was allowed on capital, and the partners shared net profits equally. In 1928 the net profit amounted to £1,368 16s. 10d. At the beginning of 1929, X withdrew £3,000 capital, and Z came into the partnership, subscribing £2,000 capital and paying for goodwill £1,750, which was divided between X and Y proportionally to their capitals, which were thereby increased. If net profits were equally divided and in 1929 they amounted to £1,428 13s. 8d., by how much was Y's income from the firm increased or reduced? What percentage return did Z receive on the total amount he paid into the firm?

(23) If instead of Z (Question (22)) actually paying £1,750 for goodwill, X's and Y's capitals had been increased proportionally, so that the combined capital of X and Y was increased by £1,750 and if the net profits had been £1,268 12s 6d., what would have been the total incomes of X and Y, and the percentage return on the amount paid into the firm by Z?

(24) Robert Davis, who owned a business, his capital being £3,500, decided to withdraw £1,500 and take Charles Young into partnership. The latter, in addition to paying a premium to Davis, subscribed £1,500 capital. The average profits for the previous five years amounted to £748 5s., and Davis expected this total profit to be approximately the same

in future years. The latter deposited his £1,500, together with the premium received from Young, in a concern paying 7½ per cent. per annum. What should the amount of the premium to the nearest £ be in order that Davis's total income should be the same as before, if Young was allowed one-third of the profits?

(25) Henry Baker is a partner in two firms having total capitals of £7,200 and £8,500, his capital being £3,500 and £3,000 respectively. He is liable for one-half the losses of the first firm and one-third those of the second. Baker becomes insolvent and the firms dissolve partnership, incurring losses of £12,375 4s. and £11,207 11s. 9d. respectively. How much of the surplus of Baker's private estate, £468 11s. 4d., is each firm entitled to receive? What additional loss does Baker's insolvency bring about to a partner in the first firm, having £2,500 capital and bearing one-third of the losses?

(26) William Bates, who became bankrupt, was an ordinary partner in three firms which were wound up. Particulars concerning the firms were as follows—

Firm.	Total Capital.	Bates's Cap.	Loss.	Bates's Share of Loss.
X & Co.	£ 7,750	£3,000	£ 5,375	one-half
Y & Co.	£11,400	£5,000	£18,632	one-half
Z & Co.	£10,800	£3,500	£19,043	one-third

The surplus of Bates's private estate, after deduction of expenses, was £294 12s. What amounts are the firms Y & Co. and Z & Co. entitled to receive?

(27) A bankrupt had an adverse balance of £202 11s. 10d. in a bank, which held securities which realized £175 14s. 3d. If the unsecured creditors of the bankrupt received 8s. 4$\frac{11}{32}$d. in the £, how much did the bank lose?

(28) N Williams was a partner in the following firms, which, owing to the bankruptcy of Williams, were wound up.

Firm.	Kind of Partner.	Capital.	Williams's Capital.	Loss.	Williams's Share of Losses
T & Co	Ordinary	£20,000	£7,000	£23,747 12s.	one-third
S & Co.	,,	£ 7,000	£4,000	£ 9,385 4s.	one-half
U & Co.	Limited	£11,000	£4,000	£ 8,710 2s.	one-third
V & Co.	,,	£ 7,500	£1,500	£11,327 13s.	one-fifth

The net surplus of his private estate was £507 3s. 10d. Assuming the remaining partners of U & Co. were solvent, what amounts were available for payments to the other firms?

(29) The firm of Carter & Salmon suspended payment on 31st December, 1931. Their liabilities and assets were given as follows—

Liabilities.		£	*Assets.*	£
Sundry Trade Creditors .		3,134	Land	1,750
,, Cash Creditors . .		945	Buildings	3,950
Loans		6,000	Machinery and Plant . .	2,750
Capital—			Stock	1,540
Carter . .	£1,200		Trade Fixtures . . .	58
Less net loss .	450		Cash in hand . . .	56
		750	Sundry Debtors . . .	1,200
Salmon . .	£700			
Less net loss .	225			
		475		
		£11,304		£11,304

The realization of the assets compared with the above were: Land, 5 per cent. above; Buildings, 25 per cent. below; Machinery and Plant, 60 per cent. below; Stock, 15 per cent. below; Trade Fixtures, £9 15s. less; Sundry Debtors, £702 10s. less. No loans were made by either partner to the firm. The loans were partly secured by assets, which realized £4,725. The expenses of the realization of the assets were £437, and certain creditors made preferential claims to the extent of £495. What was the loss?

How much in the £ should the remaining creditors receive if (i) neither partner can contribute to make up the loss, (ii) Carter's surplus or his private estate was £575 and Salmon's £634?

(30) A and B have fixed capitals of £5,000 and £3,000 respectively in their firm, which has no other partners They are entitled to monthly salaries of £30 and £25 respectively paid at the end of each month. Interest on capital is 4 per cent. per annum, and interest on credit balances in their drawings accounts, being regarded as loans to the firm, is 5 per cent. per annum, in each case payable half-yearly, namely, on 30th June and 31st December. They also share the net profits equally at the end of each half-year. The credit balances in their drawings account on 1st January, 1928, were £925 and £654 respectively, and the net profits during 1928 were £840 and £936 respectively for the six monthly periods. In addition to receiving salary, A and B withdrew £55 and £45 respectively at the end of each month. What were their credit balances on 1st July, 1928, and 1st January, 1929, respectively?

CHAPTER XIII.

LIMITED COMPANIES.

111. **A Joint Stock Company** is an association of individuals united for the purpose of engaging in business or in any undertaking. The great majority of commercial companies are incorporated by means of registration with the Registrar of Companies. An **Incorporated Company** is regarded legally as an "individual," and the members have no separate liability to the creditors of the company. The liability of a member or shareholder in a **Limited Company** is limited to the amount of the shares for which he has subscribed or accepted responsibility. As regards a **Public Limited Company**, the number of members must be not less than seven, shares are issued to the public and are freely transferable, audit of accounts is compulsory, and a balance sheet must be annually prepared and filed at the Companies Registration Office. By the Companies Act, 1929, a **Private Limited Company** may be registered by two or more persons, provided that its Articles of Association: (1) Restricts the right to transfer shares; (2) limits the number of its members (exclusive of persons who are in the employment of the company and ex-employees) to fifty; (3) prohibits any invitation to subscribe for any shares or debentures of the company. A private limited company is usually formed when sufficient capital is forthcoming from members of the same family or circle of acquaintanceship. It has an advantage over a limited partnership, for in the latter there is at least one member who is liable without limitation for the debts of the firm, whereas in a private limited company the liabilities of all members are limited. The accounts of a private limited company must be audited and an annual balance sheet prepared, but the latter need not be filed at the Companies Registration Office.

112. FORMATION OF A COMPANY.

In order to form a new company, the promoters lodge the **Memorandum of Association** and the **Articles of Association** (if any) **with the** Registrar of Companies, and, if in order and when the

legal fees and stamp duties are paid, the latter will issue a **Certificate of Incorporation.** The Memorandum of Association is the charter which declares the objects and defines the powers of the company; while; by the Articles of Association, the rules, regulations, and by-laws to govern the internal workings of the company are laid down.

The **Authorized Capital** (or nominal or registered capital) of a company is the maximum amount the directors have legal power to raise. **Issued Capital** is the amount offered and subscribed for; while **Unissued Capital** is that which the directors have power to issue at a later date. **Called-up Capital** is that part of the issued capital for which the directors have sent calls, and **Paid-up Capital** is that received by the company in answer to the calls. **Working Capital** is the amount of paid-up capital plus debentures available for carrying on business, after preliminary expenses and purchase price have been paid.

Stock represents fully-paid up shares which have been converted, and can be subdivided into fractional amounts; while **Shares,** which may or may not be fully-paid up, are numbered distinctively, are each of some definite amount, and cannot be split up.

113. SHARES are of various kinds—

Preference Shares give a preferential right either to a fixed rate of dividend or to a return of capital, or both. If the shares be **cumulative** and during a period the dividend stated on the prospectus cannot be paid in full, then in subsequent years profits must be used to pay the accumulated dividends before dividends on other shares are allowed. If **non-cumulative,** no unpaid arrears are carried forward.

Ordinary Shares entitle their owners to an unlimited or limited (according to the Articles of Association) proportional part of the profits after prior charges and dividends on preference shares have been paid.

Deferred Shares entitle their owners to dividend either after other shareholders have received a stated dividend or after the expiration of a certain period of time.

Founders' or Management Shares are usually held by the promoters, and enable them to receive a certain fraction of the net

profits after the other shareholders have received dividends up to a fixed rate.

Participating Preference Shares entitle their owners to receive additional dividends after the holders of ordinary shares have received a given percentage.

The holders of **Participating Ordinary Shares** may, if profits permit, receive additional dividends after the owners of deferred shares have received a certain percentage.

114. ISSUE OF SHARES AND STOCK.

Companies can obtain loans by the issue of **Debenture Stock**, interest on which is payable on certain dates (usually half-yearly), whether the company makes a profit or not. Owners of debentures are creditors, not members, of the company. A holder of **Mortgage Debenture** has security for the repayment of the loan. **Redeemable Debentures** provide for repayments at a given date or upon notice; while **Irredeemable Debentures** are issued only on condition that repayments will be made only on default of paying the interest stated or on the winding up of the company. Interest on debentures is usually paid half-yearly as long as there are assets out of which to pay it, whereas dividend on shares is payable only out of profits.

Invitation to the public to subscribe to a company is made by a prospectus, accompanying which is a form enabling those interested to make application for shares. The filled-up form, together with a cheque for the amount of the application money, is sent to the company's banker. Letters of allotment requesting the amount due on allotment are sent to those whose applications have been successful. Should an applicant be allowed only a part of the shares applied for, no money would be returned to him: it would be used by the company towards the amount to be subscribed as a result of future calls. Interest is allowed to shareholders who make their subscriptions before, and charged to those who subscribe after, the stated dates of the calls. A shareholder, in the event of non-payment within a reasonable time of receiving a call, would, after warning, have to forfeit his shares, which may be re-issued by the directors at a discount of not more than the amount paid up on them. A shareholder is free to transfer his shares, and this can be done by means of a share transfer deed.

The stamp duty is 1s. per £5 up to £25, 5s. for each additional £5 up to £300, 10s. for each additional £50 or part, on the price paid for shares, and the company charges a registration fee of 2s. 6d. on each transfer.

Shares and debentures may be issued at par, at a premium, or at a discount. The liability of the company is measured by the nominal amount in question. Debentures may be issued at par and redeemable at a premium, or issued at a discount and redeemable at par. The difference between the money received and that to be repaid is written off by equal annual charges against the profits of the business spread over the term of years for which the issue of debentures has to run.

115. **INCOME TAX**, in the case of dividends, is usually deducted at the source, so that, unless otherwise stated, the amount received by debenture or share owners is the declared percentage of the amounts paid up, less income tax on this at the rate for unearned incomes. Should the total income of a shareholder be such that the amount deducted for income tax exceeds that in accordance with the law, he may obtain the difference from the income tax authorities.

116. **THE MEMORANDUM OF ASSOCIATION** of a limited liability company must contain particulars of the company's name, situation of registered office, objects for which the company is being formed, a statement that the liability of members is limited, and the amount of the share capital. It must be subscribed by seven persons (or two in the case of a private company) and must bear a deed stamp of 10s. and a fee stamp of £2 if the nominal capital does not exceed £2,000. The additional fees are: For every £1,000 or part of £1,000 after the first £2,000 up to £5,000, £1; for every £1,000 or part of £1,000 after the first £5,000 up to £20,000, 5s.; and there is a graduated scale at about the same rate up to £600,000. The nominal capital may, of course, exceed this figure, but the total fees shall not exceed £50.

THE ARTICLES OF ASSOCIATION bear a deed stamp of 10s. and a fee stamp of 5s. The list of directors and the Statutory Declaration of compliance with the Companies Acts each bears a stamp of 5s. There is a duty payable upon a statement of the nominal share capital at the rate of £1 on every £100 or part of

£100 of such capital. There are other stamp duties and fees; for example, Debentures must be stamped at the rate of 2s. 6d. per cent. These fees, together with other necessary preliminary expenses, are written off as soon as possible from either the Forfeited Shares Account, or Premiums on Shares Account, or by an annual charge against the profits of the business, equally divided over a number of (not exceeding ten) years.

Example (i)—

X decided to form a business into a limited liability company, which, when formed and registered, was authorized to issue 15,000 5 per cent. preference shares of £10 each and 40,000 6½ per cent. ordinary shares at £5 each. £40,000 debenture stock was issued at £102. The shares and debenture stock were payable as follows—

		Pref. Shares	Ord. Shares	Deb. Stock.
		£ s.	£ s.	
Mar. 10.	On Application .	2 10	1 5	10%
„ 13.	On Allotment .	5 –	2 10	92%
May 2.	On First Call .	1 5	15	

The second call was to be made after an indefinite time at the discretion of the directors; the purchase price was £225,000. The shares and debenture stock were all allotted, except 10,000 ordinary shares; and all shareholders made payments in answer to the first call. The preliminary expenses, including Duty, Stamps, Printing and Advertising, Fees to Accountants, Underwriters, etc., and Commission to Brokers, amounted to £6,414. What was the nominal capital, the issued capital, the called-up capital, the paid-up capital, and the working capital respectively?

Nominal capital	= £150,000	+ £200,000	= £350,000
Issued „	= £350,000	– £50,000	= £300,000
Called-up „	= £15,000 × 8¾	+ £30,000 × 4½	= £266,250
Paid-up „			= £266,250
Working „	= £266,250 + £400 × 102 – £225,000 – £6,414		= £75,636

Example (ii)—

A company invited application for 4½% preference shares at £10 each and 7½% ordinary shares at £10. Subscriptions for both kinds of shares were to be made as follows: 7th July, on application, £3; 9th July, on allotment, £3; 31st August, on first call, £2; 31st October, on second call, £2. James Smith sent application for 4,000 ordinary shares, but was allotted only 500. William Brown applied for 2,000 preference shares and was

allotted 1,200; but did not make payment in response to the second call until 31st December, when the company paid half-yearly interim dividends on both kinds of shares at the stated rates. Interest at 5 per cent. per annum was paid or charged on subscriptions paid before or after the dates when due. What should Smith receive on 31st October and 31st December ? and what net amount shall Brown pay at the latter date, income tax being at the rate of 5s. in the £ ? (Reckon interest from 9th July.)

Amount paid by Smith on application = £12,000

„ immediately returned to Smith = £12,000 − £5,000

= £ 7,000

Interest on amount of first call from 9th July to 31st August $= £\dfrac{1000 \times 5 \times 53}{100 \times 365}$

„ „ second „ 9th July to 31st October $= £\dfrac{1000 \times 5 \times 114}{100 \times 365}$

∴ Amount paid to Smith on 31st Oct. $= £\dfrac{1000 \times 5 \times 167}{100 \times 365}$

= £22 17s. 6d.

Amount received by Smith on 31st Dec. $= \dfrac{5000 \times 7\frac{1}{2} \times \frac{1}{2}}{100} \times \frac{3}{4}$

= £140 12s 6d

Amount paid by Brown on application = £6,000

„ „ „ allotment = £1,200

„ „ „ 31st Aug. = £2,400

Interest charged on amount of second call from 31st Oct. to 31st Dec. $= £\dfrac{2400 \times 5 \times 2}{12 \times 100}$

= £20

Dividend due to Brown on 31st Dec., *less* income tax $= £\dfrac{12000 \times 4\frac{1}{2} \times \frac{1}{2}}{100} \times \frac{3}{4}$

= £202 10s.

∴ Amount Brown should pay on 31st Dec. = £2400 + £20 − £202 10s.

= £2,217 10s

NOTE 1.—Income tax is not deducted by the company in the case of interest on payment for shares made in advance.

EXAMPLE (iii)—

(*a*) A man bought 500 $4\frac{1}{2}$ per cent. preference shares at £102 per cent., payment being made as follows: 4th January, £22; 7th January, £55; 30th April, £15; 30th September, £10. Half-yearly dividends at the full rate were made on 30th June and 31st December. What amounts did he receive from the company on each date, income tax being deducted at 5s. 0d. in the £.?

(*b*) If dividend had been paid at $3\frac{1}{2}$ per cent. at the end of the complete year, what would have been the equivalent rate per cent. on his money for this year ?

(*a*) By 30th June, £92 had been paid on every £100 share; but as £2 is on account of the premium, dividend would be payable on £90.

$$\therefore \text{ Amount received on 30th June} = £\frac{90 \times 4\frac{1}{2} \times \frac{1}{2}}{100} \times \frac{3}{4} \times 500$$
$$= £759 \text{ 7s. 6d.}$$

$$\text{Amount received on 31st Dec.} = £\frac{100 \times 4\frac{1}{2} \times \frac{1}{2}}{100} \times \frac{3}{4} \times 500$$
$$= £843 \text{ 15s.}$$

(b) Dividend due on 31st Dec. at $3\frac{1}{2}\%$ $= £500 \times 3\frac{1}{2}$
$= £1,750$

Let $r \equiv$ equivalent rate per cent. per annum.

Interest on £11,000 for 361 days at $r\%$ per annum $= £110 \times \frac{361r}{365}$

" £27,500 " 358 " " $= £275 \times \frac{358r}{365}$

" £ 7,500 " 8 months " $= £\ 75 \times \frac{2r}{3}$

" £ 5,000 " 3 " " $= £\ 50 \times \frac{r}{4}$

$\therefore$ Total interest " " $= £\frac{27632r}{73} + £\frac{125r}{2}$

$$\therefore \frac{27632r}{73} + \frac{125r}{2} = 1,750$$
$$\therefore 55264r + 9125r = 255500$$
$$\therefore r = \frac{255500}{64389} = 3{\cdot}97.$$

Ansr.—(*a*) £835 6s. 3d.; £928 2s. 6d.: (*b*) 3·97% per annum.

Note 2.—Dividend on shares is based on the amount paid up on the same at the time the dividend is payable; in the case of issue at a premium, on the amount paid up less the amount of the premium. The interest on debenture stock issued at a discount is payable on the nominal value which in this case is the amount paid plus the amount of the discount.

Note 3.—Referring to (*b*), preference shareholders would be entitled to $4\frac{1}{2}$ per cent. dividend if profits permitted, although for the first year, taking into consideration the dates on which the instalments were paid, the rate would be approx. 5%.

Example (iv)—

A company's nominal capital is £50,000 made up of 2,000 5 per cent. preference shares at £10 each, 4,000 $7\frac{1}{2}$ per cent. ordinary shares at £5 each, and 100 deferred shares at £100 each. All shares have been fully paid up, and dividends are declared at the end of each working year.

(*a*) What rate of dividend could be paid on each kind of shares if the preference and ordinary shares are non-participating, and the net profits amount to (1) £750, (2) £2,150, (3) £4,420 ?

(*b*) If all shares equally participated after the deferred shareholders had received 10 per cent., what rates of dividend should the shareholders

of the different kinds of shares respectively be entitled to receive had the net profits amounted to £8,950 ?

Amount required to pay dividend on pref. shares in full = £1,000
" " " " ord. " " = £1,500
" " " " def. " at 10% = £1,000

(a) 1. Rate of dividend on pref. shares = $\frac{750}{20000} \times 100 = 3\frac{3}{4}\%$
There are no dividends on ord. and def. shares.

2. Rate of dividend on pref. shares = 5%
" " ord. " = $\frac{1150}{20000} \times 100 = 5\frac{3}{4}\%$
There is no dividend on def. shares.

3. Rate of dividend on pref. shares = 5%
" " ord. " = $7\frac{1}{2}\%$
" " def. " = $\frac{1925}{10000} \times 100 = 19\frac{1}{4}\%$

(b) Amount available for additional dividend = £5,450

∴ Rate of additional dividend = $\frac{5450}{50000} \times 100\% = 10\frac{9}{10}$

∴ Total rate of dividend on pref. shares = $5\% + 10\frac{9}{10}\% = 15\frac{9}{10}\%$
and " " ord. " = $7\frac{1}{2}\% + 10\frac{9}{10}\% = 18\frac{2}{5}\%$
also " " def. " = $10\% + 10\frac{9}{10}\% = 20\frac{9}{10}\%$

Ansr.—(a) 1. $3\frac{3}{4}\%$, 0%, 0%; 2. 5%, $5\frac{3}{4}\%$, 0%; 3. 5%, $7\frac{1}{2}\%$, $19\frac{1}{4}\%$:
(b) $15\frac{9}{10}\%$, $18\frac{2}{5}\%$, $20\frac{9}{10}\%$.

Example (v)*—

A man held 100 4 per cent. cumulative preference shares at £10 each. The dividends for the first 5 years were at the rate of 0 per cent., 2 per cent., 4 per cent., 6 per cent., and 8 per cent. respectively, paid in each case by two equal half-yearly instalments. Assuming that he invests all his dividends in a concern which also pays interest at 4 per cent. per annum payable half-yearly, what was the extent of his loss at the end of the five years, due to the fact that dividends at 4 per cent. were not paid every year ?

2nd year's dividends at the end of the 5 years amount to $£10 \times (1{\cdot}02)^7 + £10 \times (1{\cdot}02)^6$
$= £10 \times (1{\cdot}02)^6 \times 2{\cdot}02$

3rd year's dividends at the end of the 5 years amount to $= £20 \times (1{\cdot}02)^5 \times (1{\cdot}02)^4$
$= £20 \times (1{\cdot}02)^4 \times 2{\cdot}02$

4th year's dividends at the end of the 5 years amount to $£30 \times (1{\cdot}02)^2 \times 2{\cdot}02$

5th year's dividends at the end of the 5 years amount to $£40 \times 2{\cdot}02$

∴ Total amount $= £20{\cdot}2\ [4 + 3(1{\cdot}02)^2 + 2(1{\cdot}02)^4 + (1{\cdot}02)^6]$
$= £20{\cdot}2\ [4 + 3{\cdot}1212 + 2{\cdot}16486 + 1{\cdot}12616]$
$= £20{\cdot}2 \times 10{\cdot}41222 = £210{\cdot}3268$

Amount of dividends if paid at 4% per annum each year $= £1000 \times [(1{\cdot}02)^{10} - 1]$
$= £218{\cdot}995$

∴ Amount of the loss at the end of the 5 years = £ 8·6682
= £8 13s. 4d

Note 4.—As the interest on the instalments is at 4 per cent. per annum, payable half-yearly, the total amount of the dividends is the same as the interest on £1,000 for 5 years at 4 per cent. payable half-yearly. If income tax, say, at 5s. in the £ be deducted throughout, the amount of the loss would be ¾ of £8 13s. 4d.

EXAMPLE (vi)—

A manufacturing company whose capital consists of 2,750 shares of £10 each has also issued $5\frac{1}{2}$ per cent. debentures of £100 each, but four of them have to be paid off at the end of every year out of the profits of that year. At the beginning of 1932 there were 24 of these debentures. During the year the business done was £11,540; the expenses and cost of material together amounted to 76 per cent. of the business done. Find the greatest dividend per share that could be paid at the end of the year without drawing on reserve.

If a dividend of 7 per cent. had been declared, how much would be available for the reserve?

Amount available for paying off debentures, paying interest on debentures and dividends on shares

$$= £\tfrac{24}{100} \times 11540 = £2,769\ 12s.$$

$$\text{Interest on debentures} = £24 \times 5\tfrac{1}{2} = £132$$

$$\text{Amount to pay off 4 debentures} = £400$$

$$\therefore \text{Amount available for dividends} = £2,237\ 12s.$$

$$\therefore \text{Dividend per share} = \tfrac{44752}{2750}\text{ shillings}$$

$$= 16s.\ 3\tfrac{1}{4}d.$$

$$\text{Amount required to pay dividend of } 7\% = £275 \times 7$$

$$= £1,925$$

$$\therefore \text{Sum available for reserve} = £312\ 12s.$$

*EXAMPLE (vii)—

Certain debentures were issued at £95 in 1924; the interest was 4 per cent. per annum, payable half-yearly; and the debentures are redeemable at par in 1932. What rate of interest would a man who bought debentures in 1924 consider that he receives during the eight years?

£4 per £95 cash is the same as £$\frac{80}{19}$ per £100 cash.

Now, £95 at $\frac{80}{19}$% per annum payable half-yearly amounts in 8 years to $£95 \times \left(1 + \frac{40}{1900}\right)^{16} = £95 \times \left(\frac{194}{190}\right)^{16}$

$$= £132{\cdot}5846$$

In addition, at the end of 8 years the man receives an extra £5 on each £95 cash.

Thus £95 amounts to £137·5846 at the end of 8 years.

Let $£i \equiv$ interest on £1 per half-year at equivalent rate per cent.;

$$\text{then } 95 \times (1 + i)^{16} = 137{\cdot}5846$$

$$\therefore 1 + i = \sqrt[16]{\frac{137{\cdot}5846}{95}}$$

$$= 1{\cdot}023418$$

$$\therefore i = {\cdot}023418$$

$\therefore$ Equivalent rate per cent. per annum payable half-yearly $= 4{\cdot}68$

Number.	Log.
194	2·2878017
190	2·2787536
	·0090481
	·1447696
95	1·9777236
132·5846	2·1224932
137·5846	2·1385699
95	1·9777236
	·1608463
1·023418	·0100529

*117. SINKING FUND.

In the case where a company is bound to redeem debentures or other liabilities at some future date, it is usual for a Sinking

Fund to be created. This fund consists of instalments appropriated from the profits, and may or may not be invested in outside securities. It frequently happens that a company applies the instalments to the payment of the premium on a Sinking Fund Insurance Policy or, in the case of the redemption of debentures, the instalments are used to buy back some of the debentures which have to be redeemed. Whenever the liability to be redeemed is a definite one, it is customary to make the instalments each year or half-year as near as possible of the same amount: the taking out of a sinking fund insurance policy ensures this being carried out, but by the method of buying back a fixed number of debentures each year, the instalments would vary with the price of the debentures in the open market.

118. DEPRECIATION.

Wherever there be a wasting asset such as a lease or machinery, a charge called **Depreciation** is made against revenue, for the purpose of replacing this asset when necessary. Depreciation does not necessarily consist of a number of equal instalments spread over the period: the greater the revenue during any given year, the greater should be the charge set aside at the end of that year.

*EXAMPLE (viii)—

A company issued 100 £100 debentures at par in 1920, to be redeemed at £105 in 1935. If 15 instalments of equal amounts be set aside at the end of each year and invested at 4 per cent. per annum, payable yearly, what should be the value of each instalment?

Amount to be repaid after 15 years = £10,500.

Let £x ≡ the value of each instalment.

Amount of 1st instalment in 1935 $= £x \times (1{\cdot}04)^{14}$

" 2nd " " $= £x \times (1{\cdot}04)^{13}$

. . . .

" 15th " " $= £x$

$\therefore\ x\,[1 + (1{\cdot}04) + (1{\cdot}04)^2 + \ldots \text{ to 15 terms}] = 10{,}500$

Number.	Log.
1·04	·0170333
1·800941	·2554995
42000	4·6232493
80·0941	1·9036005
524·3832	2·7196488

$$\therefore\ x\,.\frac{(1{\cdot}04)^{15}-1}{{\cdot}04} = 10{,}500$$

$$\therefore\ x\,.\frac{{\cdot}800941}{{\cdot}04} = 10{,}500$$

$$\therefore\ x = \frac{42000}{80{\cdot}0941} = 524{\cdot}3832$$

$\therefore$ Value of each instalment = £524 7s. 8d.

EXAMPLE (ix)—

It was reckoned that machinery which cost £7,250 would have to be replaced after twenty years, by which time it would have brought in a total revenue of about £60,000. What percentage of revenue should be charged for depreciation? If $1\frac{1}{4}$ per cent. of the revenue be set aside for repairs, what would be the total charge on a year's revenue of £3,850? If the total revenue for the twenty years be £82,420, what sum would be available for the establishment of new machinery, assuming that the old machinery could be sold for £575?

Percentage charge on revenue for depreciation $= \frac{7250}{60000} \times 100 = \underline{12\frac{1}{12}}$

$$\text{Total charge on revenue of £3,850} = \frac{(12\frac{1}{12} + 1\frac{1}{4})}{100} \times 3850$$

$$= \frac{40 \times 3850}{3 \times 100}$$

$$= \underline{£513 \ 6s \ 8d.}$$

$$\text{Amount available for new machinery after 20 years} = £\frac{145 \times 82420}{1200} + £575$$

$$= \underline{£10,534 \ 1s \ 8d}$$

NOTE 5.—It is assumed that the instalments are a constant percentage of the yearly revenues, and that the funds are kept by the company, which allows no interest on the same. Should the instalments vary from year to year and be invested in outside securities, the amount of the fund at the end of a stated time could be obtained by finding the sum of the amounts of the instalments at the end of their respective times. There is no definite general method of charging for depreciation, except that the instalments should approximately vary with the revenue produced by the wasting asset, and should as far as possible, be just sufficient to renew the asset at the time when necessary.

*EXAMPLE (x)—

A company's sinking fund during the period 1929–1932 inclusive was increased so that the instalment at the end of a year was one-eighth of the amount of the fund at the beginning of the year. At the end of 1932 the fund amounted to £3,746 11s. 4d. What was it at the beginning of 1929, (1) assuming the fund is kept in the company and no interest is paid on it; and (2) assuming that the instalments are immediately deposited in a bank which pays $2\frac{1}{2}$ per cent. interest?

(1) **Let £x ≡ amount of fund at the beginning of 1929,**

$$\text{then } x \times (\tfrac{9}{8})^4 = 3746\tfrac{17}{30}$$

$$\therefore x = \frac{112397 \times 8^4}{30 \times 9^4} = 2338{\cdot}9631$$

∴ Amount of fund at beginning of 1929 = $\underline{£2,338 \ 19s. \ 3d.}$

(2) Let £x ≡ amount of fund at the beginning of 1929,
then amount of fund at beginning of 1930 $= £1{\cdot}025x + £0{\cdot}125x$
$= £1{\cdot}15x$
„ „ „ 1931 $= £1{\cdot}025 \times 1{\cdot}15x + £0{\cdot}125 \times 1{\cdot}15x$
$= £(1{\cdot}15)^2x$
„ „ „ 1932 $= £(1{\cdot}15)^3x$
„ „ end „ $= £(1{\cdot}15)^4x$

Number.	Log.
112397	5·0507549
30	1·4771213
1·15	·0606978
$(1·15)^4$	·2427912
$30 \times (1·15)^4$	1·7199125
2142·113	3·3308424

$$\therefore x = \frac{112397}{30 \times (1·15)^4} = 2142·113$$

∴ Amount of fund at beginning of 1929 = £2,142 2s. 3d.

119. INCREASE OF CAPITAL.

A company may under certain conditions increase its capital by the issue of new shares. It sometimes happens that a prosperous company increases its capital by the issue of **bonus shares to the** existing shareholders in proportion to the number of shares held by them (*e.g.*, one bonus share for every five shares held). If the bonus shares be issued at par, the money is simply transferred from the General Reserve Account to the Share Capital Account. If issued at a premium, the money representing the total amount of the premium is transferred to the Share Premium Account, and, of course, no dividend is payable on this amount: dividend is only payable on the money in the Share Capital Account.

If the share capital be not fully paid, the bonus could be applied in discharge of the call.

EXAMPLE (xi)—

A company had capital consisting of 225,000 £1 shares fully paid. At the beginning of 1931, it was decided to increase the capital by distributing a bonus of 20 per cent. out of the reserve by issue of 1 fully-paid share for each 5 shares held.

(*a*) If dividend at $7\frac{1}{2}$ per cent. were paid at the end of 1931 what would a man receive who held 120 shares at the end of 1930?

(*b*) If the 20 per cent. bonus had been given by distributing shares at a premium of 10s., what should he receive?

Deduct income tax at 5s. in the £.

(*a*) No. of shares held in 1931 = 120 + 24 = 144

Amount received as dividend less income tax $= £144 \times \frac{3}{40} \times \frac{3}{4}$
$= £8$ 2s.

(*b*) Value of each bonus share, including premium = £1 10s.

Thus one bonus share is given for every $7\frac{1}{2}$ shares held.

∴ No. of shares held in 1931 = 120 + 16 = 136

∴ Amount received as dividend less income tax $= £36 \times \frac{3}{40} \times \frac{3}{4}$
$= £7$ 13s.

NOTE 6.—Fractions of shares are not given: thus if a man held 119 shares, he would receive 23 bonus shares.

NOTE 7.—In case (*b*), if, say, 44,600 bonus shares had been distributed, the capital would be increased by £44,600 and £22,300 added to the share premium account.

EXAMPLE (xii)—

17s. 3d. was paid up on each £1 share of a company. What per cent. of the amount paid up on the shares should be applied as a bonus to discharge the call? If the next dividend were 10 per cent., what percentage yield would this be on the amount paid up on the shares?

2s. 9d. is given on every sum of 17s. 3d.;

$$\therefore \text{ percentage rate} = \frac{2\frac{3}{4} \times 100}{17\frac{1}{4}} = \underline{\underline{15{\cdot}94.}}$$

As the call has been discharged, the dividend will be 10 per cent. of the full nominal value of the shares, although 17s. 3d. per share has only been paid.

Thus the dividend on 17s. 3d. cash is 2s.

$$\therefore \text{ percentage yield} = \frac{200}{17\frac{1}{4}} = \underline{\underline{11{\cdot}59.}}$$

120. REDUCTION OF CAPITAL.

Companies may reduce their nominal capital by cancelling shares which have never been issued. By sanction of the Court the nominal capital may be reduced by one of the following ways—

(1) By writing off uncalled capital: this has the effect of reducing the nominal value of shares not fully paid, so that the liability of the shareholders is reduced.

(2) By cancelling lost capital: this might be done by reducing the nominal value of fully-paid shares or by cancelling a proportionate number of shares held by each shareholder.

(3) By paying back capital: this is usually done by buying back shares in the open market and then cancelling the same, but the shareholders might agree to accept cash for a proportional number of their shares or to have the nominal value of the shares reduced and be paid back cash to make up the difference.

The paid-up capital may be reduced by returning accumulated profits to reduce the amount paid up on each share; for example, if there had been 20,000 £1 shares on which 15s. had been paid and £5,000 be paid back to the shareholders, it could be then regarded that 10s. per £1 share is paid up.

EXAMPLE (xiii)—

The nominal capital of a company is £50,000, consisting of 50,000 £1 shares, of which 45,000 have been issued and 15s. per share paid. If unissued shares be cancelled and un-called capital be completely written off what is the reduced capital of the company?

Reduced capital = £50,000 − £5,000 − £¼ × 45,000 = £33,750.

EXAMPLE (xiv)—

The capital of a company consists of 5,000 preference shares of £100 each

and 75,000 ordinary shares of £10 each: all shares are issued and fully paid. If the nominal value of each preference share be reduced to £50 and one ordinary share replaced by four shares of £1 each, what is the reduced capital?

$$\text{Reduced capital} = £5{,}000 \times 50 + £75{,}000 \times 4$$
$$= \underline{£550{,}000.}$$

121. RECONSTRUCTION is the name given to the winding up of an old company and the forming of a new company to carry on the business. The purposes for reconstruction are chiefly—

(1) To raise fresh capital: this is frequently done by exchanging fully-paid shares in the old company for partly-paid shares in the new company. A shareholder in the old company who refuses to accept shares having a liability in exchange would have his interest purchased by the liquidator.

(2) For amalgamating two or more companies.

(3) To induce creditors to accept shares or debentures in payment for their claims.

EXAMPLE (xv)—

A company whose capital consisted of 72,000 £1 shares fully issued and paid went into voluntary liquidation. A new company was formed to carry on the business. The assets were sold to the new company for £42,000, payable in £1 shares of the new company, credited with 17s. 6d. per share paid and £11,375 in cash, the latter being used to defray liquidation expenses and to discharge creditors. In what way should the shares of the new company be distributed among the shareholders? If 40,000 fresh £1 shares were issued to the public and 17s. 6d. per share called, and if the duty, fees, legal and other charges amounted to £3,122, what would be the working capital (*a*) if all shareholders agreed to the reconstruction? (*b*) if shareholders representing 3,540 shares of the old company dissented and were paid for each share three-quarters of the cash value represented by the exchange of shares?

Number of shares included in purchase price $= 42{,}000 \times \frac{8}{7} = 48{,}000$

Ratio of nos. of shares in new and in old cos. $= \frac{48000}{72000} = \frac{2}{3}$

∴ 2 *shares* in the *new company* are distributed for each 3 *shares* in the *old company*.

(*a*) Amount called up on fresh shares $= £40{,}000 \times \frac{7}{8} = £35{,}000$

Total amount to be paid in cash $= £14{,}497$

∴ Working capital $= \underline{£20{,}503}$

(*b*) 1 share in old company is valued at $£\frac{1}{3} \times 2 \times \frac{7}{8}$ cash invested in new company.

∴ Amount paid to dissenting shareholders $= £3540 \times \frac{3}{4} \times \frac{7}{12}$
$= £1{,}548\ 15s.$

∴ Working capital $= £20{,}503 - £1{,}548\ 15s.$
$= \underline{£18{,}954\ 5s}$

NOTE 8.—In case (*b*), assuming the shares held by the dissenting shareholders were, after purchase, cancelled, the nominal capital of the new company would be £48,000 + £40,000 − £⅔ × 3540 = £85,640. Usually, preference shares or debentures would be publicly issued to meet the cash payments and to provide working capital.

122. **AMALGAMATION** is the name given to the combining of two or more companies into one new company, which takes over the liabilities of the old companies and issues shares in exchange for shares in the old companies. The numbers of fully-paid shares issued to the shareholders in each of the old companies should be approximately proportional to the amounts obtained by deducting the totals owing to sundry creditors from the estimated value of the assets in each case.

EXAMPLE (xvi)—

Three companies, A, B, and C, whose share capitals, in each case consisting of £1 shares fully paid, are £60,000, £45,000, and £100,000 respectively, amalgamate. The total assets are estimated at £54,000, £65,000, and £136,000 respectively; and the total amounts owing to sundry creditors are £4,000, £2,000, and £6,000 respectively. What should be the share capital of the new company in order to represent the total value of the three businesses? and in what way should the new fully-paid £1 shares be allotted:

Share capital in new company = £50,000 + £63,000 + £130,000
= £243,000.

Shareholders in	A	should be given	5	new shares for every	6 old.
,,	B	,, ,,	7	,, ,,	5 ,,
,,	C	,, ,,	13	,, ,,	10 ,,

NOTE 9.—The share capital of the new company could be made to exceed £243,000 by the issue of additional shares either to the public or to the shareholders of the old companies; but as £243,000 represents the total value of the companies, the rate of exchange of shares would not be altered.

EXAMPLE (xvii)—

Two companies A and B amalgamated. A's capital consisted of 500,000 fully-paid £1 shares; the assets were valued at £325,000, and the total owing to sundry creditors was £8,500: B's capital was 350,000 fully-paid £1 shares, the assets were valued at £125,000, and debts to sundry creditors amounted to £9,500.

(a) If 432,000 fully-paid £1 shares were divided among the shareholders of the old companies, what should be the number of new shares given in exchange for 1,000 of the old shares in each case?

(b) If shareholders of A were given 4 new shares for every 5 old, and shareholders of B 2 new shares for every 5 old, how much per 100 old shares should be paid to the new company in each case, in order that the new shares should be fully paid?

Total of assets less liabilities to sundry creditors = £325,000 + £125,000 − £8,500 − £9,500
= £432,000.

(a) ∴ Shareholders of A should be allotted 316,500 new shares,
and ,, B ,, ,, 115,500 ,,

∴ Number of new shares equivalent to 1,000 shares of A = $\frac{316500}{500}$
= 633

and ,, ,, ,, 1,000 ,, B = $\frac{115500}{350}$
= 330

(*b*) 500,000 shares in A are worth £316,500,
400,000 shares in new company, when fully paid, are worth £400,000
∴ On 500,000 shares in A, £83,500 must be paid,
∴ " 100 " A, $\frac{£83,500}{5,000}$ *i.e.*, £16 14s. must be paid.

Similarly, on 100 shares in B, £$\frac{140,000 - 115,500}{3,500}$ *i.e.*, £7 must be paid

Ansr. 80 fully-paid £1 shares in new company should be exchanged for 100 shares in A, together with £16 14s.
40 fully-paid £1 shares in new company should be exchanged for 100 shares in B, together with £7.

Example (xviii)—

The capital of X & Co., Ltd., was composed of 20,000 shares of £10 each, on which £6 17s. 6d. was paid up; and the capital of Y & Co., Ltd., consisted of 80,000 shares of £1 each, on which 17s. 6d. was paid. The companies decided to amalgamate and divide profits in proportion to the capital paid up. In what ratio would the profits be divided? What call per share should be made so that the profits should be divided in the ratio 2 : 1, the amount of the call on the shares of X & Co., Ltd., being ten times that on each share of Y & Co., Ltd.?

Paid-up capital of X & Co., Ltd. = £20,000 × $6\frac{7}{8}$ = £137,500
" " Y & Co., Ltd = £80,000 × $\frac{7}{8}$ = £70,000
∴ Profits should be divided in ratio 1375 : 700, *i.e.*, 55 : 28

Let £x ≡ amount of call on each share of Y & Co., Ltd.
then £80,000 ($\frac{7}{8}$ + x) ≡ paid-up capital of Y & Co., Ltd.,
and £20,000 ($6\frac{7}{8}$ + $10x$) ≡ " " X & Co., Ltd.

$$\therefore\quad 2 \times 80,000\left(\tfrac{7}{8} + x\right) = 20,000\left(6\tfrac{7}{8} + 10x\right)$$
$$\therefore\quad 140,000 + 160,000x = 137,500 + 200,000x$$
$$\therefore\quad 40,000x = 2,500$$
$$\therefore\quad x = \tfrac{1}{16}$$

Thus 12*s.* 6*d.* per share should be called up on each share of X & Co., Ltd.
1*s.* 3*d.* " " " " Y & Co., Ltd.

123. CARTELS AND TRUSTS.

In order to save working expenses and mutually to agree on prices to avoid excessive competition, it happens sometimes that a number of enterprises of a like character combine together to form a **Cartel.** Under this form of combination, which is usually not permanent in character, each company or firm retains its independence as regards capital and legal affairs; but certain ramifications of the businesses are taken over by a central authority, and organized in such a way as to effect economies and fix selling prices to the mutual benefit of the members of the Cartel.

A combination of a number of companies into one huge concern is known as a Trust. The various stages of production and distribution of one or more commodities are frequently carried on by a Trust formed by the amalgamation of concerns of the different types. Thus the production of mineral oil, its purification, the manufacture of by-products, the transport by sea and land, could all be carried on by a Trust. A Trust could produce a commodity to the world's markets more cheaply than would be the case if the stages in its production and distribution were carried out by a number of independent firms. Trusts, therefore, could by cutting prices destroy competition on the part of independent companies and so gain a monopoly; then they would be in a position to so regulate supply and prices so as to obtain maximum revenue.

Trusts, of course, are not the only concerns which could have a monopoly, for many companies exist having the sole right of production of certain commodities: for example, a patent medicine; while the Government and Municipal Authorities have monopoly over certain commodities and services, but in the latter case, the advantages to the public, not the gaining of maximum profits, should be the guiding motives. It is beyond the scope of this book to discuss the theory of monopoly, although a simple problem is dealt with. The table giving the quantities of a commodity which the public would purchase at different prices is called a *Demand Schedule*; while that tabulating the amounts produced with the corresponding costs per unit is known as the *Supply Schedule.*

EXAMPLE (xix)—

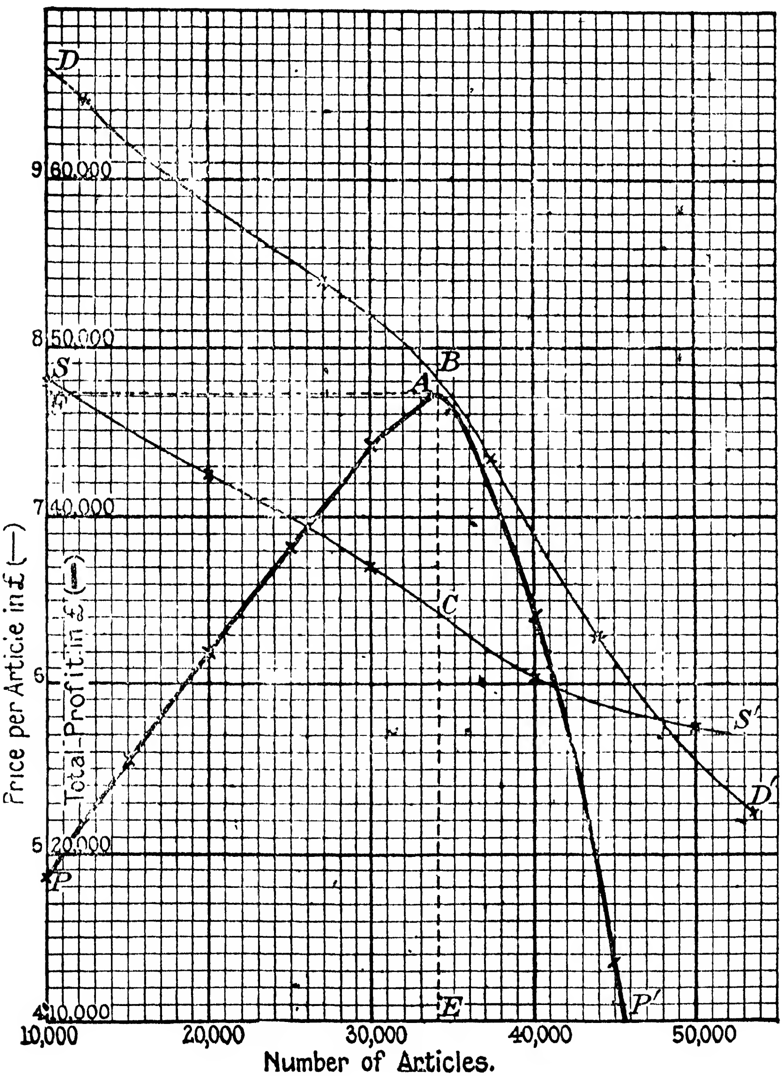
A company's paid-up capital is £86,000. During a certain year, goods to the value of £28,500 were bought and sales amounting to £154,600 were effected. The profit obtained enabled 2½ per cent. dividend to be paid and £1,526 placed in the Reserve. The next year the company was combined in a cartel: £31,500 worth of goods were bought, the sales were to the value of £168,000; but the totals of all expenses were diminished by 2 per cent. What was the greatest percentage dividend (not involving fractions) that could be paid? and in this case what would be available for reserve?

Total profit, 1st year = £$\frac{86000}{40}$ + £1,526 = £3,676
,, expenses, ,, ,, = £154,600 − £28,500 − £3,676
= £122,424
,, profit, 2nd ,, = £168,000 − £31,500 − £122,424 + £2448·48
= £16524·48
Percentage dividend possible = $\frac{1652448}{86000}$ = 19·2 . . .
∴ Maximum dividend payable is 19%

Amount available for reserve = £16524·48 − £860 × 19
= £184 9s. 7d.

EXAMPLE (xx)—

From the schedules on page 228, in which the unit of time is one year, find the number of patent appliances which should be put on the market per year and the price at which they should be offered for sale in order that maximum profits should be obtained ?



Supply Schedule.

Number of Articles.	Cost per Article.
10,000	£7 18s.
20,000	£7 5s.
30,000	£6 14s.
40,000	£6 1s.
50,000	£5 15s.

Demand Schedule.

Price per Article.	Number of Articles.
£9 9s.	12,500
£8 8s.	27,000
£7 7s.	37,500
£6 6s.	44,000
£5 5s.	53,500

Number of Articles.	Profit per Article.	Total Profit.
10,000	£1 17s.	£18,500
15,000	£1 14s.	£25,500
20,000	£1 12s.	£32,000
25,000	£1 10s. 8d.	£38,333$\frac{1}{3}$
30,000	£1 9s. 6d.	£44,250
35,000	£1 6s. 8d.	£46,666$\frac{2}{3}$
40,000	17s.	£34,000
45,000	6s.	£13,500
50,000	—	—

Ansr. 34,000 articles should be put on the market per year.
Cost price is £6 8s 6d and selling price is £7 16s 3d.
Total profits amount to £47,175

Note 10.—DD^1 is known as the Demand Curve and SS^1 as the Supply Curve. If any line be drawn parallel to the price axis, the length intercepted between the two curves determines the profit per article, so that the total profits can be tabulated with the number of articles. The curve PP^1 shows the relationship between the latter quantities, the scale for total profits being on the right of the price axis. The point A indicates the maximum total profits: by drawing perpendiculars AF and AE, it is seen that maximum profit is approximately £47,200 and that the number of articles produced should be 34,000 From B, the selling price is seen to be £7 16s. 3d., and from C the cost price is found to be £6 8s 6d

Note 11.—If by taxation or otherwise the total cost of production be increased by a fixed amount, the number of articles to produce for maximum total profit is unaltered, the latter being simply reduced by this fixed amount. If a certain per centage be charged against total profits, the production is again unaltered, the maximum total profit being now £47,175 reduced by the given percentage. If, however, the cost of producing each article be increased (*a*) by a fixed amount or (*b*) by a percentage amount, the production to give maximum total profit would differ from that formerly. By constructing the new supply curve and the new curve showing total profits (this could be drawn using the same axes as those used for the former curves), the production giving maximum profit could be determined in the same way as above.

TEST EXERCISES II, 3.

(1) A company was authorized to issue 10,000 preference shares of £5 each and 135,000 ordinary shares of £1; £50,000 debenture stock was issued

at £97 10s.; 9,000 preference shares; and 100,000 ordinary shares were issued at par, payments being made as follows—

		Pref. Shares.	Ord. Shares.	Deb. Stock.
Apr. 2.	On application . . .	£1	4s.	15 %
„ 6.	„ allotment . . .	£2	7s. 6d.	82½%
June 1.	„ first call . . .	£1 5s.	5s.	—
Aug. 31.	„ second call . .	15s.	3s. 6d.	

What were the total amounts received by the company on each of these dates?

(2) Referring to the above company, if legal expenses amounted to £559 15s., additional preliminary expenses amounted to £615 12s. 6d., and all payments were paid on the issued shares and debentures, what would be the working capital?

(3) Referring to the company of Question (1), if a man applied for 550 preference shares and 1,750 ordinary shares, what was the necessary amount of money to be paid on application?

If he were allotted 300 preference shares and 750 ordinary shares, what amount should he pay on allotment. If he had been allotted only 80 preference shares and 275 ordinary shares, what should he receive back on 6th April? What interest should he receive on 1st June and 31st August at 5 per cent. per annum, reckoning the number of days from 5th April?

(4) If a man had made the required payments on application and allotment for 110 preference shares and 420 ordinary shares of the above company, but did not make any further payment until 31st August, what amount should he forward on this date to completely pay for his shares, interest being at the rate of 5 per cent. per annum?

(5) If the interest on the above debentures were at the rate of 4½ per cent payable half-yearly, what should a man, who spent £1,170 on buying these debentures, receive each half-year, assuming income tax at 3s. 9d. in the £ to be deducted? What interest would each £100 cash yield (neglecting income tax)?

(6) A company was floated to take over a business. The purchase price was £25,000. £12,000 in ordinary shares at £1 each, £3,000 4½ per cent. debentures, and £10,000 in cash. The following were offered to the public: 70 4½ per cent debentures at £100 each, payable £15 on application, £85 on allotment; 18,000 ordinary £1 shares at £1 2s each, payable 5s. on application, 10s. on allotment, 7s. three months later; 10,000 5½ per cent. preference shares of £1 each, 5s on application, 6s. on allotment, 9s. three months later. All the shares were applied for and allotted, the preliminary expenses altogether amounting to £1,135. What was the nominal capital of the company? If, three months after allotment, all the shares were fully paid for, what was the working capital?

(7) A man applied for 10 debentures and 1,500 ordinary shares in the above company. What was the proper amount to forward on application? He was allotted 7 debentures and 840 ordinary shares. What should he pay on allotment?

(8) A man applied for and was allotted 60 of the above preference shares. He made the full payment on application, but only half of the proper amount on allotment. He made no further payments and neglected to answer the call for further payments, and his shares were forfeited. The shares were sold five months after the date of allotment at 14s. 6d. each. Reckoning interest at 5½ per cent. per annum, what did the company gain by the forfeiture?

(9) A company invited applications for 10,000 shares at £10 each: 605 persons applied for 1 share, 520 for 5 shares, 264 for 10 shares, 59 for 100 shares, and 14 for 200 shares. The directors decided to allot the shares

among the applicants for 5 shares or more proportionally, as far as possible, to the number of shares applied for, neglecting fractions of shares. Any shares remaining were to be distributed among applicants for one share. How many of the latter were successful in their application and how many were allotted to each of the other applicants?

(10) A man subscribed for 140 5 per cent. preference shares at £104 as follows: 10th September, £19 per share; 15th September, £60 per share; 16th January of the following year, £10 per share. On 4th April of the latter year, half-yearly dividends at the full rate were paid. What should he receive, income tax at 5s. in the £ being deducted?

(11) 420,000 £1 shares were issued by a company at a premium of 1s 3d. per share. By the end of the year, when a dividend of 3½ per cent. was paid, 3s. 6d. had been left unpaid on each share. What percentage interest on his money did a man who bought some of the shares receive?

(12) A man bought a number of £10 shares issued at a premium of 10s. His subscriptions were as follows: 10th May, £2 10s.; 15th May, £5; 14th August, £3. On 20th June, the next year, he received a year's dividend at 4½ per cent. for the accounting period ending 30th April. Find the equivalent rate per cent. on cash, taking account of the dates on which the instalments were paid.

(13) A cheque for £19 13s. 9d was received by a person as a year's dividend on 750 £1 shares, on each of which 17s. 6d had been paid. If income tax at 5s. in the £ had been deducted, what percentage rate of dividend had been paid?

(14) The capital of a company was 60,000 5 per cent. ordinary shares at £1 each, 18s. paid up, and 1,000 £100 deferred shares fully paid. At the end of a half-year, dividend at the rate of 8½ per cent. per annum was paid on the deferred shares. What was the total amount paid out as dividend, and what percentage per annum was this of the paid-up capital?

(15) X owned 15 4½ per cent. debentures issued at £96, 1,200 6 per cent. preference shares at £5 each issued at par, £4 5s. per share paid up; and 7,000 ordinary shares at £1, 17s. 6d. per share paid up. The dividend on the ordinary shares during a certain year was 4 per cent. What was X's total income due to the investment? and what per cent. on the money invested was this?

(16) A dividend of 17½ per cent. free of income tax was paid on the ordinary shares of a rubber company. If, at the time, income tax was 4s. 6d. in the £, what was the percentage dividend before income tax was deducted?

(17) The nominal capital of a company was £140,000 made up of 4,000 5 per cent. preference shares at £10 each, 80,000 7 per cent. ordinary shares at £1 each, and 200 deferred shares at £100 each. The shares were completely paid up, and the preference and ordinary shares were non-participating. The profits available for dividend for a certain half-year were £3,555. What were the percentage dividends per annum on each kind of shares? What would the latter have been if the profits had amounted to (*a*) £894? (*b*) £1,437? (*c*) £5,000?

(18) Referring to the above question, what would the percentage dividends per annum have been in each case had £8 15s. and 16s. 6d. respectively been paid on the preference and ordinary shares?

(19) A company has issued 450 4 per cent. debentures at £100, 5,000 6 per cent. preference shares at £10, and 100,000 ordinary shares at £1 each. In each case the full payment has been made, and the working capital is £95,000. The profits during certain half-years, after all charges had been deducted excepting interest on debentures, were (i) 1·47 per cent., (ii) 2·53 per cent., and (iii) 3·22 per cent. respectively of the working capital. What percentage dividends per annum could have been paid on the preference and ordinary shares?

(20) Referring to Question (17), if the profits available for dividends had been £5,695, what percentage dividends per annum could be paid if (*a*) the ordinary and deferred shareholder equally participated after the latter had received 10 per cent. per annum? (*b*) all shareholders equally participated after the dividend allowed in deferred shares had been 10 per cent. per annum?

(21) If the preference and ordinary shares of the company whose capital is stated in Question (19) be equally participating after a dividend of 12½ per cent. on ordinary shares is allowed, what would be the total percentage dividends on each for a half-year when the net profits (interest on debentures paid) amounted to £21,945?

(22) A company whose capital was 25,000 4½ per cent. cumulative preference shares at £5 each, 200,000 ordinary shares at £1 each, was formed at the end of 1908. The net profits, which were available for the payment of dividend for the period 1926–1931 inclusive, were £937 10s., £3,750, £8,000, £15,000, £18,000, and £28,000 respectively. What percentage dividends could be paid at the end of each year?

(23) Referring to Question (22), what percentage dividends should have been paid if 5,000 preference and 40,000 ordinary shares had not been issued, and on the issued shares £3 17s. 6d. per preference share and 16s. per ordinary share had been called up and paid?

(24) A man bought 2,500 6 per cent. cumulative preference shares at £5 each, which were issued at a premium of 15s. The dividends for the first seven half-yearly periods were at the rate of 4 per cent., 0 per cent., 4½ per cent., 5½ per cent., 5 per cent., 7½ per cent., and 7½ per cent. respectively. What percentage dividend should be paid the next period to wipe off the arrears of dividend? What amounts did the man receive each half-year, income tax being deducted at 1s. 2d., 1s. 2d., 1s. 2d., 2s. 6d., 3s. 6d., 3s. 6d., 5s., and 5s. in the £ respectively? What percentage yield free of income tax on the money invested did he receive in each case?

(25) A held 12 4½ per cent. cumulative preference shares at £100, which he bought at par at the time of issue. For the first two years, half-yearly dividends at 2½ per cent. per annum were paid; for the next two years, the dividends were at 6½ per cent. per annum. If income tax at 1s. 2d. in the £ be deducted, and A had deposited the amounts in a bank paying interest at 3 per cent. per annum payable half-yearly, what would the total amount be at the end of the four years? Had eight dividends at 4½ per cent. per annum been paid, what would the total amount have been?

(26) A company which was formed at the end of 1923 issued 75 5 per cent. debentures of £100 each and 5,000 ordinary shares at £10 each, all of which were fully paid up. The debentures were paid off at the rate of five each year at £105, the money for this being charged against profits. During the year 1930 the business done was £13,245, and the expenses and cost of materials amounted to 78 per cent. of the business done. Find the greatest percentage dividend that could be paid out of the profits. In 1931 the figures were £14,179 and 76 per cent. What was the greatest dividend per share that could be paid without drawing on reserve?

(27) X had allotted to him on 25th June, 1926, 750 cumulative 6 per cent. preference shares of £1 each issued by a company, the terms of issue being: 2s. per share on application; 2s. 6d. per share on allotment; 5s. per share on 1st January, 1927; 5s. per share on 1st July, 1915; and 5s. 6d. per share on 1st January, 1928. He was to receive a fixed cumulative preferential dividend of 5 per cent. per annum on the capital paid up, calculated from the dates fixed for the payment of instalments; and interest at the rate of 4 per cent. was to be allowed on instalments paid in advance from the date of payment until the due dates respectively, 6 per cent. dividend not beginning until 1st October, 1928. X paid up in full on 25th June, 1926; and the above preferential dividends were paid half-yearly on 1st June

and 1st December each year. What did he receive on 1st December, 1926; 1st June, 1927; 1st December, 1927; 1st June, 1928; and 1st December, 1928, respectively?

(28) A man who held a number of £1 shares in a company received a half-yearly dividend at the rate of 35 per cent. per annum, which, with income tax at 3s. 9d. in the £, amounted to £284 7s. 6d. If 16s. per share had been paid up, how many shares did he hold?

(29) A man held 560 £1 shares and received £41 6s. 10d. as a half-yearly dividend at the rate of $22\frac{1}{2}$ per cent. per annum, income tax at 5s. in the £ being deducted. How much per share was paid up?

*(30) Certain 4 per cent. debentures were issued at par in 1920 and were redeemed in 1930 at 104, and interest was paid half-yearly. What was the equivalent rate of interest for the whole period of ten years?

(31) A certain company paid a dividend on stock at the rate of 9 per cent. per annum free of income tax. What was the equivalent rate before deducting income tax if the latter was at the rate of (i) 3s. 9d. in the £? (ii) 5s. in the £?

*(32) A company has to redeem debentures at par to the value of £75,000 in ten years' time. If twenty equal half-yearly instalments be set aside for this purpose and each invested at $3\frac{1}{2}$ per cent. per annum, interest payable half-yearly, what should be the amount of each instalment? What would be the amount of each half-yearly instalment, if interest was payable yearly?

(*N.B.*—The amount of two instalments of £x paid in at the end of six months and one year respectively is the same as one instalment of £$2{\cdot}0175x$ at the end of the year. Thus, in latter part of question, consider the payments as consisting of ten equal instalments of £$2{\cdot}0175x$.)

(33) The total cost of erecting machinery in a factory was £10,540. It was estimated that in eighteen years' time it would cost £8,725 to replace it by new machinery, by which time the original machinery would on an average each year have manufactured goods to the value of £50,000 from raw material costing £17,500. It was decided to set aside a certain percentage of this increase in the value of goods each year to meet the cost of the new machinery, the money to be kept by the company and interest not charged upon it. What should this rate per cent. be? What amounts should have been set aside at the end of years when (i) £15,835 of raw material was converted into £47,440 of manufactured goods? (ii) £21,207 of raw material was converted into £58,326 of manufactured goods?

(34) It was reckoned that plant which cost £25,000 to erect would need to be replaced after 25 years at a cost of £20,000. It was estimated that the total revenue produced during this period would be £325,000. If depreciation be charged against revenue at a constant rate, what percentage rate should this be? The revenue during a certain year was £35,295: if $1\frac{1}{8}$ per cent. of this be set aside for repairs, what was the total amount charged against revenue for repairs and depreciation? If after 25 years the total revenue be £286,450, and the instalments for depreciation have been kept by the company and interest not allowed, after how long could the machinery be replaced, the cost being £19,000 if the total of the instalments be invested at 4 per cent. per annum?

*(35) The sinking fund of a company was started at the end of 1925 by setting aside £1,200. At the end of each of the next six years it was increased by one-fifth of the amount of the fund at the beginning of each year respectively. What was the amount of the fund? What would have been the amount had the instalments been invested at 3 per cent. per annum, interest payable yearly?

*(36) The sinking fund of a company was £5,725 at the end of 1920. By what percentage should it have been increased at the end of each

subsequent year, so as to amount to £10,000 by the end of 1931? What would the yearly percentage increase have been had the fund been deposited so as to accumulate at $2\frac{1}{2}$ per cent. per annum?

*(37) The sinking fund of a company was increased by 10 per cent. of the amount at the end of each year, after interest at 2 per cent. on the amount at the beginning of the year had been added. At the end of 1930 the amount of the fund was £5,826 11s. 3d. What did the fund amount to at the beginning of 1923?

[*N.B.*—£1 at the beginning of any year amounts to £1·02 × 1·1, *i.e.*, £1·122 at the end of the year.]

(38) A company was wound up and the liquidator was left with £95,474 13s. 6d. to return to the shareholders as repayment of capital. The capital was £75,000 preference stock and £135,000 ordinary stock. What amount should be returned per £100 stock in each case (*a*) if holders of preference stock have a preferential right to the return of capital? (*b*) if they do not?

(39) A company distributed a bonus of $12\frac{1}{2}$ per cent. out of the reserve by issue at par of one fully-paid share for every 8 shares held. The dividend before the distribution was at the rate of 18 per cent., and the year after it was at the rate of 15 per cent. By what ratio was the dividend received by a shareholder thus increased or diminished?

(40) One bonus £1 share at a premium of 7s. 6d. was distributed by a company for each eight shares held. At what rate per cent. was the bonus? A man previous to the distribution held 175 shares, and the dividend paid was at the rate of $22\frac{1}{2}$ per cent. and income tax was at 2s 6d. in the £. Afterwards the dividend was at the rate of 20 per cent and income tax was at 3s. 6d. in the £. By how much was the amount received by the man after the distribution greater or less than that before?

(41) £8 12s. 6d. was paid up on each £10 share of a company. A bonus was given so as to discharge the call, and the next dividend was at the same rate as that before the giving of the bonus. In what ratio is the amount received as dividend by a shareholder increased? If 15,000 shares had been subscribed for, how much is required in order to give the bonus? What per cent. is this of the paid-up capital before the bonus was given?

(42) Ordinary £10 shares, on which £9 2s. 6d is paid up, are replaced by three £1 shares fully paid. By what amount is the paid-up capital reduced, the original number of shares being 1,250?

(43) The nominal capital of a company consisted of 225,000 £1 shares, of which 72,500 were unissued and 17s. 9d. had been paid up on each of the issued shares. The unissued shares were cancelled and money returned to the shareholders, so that 12s. 6d. was now paid up on each share. What was the new nominal capital and the new paid-up capital?

(44) A shareholder held 520 shares in the above company before the reduction of capital, the company paid $12\frac{1}{2}$ per cent. dividend and afterwards $17\frac{1}{2}$ per cent. dividend. By how much was the amount received by him after greater or less than that before the reduction of capital, income tax being at 1s. 2d. in the £?

(45) The subscribed capital of a company is: £45,000 in £5 preference shares, £4 2s. 6d. per share called up and paid; and £105,000 in £1 ordinary shares, 16s. 8d. per share called up and paid. It was agreed to reduce capital by replacing 5 old preference shares by 3 new fully-paid £5 preference shares and 8 ordinary shares by 5 new fully-paid £1 ordinary shares. By what per cent. has the paid-up capital (*a*) represented by preference shares, (*b*) represented by ordinary shares, been reduced? and by how much has the total paid-up capital been reduced?

(46) A new company was formed to carry on the business of A & Co., Ltd., whose capital was £165,000, composed of £1 shares fully subscribed

and paid. The assets were sold for £55,000, payable in £1 shares of the new company, credited with 16s. 8d. per share paid and £13,750 in cash. In what way should the shares of the new company be distributed? What are the liabilities of shareholders who before the reconstruction held 75, 124, and 317 shares respectively?

(47) Owing to reconstruction, 350,000 £1 shares are to be divided among shareholders of a company which went into voluntary liquidation. The capital of the latter consisted of £560,000 fully-paid £1 shares, and the assets were sold to the new company for £9,375 cash and £301,875 payable in £1 shares of the new company. In what way should the shares of the new company be distributed and how much is credited on each?

(48) The subscribed capital of a company was £305,000 in £1 shares, on which 17s. 9d. per share was paid up. Capital was reduced by returning to the shareholders the amount paid up on every one share out of five held and, in addition, 2s. 6d. on each of the remaining shares. What was the new subscribed capital and the new paid-up capital. If a half-yearly dividend at 3½ per cent. be declared after the reduction of capital, what total amount from the time just before the reduction of capital was made would a shareholder receive who formerly held 1,435 shares, income tax being at the rate of 3s. 9d. in the £?

(*N.B.*—Return of capital is not income, and thus is not subjected to tax.)

(49) The capitals of three companies—A & Co., Ltd.; B & Co., Ltd.; and C & Co., Ltd.—are: 75,000 shares of £1 each, on which 15s. is paid up; 2,500 shares of £10 each, on which £7 15s. is paid up; and 55,000 shares of £1, fully paid, respectively. The companies amalgamate and mutually decide that no fresh capital shall be issued or called up, and that profits shall be divided in proportion to the paid-up capital. The year after the amalgamation the total profits available for dividend amounted to £38,574. How should this be divided among the three companies?

(50) The capital of W. Smith & Co., Ltd., consisted of £70,000 in fully-paid £1 shares; that of R. Jones & Co., Ltd., comprised 55,000 shares of £1 each fully paid. The total assets were estimated to be £80,244 and £38,272 respectively, and the sums owing to sundry creditors £1,494 and £5,272 respectively. These companies were amalgamated by the formation of a new company, the capital of which consisted of fully-paid £1 shares to the amount of the total estimated value of the two businesses. In what way should the shares of the new company be allotted among the shareholders of the old companies? If the paid-up capital had been the same as above, but consisting of £1 shares credited with 16s. 8d. per share paid, in what way should the shares be allotted?

(51) The capital of a company consisted of 5,250 shares of £10 each, on which £6 15s. was paid up: its assets were estimated at £85,000 and the total owing to sundry creditors was £2,125. The capital in the case of a second company was 47,500 shares of £1 each, on which 13s. 6d. was paid up: its assets were estimated at £22,500 and the total owing to sundry creditors was £3,720. The companies were amalgamated, but no fresh capital was called up or issued. A year later a dividend of 4½ per cent. was declared. What were the percentage rates of dividend on the amounts of capital paid up in the case of each company respectively, it being assumed the division of profits to be proportional to the values of the respective companies at the time of amalgamation?

(52) Referring to the two companies of Question (51), if the new company's paid-up capital be considered as £101,655 and consisting of £1 shares on which 17s. 4d. per share is paid up, and that shares in the old companies be exchanged for these new shares, how many of the latter should be exchanged for 100 shares of each of the old companies?

(53) Referring to the companies of Question (51), if shareholders had

been given 20 new shares for 1 old share and 4 new shares for 5 old shares respectively, the new shares being £1 each credited with 17s. 6d. paid up, how much per 100 old shares should be paid to the new company in each case ?

(54) Two companies propose to amalgamate. The capital of the first consists of 135,000 shares of £1 each, on which 15s. 9d. is paid up; and that of the second consists of 70,000 shares of £1 each, on which 11s. 3d. is paid up. If profits are divided in proportion to the paid-up capitals, in what ratio will the profits have to be divided between the two companies ? If the profits to be divided amount to $4\frac{1}{2}$ per cent. of the total paid-up capital, what dividend should be given to holders of 100 shares of each of the above companies respectively ? What dividends on 100 shares of each company respectively should be given if the profits divided amounted to £14,568 15s. ?

(55) Referring to the above companies, what call per share (equal in each case) must be made in order that profits should be divided in the ratio of 5 : 2 ? What will be the total paid-up capital after this call has been made ?

(56) The gross receipts of a company were 4 per cent. greater than the total of all amounts paid out, one-third of which was for the purchase of raw material. The cost per unit of raw material was increased by 2 per cent., but by joining a cartel the total of all other expenses was reduced by 5 per cent. and the selling price per unit was increased by 3 per cent. Find by how much per cent. the total net profit was increased (*a*) if the same quantity of the commodity was bought and sold as before, and (*b*) if the quantity bought and sold was increased by 5 per cent.

(*N.B.*—Start with 100 units of the commodity costing 1 unit of money each.)

(57) A company during a certain year bought goods for £74,000 and, after subjecting them to a certain process, sold them for £157,500, gaining £5,416 net profit. During the following year, the company having become a member of a cartel, the quantity of goods bought and sold decreased by 5 per cent.; the cost price and selling price per unit were increased by 5 per cent. and $7\frac{1}{2}$ per cent. respectively; and the total of expenses and charges, exclusive of payment for goods bought, was diminished by 8 per cent. How much profit was made during this year ?

(*N.B.*—Start with 100 units of goods costing £740 per unit.)

(58) The cost of maintaining a bridge was £1,740. Tolls were charged on vehicles using the bridge, the demand schedule being—

Toll per Vehicle	Number of Vehicles.			
	Jan.-Mar.	Apr.-June.	July-Sept.	Oct.-Dec.
1s.	10,440	11,190	11,560	9,560
1s. 6d.	8,020	9,380	10,490	8,040
1s. 9d.	6,110	8,230	9,580	6,660
2s.	4,980	6,570	8,710	5,600
2s. 6d.	3,850	4,740	6,700	4,220

Find to the nearest penny what toll should be charged each quarter, in order to obtain maximum revenue; and find the approximate number of vehicles using the bridge each quarter and the maximum total profit for the year. If the same toll was charged throughout the year, what should this be to the nearest penny to obtain maximum revenue ? Find the maximum total profit approximately in this case.

(59) The demand schedule and supply schedule in reference to the yearly output and sale of a certain patent article is as follows—

SUPPLY SCHEDULE.

No. of Articles.	Cost per Article.
25,000	£2 12s.
40,000	£2 2s. 6d.
75,000	£1 18s.
100,000	£1 16s. 6d.

DEMAND SCHEDULE.

Price per Article.	No. of Articles.
£3	22,000
£2 15s.	29,000
£2 10s.	38,000
£2 5s.	50,000
£2	74,000
£1 15s.	111,000

Find what output would produce the greatest revenue, and find the maximum profit approximately; also find the corresponding cost price and selling price to the nearest 3d.

Find approximately the output, cost price, and selling price to give maximum profit, if (i) A tax of 2s. 6d. were put on each article; and (ii) the cost of production were increased by 10 per cent.

SECTION III.

TRADE.

INTRODUCTION.

As human needs are varied, and as no individual or community can produce or manufacture all those commodities necessary to satisfy all requirements, it follows that an exchange of goods between districts and between countries must be carried out. In general, producers or manufacturers whose efforts are concentrated on solving the difficulties of production or manufacture do not wish to concern themselves with the distribution of their goods to the retail shops, so that this latter and important function is left to the wholesale dealers. The latter, in general, do not wish to carry on the house-to-house distribution, this function being in the hands of retailers. The exchange and distribution of commodities is termed **Trade**, engaged in which are vast numbers of persons carrying on specialized work.

The kind and the amount of trade carried on by the business men of a country has its effect on the well-being of the nation in question. The **Board of Trade** has been established in this country so that British trade can be regarded as a whole, and it has made regulations with the idea of conferring mutual benefit to all concerned. By the work of the Statistical Department, the commercial prosperity of the British Isles and the British Empire can be determined and recorded.

Most manufacturers distinguish their goods from similar goods made by others by means of a **Trade Mark**, which gives to the purchaser a satisfactory assurance of the make and quality of the goods he is buying.

Just as the value of a sum of money is dependent on the time of payment (*e.g.*, £100 now is of greater value than £100 in one year's time), so is the value of goods dependent on the locality at the time of sale (*e.g.*, 100 pieces of silk in a London warehouse are of greater value to the London merchant than 100 similar pieces on board ship). The value of goods to the buyer also depends on the time of delivery, for his opportunity to effect a sale is likely to be lost by delay. Thus in buying goods, the dealer must ascertain (1) the locality of the goods and the conditions of payment as regards freight, insurance, Customs, etc.; (2) the time at which payment **must be made**;

and (3) the time at which the goods can be delivered to him. In quoting prices of goods, some of the abbreviations used are as follows—

C.W.O. ≡ Cash with Order	C.O.D. ≡ Cash on Delivery
C. & F. ≡ Cost and Freight	C.I.F. ≡ Cost, Insurance, and Freight
F.A.S. ≡ Free alongside Ship	F.A.Q. ≡ Free alongside Quay
F.O.B. ≡ Free on Board	F.O.R. ≡ Free on Rail

The Base Price of goods is the price of the seller's goods wherever they may be at the time of sale: for example, if the base price be indicated "*Ex* Quay," it is understood that the buyer bears the expense of their conveyance from the quay to his address.

The actual weight of goods; the total weight of goods and the cases, bags, wrappers, etc., in which they are packed; and the weight of the cases, bags, wrappers, etc., are respectively known as the **Net Weight**, the **Gross Weight**, and the **Tare**,

In this section, brief outlines are given of certain commercial transactions and the methods of making the necessary calculations are considered. These calculations are comparatively easy, and should present little difficulty to the student.

CHAPTER XIV.

HOME TRADE.

124. BRIEF OUTLINE OF A HOME TRADE TRANSACTION.

MESSRS. R. REYNOLDS & Co., 268 Leicester Street, Dover, wished to make purchases of calico, etc.; and sent to Fane & Co., Ltd., of 57 Wallsend Street, London, E.C., asking for their price list or for a quotation. The former decided to order certain goods, a list of which was made out on an **Order Note** signed by R. Reynolds and forwarded to Fane & Co., Ltd. The latter accepted the order, agreed to pay cost of carriage, and sent Messrs. R. Reynolds & Co. a **Bought Note** specifying exact particulars concerning the price, quality, and delivery of the goods, and the terms of payment.

A **Consignment Note**, on which the Southern Railway was requested to receive and forward the goods, was sent by Fane & Co., Ltd., to the office of the Goods Department of the railway company. The carrier who called for the goods, before removing them from the warehouse, signed a **Delivery Note**, thereby making the railway company responsible for their delivery to Messrs. R. Reynolds & Co. The goods were delivered to the latter, who signed the carter's **Delivery Sheet**, thus relieving the railway company of responsibility.

On dispatch of the goods, the **Invoice** was posted to Messrs. R. Reynolds & Co., and was as shown on the next page.

The stock-keeper of Messrs. R. Reynolds & Co. checked the invoice with the Order Note and the goods which had arrived, and found there was no discrepancy. As, however, it was discovered that about 80 yards of sheeting (A 57) was soiled, a **Debit Note** claiming a reduction of £1 15s. was sent to Fane & Co., Ltd. After consideration, the latter forwarded a **Credit Note** to Messrs. R. Reynolds & Co. allowing the reduction.

On the 1st day of October a **Statement** was sent to Messrs. R. Reynolds & Co. summarizing particulars of goods purchased and allowances made since the last day of settlement.

On 6th September, goods to the value of £39 3s. 9d. were purchased, but by an error £38 13s. 9d. was the total on the invoice. Fane & Co., Ltd., finding out the mistake, sent a Debit Note

INVOICE.

Telephone: No. 4136. 57 WALLSEND STREET,
LONDON, E.C. 2

MESSRS. R. REYNOLDS & Co. *13th September*, 19..
268 Leicester Street, Dover.

Bought of FANE & CO., Ltd.,

WHOLESALE DRAPERS.

TERMS: 2% CASH.

				£	s.	d.	£	s.	d.
2	pcs. Bleached Calico 1579 .	186 yds.	4⅝d.	3	11	8			
1	pcs. Unbleached Calico 1612 .	92½ „	3½d.	1	7	–			
				4	18	8			
		Less 7½%			7	5			
							4	11	3
3	pcs. 24″ Flannel 795	198 yds.	10¾d.	8	17	5			
6	pcs. 24″ Flannellette 1054 . .	394½ „	6⅜d.	10	9	7			
				19	7	–			
		Less 5%			19	4			
							18	7	8
4	pcs. 54″ Sheeting A 57	252½ yds.	10¼d.	10	15	8			
2	pcs. 70″ Sheeting A 59	125 „	1/2⅜	7	9	9			
		Net					18	5	5
	Per S R. Carr. Pd.						41	4	4

claiming the additional 10s. Messrs. R. Reynolds & Co. acknowledged the same by returning a Credit Note. On 20th September, goods were also purchased, the invoice price being £97 11s. 10d., which, however, included 5s. for empty cases, which Messrs. R. Reynolds returned, sending a Debit Note claiming 5s. for repayment. The Statement was as shown on the next page.

Messrs. R. Reynolds sent a **Cheque** for £172 9s. 6d. to Fane & Co., Ltd., who returned a **Receipt** acknowledging that the payment had been made.

N.B.—An invoice should give details concerning the quantity, quality, and price of the goods; particulars as to delivery; and the terms of payment. In the case when goods are sold by weight, the gross weight, tare, and net weight should be stated. *E. & O.E.*, which means *Errors and Omissions Excepted*, is sometimes written on an invoice by the seller: so that should the invoice total through error or omission be less than the total amount as regards the goods actually delivered, he could claim the difference. In general, this is not done, for no reputable business house would take

STATEMENT.

Telephone: *No.* 4136. 57 WALLSEND STREET,
LONDON, E.C. 2
1st October, 19..

MESSRS. R. REYNOLDS & Co.,
268 Leicester Street, Dover.

Dr. to FANE & CO., Ltd.

Folio 149. TERMS: NET ONE MONTH.

		£	s.	d.	£	s.	d.
Sept. 6	To Goods	38	13	9			
,, 12	,, Allowance as per Debit Note .		10	–			
,, 13	,, Goods	41	4	4			
,, 20	,, ,, ...	97	11	10			
		177	19	11			
,, 15	By allowance as per Credit Note .	1	15	–			
,, 22	,, ,, ,, ,, .		5	–			
		2	–	–			
		175	19	11			
	Less 2% discount . .	3	10	5			
					172	9	6

advantage of a clerical error on the part of the firm with which they are dealing.

Instead of altering an invoice, adjustments brought about by (1) damage to goods, (2) the non-arrival of certain of the articles included in the invoice, (3) clerical errors, (4) goods being not up to sample quality, (5) return of empties, are made by means of credit notes and debit notes. The buyer would forward to the seller a debit note or a credit note according as to whether the incorrect invoice total was greater or less than the correct amount.

In the case of an isolated purchase, the number of days' credit would be counted from the date of the invoice or the date of delivery of the goods. As regards regular custom, a monthly or quarterly account would be sent to the customer. Suppose cash discount were 2 per cent. one month, then this reduction would be allowed on the invoice prices of all goods bought after the previous account had been settled. Should payment not be made, request for payment would be made on the next monthly statement, but cash discount would not be allowed. If payment were delayed several months, the seller could not by law charge interest on the amount unpaid. A good business man, however, would always pay up to time if possible, for by so doing he would enhance his reputation with the businesses with which he deals.

Payments are usually made by cheque, but often by means of Bills of Exchange or Inland Promissory Notes. This subject is dealt with in the section on Banking and Finance.

Calculating discounts is most readily done by decimalizing by inspection, for example, $1\frac{1}{2}\%$ of £135 11s. 8d. = £$1\frac{1}{2} \times 1{\cdot}35583$ = £1·3558 + £·6779 = £2·0337 = £2 0s. 8d.

125. CUSTOMARY TRADE UNITS.

Although the units pound (*Avoirdupois*), yard, gallon must be uniformly the same in all localities in the British Isles and with respect to trade in all commodities, yet in certain trades there are employed certain units peculiar to the trades in question. A selection of some of the customary trade units is as follows—

Flour Weight.

14	pounds	= 1	peck *or* stone
56	,,	= 1	bushel
40	,,	= 1	boll
196	,,	= 1	barrel
280	,,	= 1	sack

Wool Weight.

7	pounds	= 1	clove
2	cloves	= 1	stone
2	stones	= 1	tod
6½	tods (182 lb.)	= 1	wey
2	weys	= 1	sack
12	sacks	= 1	last
20	pounds	= 1	score
12	score	= 1	pack

Cloth Measure.

2¼	inches	= 1	nail
4	nails	= 1	quarter (of a yd.)
3	quarters	= 1	Flemish ell
4	,,	= 1	yard
5	,,	= 1	English ell
6	,,	= 1	French ell

Linen Yarn Measure.

300	yards	= 1	cut
12	cuts	= 1	hank
16	hanks	= 1	bundle

Ale and Beer Measure.

9	gallons	= 1	firkin
4	firkins	= 1	barrel
1½	barrel	= 1	hogshead
2	hogsheads	= 1	butt
2	butts	= 1	tun

Hay Weight.

56	pounds	= 1	truss of old hay
60	,,	= 1	,, new ,,
36	trusses	= 1	load

Straw Weight.

36	pounds	= 1	truss
11	cwt. 64 lb.	= 1	load
36	trusses	= 1	load

Butter and Cheese Weight.

8	pounds	= 1	clove
56	,,	= 1	firkin
84	,,	= 1	tub
112	,,	= 1	Dutch cask
224	,,	= 1	barrel
256	,,	= 1	Suffolk wey
336	,,	= 1	Essex ,,

Cotton Yarn Measure.

120	yards	= 1	skein
7	skeins	= 1	hank
18	hanks	= 1	spindle

Worsted Yarn Measure.

80	yards	= 1	skein
7	skeins	= 1	hank
144	hanks	= 1	gross

Wine Measure.

10	gallons	= 1	anker
18	,,	= 1	runlet
31½	,,	= 1	barrel
42	,,	= 1	tierce
63	,,	= 1	hogshead
84	,,	= 1	puncheon
2	hogsheads	= 1	pipe *or* butt
2	pipes	= 1	tun

Cereals are sold by weight, the **bushel** being reckoned as follows: Wheat—English, 60 lb.; Foreign, 62 lb. Barley—English, 50 lb.; French, 52½ lb.; Mediterranean, 50 lb. Oats—English, 39 lb.; Foreign, 38 and 40 lb. Rye and Maize, 60 lb. Buckwheat, 52 lb.

Fish.—Scotland and parts of England, 1 Cran = 37½ gall.; Ireland and Isle of Man, 1 Maze = 5 long hundreds of 126 each; East Coast of England, 1 Last = 13,200 fish, 1 long hundred = 132, 1 thousand = 1,320. Cured herrings are sold by the barrel, the capacity of which is 26⅔ gall.

Timber and **Wood.**—50 cub. ft. of planks = 1 load; the St. Petersburg standard consists of 165 cub. ft., or 120 pieces 1½″ × 11″ × 12′, or 120 pieces 3″ × 11″ × 6′. A cord of wood is 2½ tons or 128 cub. ft.

126. PAYMENT FOR GOODS.

Merchants and manufacturers usually print catalogues or trade lists; but unless the prices are marked net prices, persons engaged in the trade can make purchases for less than the gross prices which are there stated. This difference is known as **Trade Discount**, and is usually calculated as a percentage of the gross prices. This percentage varies considerably in different trades, and fluctuates from time to time according to a rise or fall in the cost of production or manufacture. The manufacturer varies his prices by altering the trade discount, keeping the price list the same. A merchant, in order to secure a large order, might offer a special increased rate of discount. Except in retail business to customers, it is customary to buy and sell goods on **credit**, that is to say, the buyer, instead of giving cash at once for his goods, undertakes to pay the amount at or within some definite fixed time in the future. When the buyer is ready to make payment before the expiration of the time agreed upon for payment, a **Cash Discount** is allowed him, which is usually calculated as a percentage of the invoice price (*i.e.*, the price after deducting trade discount), taking into consideration the time between the dates of payment and the date when the payment is due.

Prompt cash means payment within a day or two without discount.

Net cash means payment without discount in from five to ten days.

2 per cent. one month means payment in one month less 2 per cent. discount.

Prompt cash less 3 months means payment within a day or two, less three months' interest at 5 per cent. per annum.

A merchant might have several payments to make to the same firm or company, the payments being due at different dates. The date on which the total payment could be paid without loss to either side is known as the **equated time** or the **average due date**. This may be calculated by a simple rule, which is obtained as follows—

Let $P_1, P_2, P_3 \ldots \equiv$ amounts owed by the merchant,

,, $n_1, n_2, n_3 \ldots \equiv$ number of days after zero date when payable respectively,

,, $n \equiv$,, ,, total is payable,

,, $r \equiv$ rate of discount per cent. per annum.

Then the value of $P_1, P_2, P_3 \ldots$ on zero date

$$= P_1\left(1 - \frac{n_1 r}{36500}\right), P_2\left(1 - \frac{n_2 r}{36500}\right), P_3\left(1 - \frac{n_3 r}{36500}\right) \ldots \text{ respectively}$$

and the value of $(P_1 + P_2 + P_3 + \ldots)$ on zero date

$$= \left(P_1 + P_2 + P_3 + \ldots\right)\left(1 - \frac{nr}{36500}\right)$$

Now, the total amount should be paid on such a day that its value on the zero date should be the same as the sum of the values of the separate amounts on the zero date.

$$\text{Thus } \left(P_1+P_2+P_3+\ldots\right)\left(1-\frac{nr}{36500}\right) = P_1\left(1-\frac{n_1r}{36500}\right)+P_2\left(1-\frac{n_2r}{36500}\right)$$
$$+ P_3\left(1-\frac{n_3r}{36500}\right)+\ldots$$

$$\therefore -\frac{nr}{36500}\left(P_1+P_2+P_3+\ldots\right) = -\frac{r}{36500}\left(P_1n_1+P_2n_2+P_3n_3+\ldots\right)$$

by removing brackets, and taking $(P_1+P_2+P_3+\ldots)$ from each side.

$$\therefore \quad n(P_1+P_2+P_3+\ldots) = P_1n_1+P_2n_2+P_3n_3+\ldots$$

$$\therefore \quad n = \frac{P_1n_1+P_2n_2+P_3n_3+\ldots}{P_1+P_2+P_3+\ldots}$$

It should be noted that the average due date is independent of the rate per cent. per annum, but the latter must be the same in the case of all the separate payments, for, if not, the above rule does not hold good.

EXAMPLE (i)—

A retailer bought 1 barrel of butter, the list price being £16 15s He was allowed $7\frac{1}{2}$ per cent. trade discount and $2\frac{1}{2}$ per cent. one month. What was his gross profit if he sold all the butter at 1s. 10d. per lb. ?

		£	s.	d.
Trade discount = £$16\frac{3}{4} \times \frac{3}{40}$	=	1	5	1
Price, less trade discount	=	15	9	11
Cash discount = $\frac{1}{40}$ of £15 9s. 11d.	=		7	9
Price paid by retailer	=	15	2	2
Total selling price = 224 × 1s. 10d.	=	20	10	8
Gross profit	=	5	8	6

NOTE 1.—Discounts of $2\frac{1}{2}$ per cent., 5 per cent., $7\frac{1}{2}$ per cent , 10 per cent., $12\frac{1}{2}$ per cent., etc., may be mentally reckoned, if it be remembered that they are 6d., 1s., 1s. 6d., 2s., 2s. 6d., etc., in the £ respectively. In the case of large payments, discount is sometimes based on the whole number of £.

EXAMPLE (ii)—

In a manufacturer's price list an article is listed at 55 guineas. The manufacturer allows to the retailer a trade discount of 25 per cent. and 6 months' credit. He wishes to change his system to 3 months' credit and a trade discount of 20 per cent. Reckoning interest at 4 per cent. per annum, what would be the equivalent price in the new price list ?

Retailer's price, prompt cash = £$57\frac{3}{4} \times \frac{3}{4} \times \frac{98}{100}$

Let £x ≡ equivalent price in new price list

then retailer's price, prompt cash = £$x \times \frac{4}{5} \times \frac{99}{100}$

As the price for prompt cash should be the same in each case, it follows that $x \times \frac{4}{5} \times \frac{99}{100} = \frac{231}{4} \times \frac{3}{4} \times \frac{98}{100}$

$$\therefore x = \frac{231 \times 3 \times 98 \times 5}{4 \times 4 \times 4 \times 99} = 53\tfrac{95}{160}$$

$\therefore$ New List Price = £53 11s. $10\frac{1}{2}$d.

EXAMPLE (iii)—

A dealer bought a piano for £37 15s. to be paid for in 1 month. He sold it immediately for £45, payable in 3 months. Reckoning discount at $4\frac{1}{2}$ per cent. per annum, find his gain per cent. on the cost price.

Cost price, deducting disct. for 1 mon. at $4\frac{1}{2}$% per annum $= £37\frac{3}{4} \times \frac{99\frac{5}{8}}{100}$

Selling „ „ „ 3 mon. „ „ $= £45 \times \frac{98\frac{7}{8}}{100}$

$$\therefore \text{ Profit} = £\frac{45 \times 791}{800} - £\frac{151 \times 797}{4 \times 800} = £\frac{22033}{4 \times 800}$$

$$\therefore \text{ Percentage gain on cost price} = \frac{22033 \times 4 \times 800 \times 100}{4 \times 800 \times 151 \times 797}$$

$$= 18{\cdot}3\%.$$

EXAMPLE (iv)—

A retailer owes the following amounts to a wholesaler: £175 payable on 15th May, £383 10s. payable on 19th June, and £258 15s. payable on 7th July. On what day should the total amount, viz., £817 5s., be paid, in order that the debts should be completely cleared?

Let the zero date be 15th May.

£175 is payable 0 days after zero date.

$£383\frac{1}{2}$ „ 35 „ „

$£258\frac{3}{4}$ „ 53 „ „

$\therefore$ number of days after zero date $£817\frac{1}{4}$ is payable

$$= \frac{175 \times 0 + 383\frac{1}{2} \times 35 + 258\frac{3}{4} \times 53}{817\frac{1}{4}}$$

$$= \frac{767 \times 70 + 1035 \times 53}{3269}$$

$= 33$ to nearest day

Thus £817 5s. paid on 17th June completely clears the debts.

EXAMPLE (v)—

How much per cent. must the list price of an article be greater than the cost of manufacture, such that by selling at a trade discount of 15 per cent. and allowing a cash discount of $2\frac{1}{2}$ per cent., the manufacturer would make a profit of $7\frac{1}{2}$ per cent. on the cost of manufacture. What percentage of the manufacturer's selling price would this profit be?

Let $x \equiv$ the percentage excess of list over the cost to manufacture, then if the cost to manufacture = 100 units,

the list price $= 100 + x$ units;

then net selling price $= (100 + x) \times \frac{85}{100} \times \frac{97\frac{1}{2}}{100}$

so that $(100 + x) \times \frac{85}{100} \times \frac{195}{200} = 107\frac{1}{2}$

$$\therefore 100 + x = \frac{215 \times 100 \times 200}{2 \times 85 \times 195}$$

$$= 129{\cdot}7$$

$$\therefore x = 29{\cdot}7$$

Percentage profit on manufacturer's selling price $= \frac{29{\cdot}7 \times 100}{107\frac{1}{2}}$

$= 27{\cdot}6$

Ansr. = 29·7%; 27·6%.

127. PERCENTAGE GAINS AND LOSSES.

In order to minimize the amount of arithmetical work in the solution of certain types of problems, the following generalizations are sometimes of considerable use—

I. When *all* the goods bought at a certain amount per unit by a dealer are sold by him at a uniform price per unit, the percentage gain or loss is independent of the quantity of goods thus bought and sold.

Proof.—Let £$x \equiv$ cost price per unit and £$y \equiv$ selling price per unit.

Let a units $\equiv$ quantity of goods bought.

Then percentage gain on cost price $= \dfrac{ay - ax}{ax} \times 100 = \dfrac{a(y - x)}{ax} \times 100$

$= \dfrac{(y-x) \times 100}{x}$

= percentage gain by buying and selling 1 unit.

Similarly $\dfrac{(x-y)}{x} \times 100$ = percentage loss by buying and selling 1 unit.

Similarly, $\dfrac{(y-x) \times 100}{y}$ or $\dfrac{(x-y) \times 100}{y}$

= percentage gain or loss respectively on selling price, whatever quantity be bought and sold.

EXAMPLE (vi)—

A retailer bought 5 cwt. 3 qr. 16 lb. of butter at £7 13s. 9d. per cwt. and sold it at 1s. 10½d. per lb. Find the percentage gain based (1) on cost price, (2) on selling price.

Selling price of 1 cwt. of butter $= 1\frac{7}{8} \times 112$ shillings

$= 210$ „

∴ Gain by selling 1 cwt. of butter $= 56\frac{1}{4}$ „

∴ Percentage gain on cost price $= \dfrac{56\frac{1}{4} \times 100}{153\frac{3}{4}} = 36{\cdot}6$ approx.

and „ „ selling „ $= \dfrac{225 \times 100}{4 \times 210} = 26{\cdot}8$ „

Ansr.—36·6%; 26·8%.

II. If $a_1, a_2, a_3 \ldots$ be the cost prices of goods respectively, and $r_1, r_2, r_3 \ldots$ be the percentage gains on the cost prices respectively, then the percentage gain on the total outlay is $\dfrac{a_1r_1 + a_2r_2 + a_3r_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}$

Proof.— Total gain $= \dfrac{a_1r_1}{100} + \dfrac{a_2r_2}{100} + \dfrac{a_3r_3}{100} + \ldots$

$= \dfrac{a_1r_1 + a_2r_2 + a_3r_3 + \ldots}{100}$

$$\therefore \text{ Percentage gain on total cost} = \frac{a_1r_1 + a_2r_2 + a_3r_3 + \ldots}{100(a_1 + a_2 + a_3 + \ldots)} \times 100$$

$$= \frac{a_1r_1 + a_2r_2 + a_3r_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}$$

If $b_1, b_2, b_3 \ldots$ be the selling prices respectively and $s_1, s_2, s_3 \ldots$ be the percentage gains on the selling prices respectively,

then, similarly, percentage gain on total selling price $= \dfrac{b_1s_1 + b_2s_2 + b_3s_3 + \ldots}{b_1 + b_2 + b_3 + \ldots}$

If, say, the goods bought for a_2 units had been sold at a loss of r_2 per cent. of the cost price, then

$$\text{percentage gain on total cost} = \frac{a_1r_1 - a_2r_2 + a_3r_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}$$

$$\text{or} \quad \text{,, loss ,,} = \frac{-a_1r_1 + a_2r_2 - a_3r_3 + \ldots}{a_1 + a_2 + a_3}$$

EXAMPLE (vii)—

A dealer bought a certain number of barrels of flour at a uniform price: he sold one-half the quantity, thereby gaining 8 per cent. on cost price; one-third, thereby gaining 10 per cent. on cost price; and the remainder, thereby gaining 12 per cent. What percentage was the total gain of the total cost price ?

Let 1 unit $\equiv$ total cost of flour.

$$\text{The percentage gain on total cost} = \frac{\frac{1}{2} \times 8 + \frac{1}{3} \times 10 + \frac{1}{6} \times 12}{1}$$

$$= \underline{\underline{9\tfrac{1}{3}}}$$

EXAMPLE (viii)—

A dealer sold three pianos for 30, 45, 50 guineas respectively, thereby gaining 5 per cent. on the total selling price. He lost 4 per cent. by the sale of the first and gained $7\frac{1}{5}$ per cent. by selling the second, the percentages being based on selling prices. What percentage gain did he obtain by selling the third ?

Let $r \equiv$ percentage gain by selling the third piano, based on selling price,

$$\text{then} \quad \frac{-30 \times 4 + 45 \times 7\frac{1}{5} + 50r}{125} = 5$$

$$\therefore \; -120 + 324 + 50r = 625$$

$$\therefore \; 50r = 625 + 120 - 324$$

$$\therefore \; r = \underline{\underline{8{\cdot}42.}}$$

III. If goods bought for a units are sold for b units and the percentage gain on cost price be r, then $r \times \dfrac{a}{b}$ is the percentage gain on the selling price.

Proof.—Gain per cent. on selling price $= \dfrac{(b-a) \times 100}{b}$

$$\text{but } \frac{(b-a) \times 100}{a} = r, \text{ i.e., } (b-a) \times 100 = ar$$

$$\therefore \text{ Gain per cent. on selling price} = \frac{ar}{b}$$

Similarly, if $s \equiv$ percentage gain on selling price,

then $\dfrac{bs}{a} =$,, ,, cost ,,

Example (ix)—

A puncheon of wine is bought and sold and the gain was 15 per cent. on cost price. What percentage gain is this on the selling price?

If 100 units be the cost price, then the selling price is 115 units;
then percentage gain on selling price $= 15 \times \frac{100}{115} = 13\frac{1}{23}$

Answr—$13\frac{1}{23}\%$.

IV. (*a*) If the cost price and selling price of an article be each increased by i per cent, the percentage gain will be unaltered; but the actual gain will be increased by i per cent.

Proof—Let a and b units be the cost and selling price respectively, then $b - a \equiv$ number of units of money gained

and $\dfrac{(b-a) \times 100}{a} \equiv$ percentage gain on cost price.

Also $\dfrac{a(100+i)}{100}$ and $\dfrac{b(100+i)}{100}$ units are the cost and selling prices after increase, then actual gain $= \dfrac{(100+i)}{100} \times (b-a) =$ original gain increased by i per cent. and percentage gain $= \dfrac{\dfrac{(100+i)(b-a)}{100}}{\dfrac{a(100+i)}{100}} \times 100 = \dfrac{(b-a) \times 100}{a}$

= original percentage gain. Similarly, the percentage gain on selling price is unaltered.

Had cost and selling prices been reduced by d per cent., it is easily seen that the actual gain $= \dfrac{100-d}{100} \times (b-a) =$ original gain decreased by d per cent.

(*b*) If the cost price and selling price be each increased by the same amount, the actual gain will be unaltered, but the percentage gain on cost price will be reduced in the ratio of the original cost price to the increased cost price, and the percentage gain on selling price will be reduced in the ratio of the original selling price to the increased selling price.

Proof.—Let a, b units $\equiv$ original cost and selling prices respectively,
" x units $\equiv$ increase in price of each;
then actual gain after increase $= b + x - (a + x)$ units
$= b - a$ units = original gain.

Let $r_1, r_2 \equiv$ percentage gains on cost price before and after increase respectively;

then $r_1 = \dfrac{(b-a) \times 100}{a}$ and $r_2 = \dfrac{(b-a) \times 100}{a+x}$

$\therefore \dfrac{r_2}{r_1} = \dfrac{a}{a+x}$ i.e., $r_2 = r_1 \times \dfrac{a}{a+x}$
= original percentage gain on cost price reduced in ratio $a:(a+x)$

Similarly, if the cost and selling price be each reduced by y units, the gain per cent. on cost price will be increased in the ratio $a:(a-y)$.

Similarly, the percentage gain on selling price will be reduced in ratio $b:(b+x)$ or increased in ratio $b:(b-y)$, according as the original prices are increased by x units or diminished by y units.

EXAMPLE (x)—

A man bought goods for £85 and sold them, gaining 10 per cent. on selling price. The cost price is now increased by 10 per cent., and he increased his selling price also by 10 per cent. What will now be the gain obtained by selling the article?

$$\text{Gain, before increase} = £85 \times \left(\tfrac{100}{90} - 1\right) = £\tfrac{85}{9}$$

$$\therefore \text{Gain, after} \quad ,, \quad = £\tfrac{85}{9} \times \tfrac{110}{100} = £10\tfrac{7}{18}$$

Ansr.—£10 7s. 9d.

EXAMPLE (xi)—

A greengrocer bought potatoes at £1 1s per sack and sold them, gaining $16\frac{2}{3}$ per cent. on his outlay. Later he found the price had increased by 3s. 6d. per sack, and he increased his selling price by $\frac{1}{4}$d. per lb. Find the gain per cent. on his outlay?

£1 1s. per 168 lb $= 1\frac{1}{2}$d per lb.
Increase of cost price $= \frac{1}{4}$d. ,,

$$\therefore \text{percentage gain on outlay} = 16\tfrac{2}{3} \times \frac{1\frac{1}{2}}{1\frac{3}{4}} = 14\tfrac{2}{7}$$

Ansr.—$14\frac{2}{7}$%.

V. Keeping the expenditure the same, if the price per unit of any commodity be increased by r per cent., the quantity that can be bought is reduced in the ratio $100:(100+r)$, that is, by $\dfrac{100r}{100+r}$ %.

Proof.—Let $£x \equiv$ cost per unit,
,, $n \equiv$ number of units of the commodity bought;
then $£nx =$ cost of n units at original price.

Now $£\dfrac{x(100+r)}{100} =$ cost per unit after increase

$\therefore \; nx \times \dfrac{100}{x(100+r)} =$ number of units bought after increase of price,

i.e., $n \times \dfrac{100}{100+r}$ or n decreased in ratio $100:(100+r)$ is the quantity that can be bought after the increase in price.

Now on $100 + r$ units the reduction is r units

$\therefore$,, 100 ,, ,, $\dfrac{100r}{100+r}$ units,

thus the quantity bought is decreased by $\dfrac{100r}{100+r}$ %.

Similarly, if the price per unit had been decreased by r%, the quantity bought for the same money would be increased in the ratio $100:(100-r)$, that is by $\dfrac{100r}{100-r}$ %.

EXAMPLE (xii)—

If the price per ton of coal be increased by 10 per cent., by what percentage must the quantity bought be reduced so that the expenditure would be the same? If after this reduction the price be reduced by 10 per cent., by what per cent. would the quantity bought be greater than that bought

after the former increase of price? Also find how much per cent. the latter quantity is greater than the original quantity?

Percentage reduction in quantity bought after rise in price of 10% $= \dfrac{100 \times 10}{110} = \underline{\underline{9\frac{1}{11}}}$

Percentage increase of the above quantity after reduction in price of 10% $= \dfrac{100 \times 10}{90} = \underline{\underline{11\frac{1}{9}}}$

∴ Let 100 units be the quantity bought originally,
then $100 \times \frac{100}{110} \times \frac{100}{90}$ units is the quantity bought after the rise and fall of price.

∴ Percentage increase $= 101\frac{1}{99} - 100 = \underline{\underline{1\frac{1}{99}}}$.

TEST EXERCISES III, 1.

(1) From the following particulars calculate the percentage gain (i) on the cost price, (ii) on the selling price.

Piano	£45 18s.	60 guineas
Book-case. . . .	£19 12s. 6d.	22½ ,,
Drawing-room Suite . .	£21 15s. 9d.	25 ,,

(2) If 5 per cent. cash discount were deducted from the above selling prices, what would be the percentage gains then? (Derive the answers from those of Question (1).)

(3) At the beginning of a month a coal merchant had in stock 175 tons of coal which he had bought at an average cost of 19s. 4d. per ton. During the month he bought 437 tons at 20s. 9d. per ton and sold 528 tons at 27s. 6d. per ton. Reckoning stock at the end of the month at 20s. 9d. per ton, find his gross profit for the month?

(4) A tradesman stated that his profit for the year was 15 per cent. on the selling price of his goods. What was his gain per cent. on the cost price?

(5) A merchant's terms were 20 per cent. trade discount, net cash off list prices, but he makes an advance of 10 per cent. on these cash prices. Under this new arrangement a customer pays £19 8s. 8d. for goods purchased from the merchant. What were the goods listed at and what percentage discount from the list prices is now allowed? Also if the list prices are 25 per cent. above the cost prices to the merchant, what percentage gain does this arrangement give the latter on the cost price?

(6) A motor-car was bought for 350 guineas and after a few months was sold second-hand for £275. What was the percentage depreciation?

(7) A merchant increased his prices by 5 per cent. By what percentage should he decrease these prices so as to return to the original prices?

(*N.B.*—Decrease on 105 units of money is 5 units of money: find decrease on 100 units.)

(8) A dealer altered his trade discounts from 15 per cent. to 10 per cent. By what percentage were the selling prices increased, the percentage cash discounts remaining the same as before?

(9) A retailer bought 6 casks of butter, the average gross weight being 1 cwt. 1 qr. 12 lb. and the average tare 16½ lb. The list price was 195s. per cwt., the trade discount 5 per cent., and the cash discount 2½ per cent. What was actually paid for the butter? If the retailer sold it at 2s. 4d. per lb., what was the extent of his gain?

(10) A man bought a last of herrings for £83 15s. He sold 150 doz. at 1s. 9d. per dozen, 350 doz. at 1s. 9½d. per dozen, 280 doz. at 1s. 10d. per dozen in each case, allowing 5 per cent. trade discount, and he sold the remainder at 1s. 7½d. net. How much did he gain in all?

(11) The list price per barrel of wine was £4 15s. A man bought

4 barrels, and was allowed a trade discount of 12½ per cent. and a cash discount of 1½ per cent. How much did he pay?

(12) A dealer was offered new hay at 15s. 6d. per truss carriage forward, or at 17s. 9d. per truss carriage paid. The cash discount was 5 per cent. in each case, and the carriage was at 2s. 3d. per truss. What would be his advantage in accepting the latter terms if he purchased 1¾ loads?

(13) A dealer bought furniture from a manufacturer, who allowed him 10 per cent. trade discount and 4 months' credit at 5 per cent. per annum. He made out a price list by increasing the manufacturer's invoice price by 50 per cent. In selling to retailers, he allowed 20 per cent. off his list prices and 3 months' credit at 5 per cent. per annum. Find in the following columns—

	Manufacturer's List Price.	Dealer's List Price.	Dealer's Buying Price for Cash.	Dealer's Selling Price for Cash.
Piano . . .	£85			
Suite . . .	£28			
Sideboard . .	£35			
Cabinet . . .	£40			

(14) Find the cost of 445 ton 7 cwt. 70 lb. of pig iron at £4 3s. 10½d. per ton.

(15) A merchant's terms were 20 per cent. discount off list prices for cash, but he makes a reduction of 10 per cent. on these cash prices. Under this new arrangement a customer pays £42 12s. for goods purchased from the merchant. What were the goods listed at and what percentage discount from the list prices is now allowed?

(16) Goods were bought for £56 with 4 months' credit by a dealer, who sold them immediately for £62 18s. If the dealer wished to gain at least 12½ per cent. profit on the cost, find the greatest number of days' credit he could give, reckoning discount at 5 per cent. per annum in each case?

(17) A retailer gets 33⅓ per cent. trade discount off the prices in a manufacturer's list, and is allowed 5 per cent. discount for cash. He wished to price his articles in such a way that, after allowing his customers 7½ per cent. discount for cash, he will still make a profit of 15 per cent. on what the goods cost him. Find by how much per cent. the prices he marks the articles are greater or less than the manufacturer's list prices.

(18) Certain goods cost a manufacturer £725 to make. His list price was 25 per cent. above this, and his trade discount was 15 per cent. and cash discount 2 per cent. A dealer bought the goods under these terms and made his list price 20 per cent. above the actual price he gave. A retailer bought the goods, being allowed 10 per cent. trade discount and 1½ per cent. cash discount. If the retailer could have bought the goods, including cost of carriage, direct from the manufacturer at 12½ per cent. discount off list price for cash, what amount would he have saved by so doing?

(19) The gross weight of certain goods was 9 ton 13 cwt. 2 qr. 50 lb. and the tare was 17 cwt. 3 qr. 12 lb. A merchant offered to sell at £2 18s. 9d. per ton net cash, carriage forward. What should he charge per ton net cash, carriage paid, if the carriage be 17s. 6d. per ton?

(*N.B.*—The increase in price of 1 ton of the goods due to carriage is 17s. 6d. increased in the ratio of net weight to gross weight.)

(20) A merchant sold goods to the value of £750, £345 10s., and £672 10s. to Thomas Goode, the accounts to be paid on 4th June, 10th July, and 25th August respectively. Find on what date the total sum should be paid, in order to settle the debts: hence find what sum Thomas Goode should pay on 1st June to completely clear his accounts with the merchant, reckoning discount at 5 per cent. per annum?

(21) X bought goods from Y on 5th May and 13th July, the invoice prices being £425 and £265 respectively, and in each case 3 months' credit was allowed. On what day must he pay the total, in order to settle the accounts ?

(22) Find the equated time of paying the total of the following account—

Dr.

Jan. 9. 1932 . . .	Goods,	£245;	credit	3 months
„ 27, 1932 . . .	„	£312;	„	2 „
Mar. 16, 1932 . . .	„	£107 10s.;	„	3 „

Reckoning discount at 5 per cent. per annum, what should be paid on 16th March, 1932, so as to complete the total payment?

(23) A retailer bought 3 cwt. 2 qr. 18 lb. of coffee at £6 11s. 9d. per cwt. and 7 cwt. 1 qr. 24 lb. of tea at £10 14s. 6d. per cwt. He sold all the tea at 3s. 2d. per lb. and all the coffee at 1s. 9d. per lb. Find the percentage gain, based on cost price, (i) by selling the coffee, (ii) by selling the tea, (iii) by selling the tea and the coffee.

(24) A dealer bought a quantity of rice at £1 15s. per cwt. He sold one-third of the quantity at 4½d. per lb., one-quarter of the quantity at 2½d. per lb. At what price per lb. must he sell the remainder so as to gain at least 12 per cent. on cost price after selling all the rice ?

(25) A dealer purchased a quantity of wheat at a certain price per quarter. After a week he had sold 44 per cent. of the quantity, gaining 12 per cent. on cost price; during the next week he sold 53⅓ per cent. of the remainder, gaining 9 per cent. on cost price. The following week he sold the remaining wheat at a loss, such that the percentage gain in selling all the wheat was 6 per cent. on cost price. What was the loss per cent. on cost price incurred by the sale of wheat during the third week ?

(26) After selling 85 per cent. of a certain commodity at a fixed price per unit, a man had received 5 per cent. more money than he had paid for the total quantity. Assuming he sold the remainder at the same fixed price, what was the percentage gain based on selling price ?

(27) A tobacco manufacturer sold tobacco at 7s. 4d. per lb., thereby gaining 10 per cent. on total cost. The retailer sold it at 7d. an oz. Owing to increase in duty, the cost to the manufacturer increases by 11d. per lb. If the latter were to sell so as to gain the same percentage profit as before, and if the retailer raised his selling price also to gain the same percentage profit as before, what would be the increased retail price per ounce ?

(28) A retailer bought certain articles for 2s. 9d. each and sold them for 3s. 6d. After a time the cost price is reduced 3d., and he lowers the retail price 3d. By what per cent. must the average weekly number of articles bought and sold be increased so that the average weekly profit should be increased by 20 per cent. ?

(29) During a certain year a manufacturer purchased 276 tons of coal at a contract price of 21s. 6d. per ton. For the next year the coal merchant increased his price by 15 per cent. If the manufacturer's outlay for coal was the same as before, how many less tons could he buy ? If he had purchased 225 tons at the increased price, by what per cent. would his outlay on coal be increased or decreased ?

(30) James White & Co., Bedford, purchased from William Turner & Son the following: 4 pcs. Calico (T 179), 395 yd. at 3$\frac{7}{16}$d.; 3 pcs. 42 in., Long Cloth (F 23), 75½ yd. at 11¼d.; 2 pcs. 30 in. Flannel (B 57), 128 yd. at 9¼d.; 2 pcs. 33 in. Flannel (B 59), 127 yd. at 10d.; 5 pcs. Flannelette (C 39), 302 yd. at 4¾d. Make out the invoice, dated 19th December, 19..; 7½ per cent. trade discount, carriage paid; and terms 1¼ per cent. discount for cash. Calculate the net amount for cash.

(31) The Truefit Boot Co., Nottingham, forwarded the following to Abel & Son, London, on 30th November, 19..: 12 prs. Boots (A 713) at 8s. 6d. pair; 8 prs. Boots (A 717) at 9s. 3d. pair; 18 prs. Boots (A 723) at 11s. 6d. pair; 20 prs. Shoes (D 25) at 7s. 9d. pair; 30 prs. Shoes (D 78) at 6s. 10½d.

pair. The trade price was 5 per cent. above list prices, and the terms were prompt cash less 3 months. Make out the invoice and calculate the net total.

(32) In the above question the shoes (D 78) should have been 6s. 4½d. Abel & Son wrote pointing this out, and the Boot Company forwarded a Credit Note. Make out the Credit Note in the proper form, stating the amount that Abel & Son have been overcharged. What would the net total now amount to?

(33) S Roberts & Son forwarded the following to Messrs. Howard & Atkins: 28 lb. Digestive Biscuits at 9½d. per lb.; 14 lb. Gingernuts at 8¼d. lb.; 10½ lb. Shortbread at 10¼d. lb.; 2 cwt. 8 lb. Genoa Cake at 6½d. lb. A trade discount of 5 per cent. was allowed, and 3 tins at 6d and 1 tin at 9d. was charged Prepare an invoice. Messrs Howard & Atkins returned the tins and were credited with the amount. Make out the Credit Note and find the net total if 1½ per cent. discount was deducted for prompt cash.

(34) Goods were delivered by Messrs. Bond, Trott & Co., of London, to R. Fox & Sons, Tonbridge, as follows—

4th November, £71 15s 4d; 7th November, £185 3s 8d, 11th November, £57 14s 2d; 15th November, £78 11s. 7d.; 19th November, £114 3s. 11d.; 23rd November, £39 4s. 9d.

Messrs Bond, Trott & Co sent Credit Notes for return of empties, etc., as follows—

15th November, £2 3s. 8d.; 21st November, £1 11s 9d.

Make out the statement for the end of the month, and calculate the net amount to be paid, the terms being 1½ per cent. one month.

(35) Make out a statement rendered by Brand & Sons, of Birmingham, on 30th April for the following transactions in February with William Kemp, of Bristol: Goods delivered on the 5th, £73 9s. 10d., on the 10th, £29 4s. 3d.; on the 17th, £31 3s. 6d.; on the 24th, £17 4s 5d.; on the 26th, £35 3s. 8d.; goods returned by Kemp on the 18th, £7 3s. 11d.; cash received on account on the 28th, £75. Allow 1¼ per cent discount on the net goods supplied.

(36) Make out an invoice for the following goods dispatched by N Harris & Co. to J. Higgins & Son: 3 quires P.O.P. at 18s per quire, discount 16⅔ per cent.; 1½ doz. 15-gr tubes Chloride of Gold at 27s. 6d per doz net; 2 casks Carbonate of Soda at 35s. per cwt. discount, 10 per cent.; No. 1, 1 cwt. 2 qr. 8 lb., tare 15 lb.; No. 2, 1 cwt. 2 qr. 10 lb., tare 17 lb.: 2 Casks at 2s. 6d; 1 Hamper at 4s. 6d. J. Higgins & Son returned the casks and hampers, and cash discount at 1¾ per cent. was allowed. What was the net amount paid for the goods?

CHAPTER XV.

IMPORT TRADE.

128. METHODS OF IMPORTING GOODS.

THERE are a variety of ways by which commodities are imported into this country, the transactions differing in detail according to the class of commodity imported. In general, there are two fundamental methods : (1) Commodities imported on **Consignment** and (2) importation by **Firm Contracts.**

By the first method, the commodities are forwarded by the consignor, that is, the foreign exporter, to an agent in this country, the function of the latter being to receive the goods on arrival at a home port and to effect a sale in this country. This he may do himself, or by employing a broker who may or may not sell the goods by auction in one of the great exchanges. After deducting charges, which may include dock charges, insurance, warehousing, cartage for delivery, brokerage (if any), freight (if not already paid by the consignor), and his own commission from the amount received by selling the goods, the agent remits the net proceeds to the consignor, or carries the amount to the latter's credit in the case of a periodic settlement. Particulars concerning the sale and the necessary charges are made out on an **Account Sales,** which is forwarded to the foreign exporter. Thus the goods are brought into the country before a sale is arranged, and the importer does not buy the goods himself, for he simply renders service to the consignor, for which he receives a percentage commission based on the amount for which the goods are sold.

By the second method, a merchant in this country either makes an offer to the foreign merchant which is accepted, or accepts an offer made to him by the latter. In some cases an agent is employed who simply brings the two merchants into communication one with the other. The sale having been arranged, the exporter may employ a forwarding agent to see to the placing of the goods on board ship, or he may see to this himself. On the arrival of the goods at a home port, the importer may employ a

Telegraphic Cypher..........

No....1778......

INVOICE of 15 *cases Pongees* shipped per ss. "*Novara*" to *London* on account and risk of the Concerned, consigned to *Messrs. Brockwell & Jeffrey, London.*

Financed by our drafts on them against documents through ....*Hongkong & Shanghai Banking Corporation.*........

		£	s.	d	£	s.	d.
B J. 19/33	15 cases containing 300 pieces Ninghai Pongees No. 183 33/4″ 48/50 yds. 95/100 ozs. @	106/-			1,590	-	
	Balance of 30*th Nov.*, 19..						
	Gross weight 177½ lb. Net weight 122½ lb.						
	„ 178½ „ „ 124 „						
	„ 177½ „ „ 124 „						
	„ 177½ „ „ 125½ „						
	„ 180 „ „ 125½ „						
	„ 180 „ „ 125½ „						
	„ 180 „ „ 124 „						
	„ 178½ „ „ 125 „						
	„ 176 „ „ 124 „						
	„ 176 „ „ 122½ „						
	„ 178½ „ „ 126½ „						
	„ 177½ „ „ 124 „						
	„ 176 „ „ 122½ „						
	„ 180 „ „ 124 „						
	„ 174½ „ „ 121½ „						
	Insured with Phoenix Assurance Co., Ltd., for	£1,800	-	-			
	Drawn through Hongkong & Shanghai Banking Corporation for £1,590 @ 4 m/s on ...*Messrs. Brockwell & Jeffrey,*... ....*London.*....						
	(Shipment Sample No. D 065.) E. & O. E. CHEFOO, ...*7th Dec.*, 19..						

Yamasaki & Co.

forwarding agent to deal with the work in connection with the transference of the goods from the docks to the former's warehouse, or this may be done by a department of the merchant's business. Thus the goods are brought into the country after a sale has been arranged; and the merchant at home is the importer, as the goods are shipped to his order. In arranging the sale, details concerning payment, quality of goods, and terms of delivery should be made quite clear. This question is fully dealt with in the next chapter.

A modification of the second method is brought about by the authorizing by the merchant at home of an agent abroad to purchase for him certain commodities in one of the foreign exchanges and to forward the same. In this case, the merchant would probably state a maximum price c.i.f. (*i.e.*, price including cost, insurance, and freight) and the agent would buy at a lower price, so that after paying the various charges necessary he would still be able to make a fair profit for himself. As before, the merchant is the importer; and arrangements for the sale of the goods are made before importation.

129. A *brief outline* of an *importing operation* carried out by the second method is described as follows—

(1) A silk importer of London arranged to purchase from a firm of silk exporters of Chefoo, China, a number of pieces of Ninghai Pongees (as per sample) at 106s. per piece of from 48 to 50 yards c.i.f. After effecting insurance of the silk for £1,800 and arranging for the shipment, the invoice on page 255 was made out and a copy forwarded to the English firm.

(2) When the silk was placed on board the ss. *Novara*, the captain, mate, or purser signed three copies of the *Bill of Lading* and these were indorsed "*per pro.*" by the exporters. One copy was retained on the ship and two were placed in the possession of the latter firm, who then drew a bill of exchange in duplicate on the importer and *advised* him as shown on page 257.

(3) The exporter then *negotiated* the bill of exchange through the Hongkong and Shanghai Banking Corporation; that is to say, the firm handed over to the Bank copies (each in duplicate) of the invoice, bill of exchange, bill of lading, and insurance certificate, and received an amount of money in dollars on the £1,590 at a rate of exchange quoted by the banker. Having received

CHEFOO, *7th December*, 19..

Messrs. Brockwell & Jeffrey,

London.

DEAR SIRS,—

We beg to advise having drawn on your goodselves the undermentioned Draft at *Four Months'* sight against shipments of 15 *cases Pongees B.J.* 19/33 per ss. *"Novara" to London* for account and risk of *your goodselves.*

Recommending our signature to your kind protection,

We are, Dear Sirs,

Yours faithfully,

Yamasaki & Co.

No.	Draft Date.	Amount.	Negotiated through	Date & No. of Credit.
P. 64	7/12/19..	£1,590 : 0 . 0	Hongkong & Shanghai Banking Corporation	

payment, the transactions were ended as far as the exporters were concerned, and the bank which paid the money now held the sets of documents or *Collaterals* and thus held the goods as security.

(4) The banker sent, by different mails to avoid loss, the two sets of documents to the London branch of the Hongkong & Shanghai Banking Corporation, and the first to arrive was then sent to the office of the importer for the firm to *accept* the bill of exchange, which was payable four months after sight (*i.e.*, four months and three days after the date on which the firm first saw the bill). The bill was accepted and the documents returned to the London branch of the Bank, as the goods were sold *D.P.* (i.e., *Documents against Payment*). Had the terms of sale been *D.A.* (i.e., *Documents against Acceptance*), the documents could

have been retained by the firm after the bill had been accepted, and the bill of exchange only returned to the bank.

(5) Meanwhile the goods were transferred at Shanghai to ss. *Novara*, which proceeded to England. Before arrival at the London Docks, the importer notified the Port of London Authority that silk in thirty-two cases marked B.J. 19/50 had been shipped to him on the ss. *Novara*. These cases included the fifteen marked "B.J. 19/33," as per invoice. The Port of London Authority received the goods and prepared a **Landing Note** and a **Weight Note** (sometimes these notes are combined in one document, as in this particular case, see next page). The Customs Officers examined the cases, after which they were placed in Bond until the Import Duty was paid.

(6) In order to obtain the goods, the importer **retired the bill**; that is, paid the amount of the bill of exchange less rebate to the London branch of the Hong Kong & Shanghai Banking Corporation and, in return, received the documents. On producing the bill of lading (duly indorsed) before the steamship company, the latter, as the freight had already been paid, **released** the bill of lading; that is, issued to the importer a **Delivery Order**, without which he could not have obtained the goods.

(7) After paying Import Duty and the charges due to the Dock Authority and presenting the Delivery Order before the Master Porter, the importer gave instructions to the latter to cart the silk to his warehouse, and this was accomplished.

130. The **expenses incurred** from the time a ship arrives in port to the time of delivery of the goods in the importer's warehouse include: (1) Landing dues or wharfage; (2) cost of opening cases, bags or barrels, etc., for inspection of the goods by the officers of Customs, and then repacking; (3) delivery, that is, the total cost of moving the goods from the time of the landing to the time of placing on carts, barges, or trucks for the purpose of conveying the goods to the importer's warehouse; (4) cost of warehousing (if any); (5) cost of insurance against fire while the goods are in the charge of the dock authority; (6) import duty (if any); (7) cartage or other transport of the goods from the docks to their destination.

[FRONT]

PORT OF LONDON AUTHORITY.

_____________________*Dock.*

_____________________*Department.*

____ *27th Feb.*, 19.. ______

LANDING ACCOUNT

Of 32 *Cases Silks*____________________

Entered by____*Port of London Authority*____

on the 15*th Feby.*, 19..__________

Per the Ship____*Novara*______

Master________________ Rotn. No. 17 1—102

from____ *Yokohama, etc.*___

Rent commences on the___15*th Feby.*, 19..___

J. B. 19/50

Dock Nos.	Quantity	Total Landing Weight Cwt.	qrs.	lbs.	
	32	50	1	10	Gross
					Tare
					Net

——*Wm. Brown*,——Clerk.

Examined——

[BACK]

Original No.	Dock No.	Gross Landing Weight cwt	qrs	lbs	Mints.			Remarks
	19	1	2	6				
	20	1	2	8				
	1	1	2	7				Coopered
	2	1	2	7				
	3	1	2	9				
	4	1	2	8				
	5	1	2	5				
	6	1	2	8				
	7	1	2	5				
	8	1	2	7				
	9	1	2	5				Coopered
	30	1	2	6				
	1	1	2	6				
	2	1	2	10				Stained
	3	1	2	6				
	4	1	2	11				
	5	1	2	13	3/1	1/9 each	1/7	Coopered
	6	1	2	10				
	7	1	2	11				
	8	1	2	14				
	9	1	2	9				
	40	1	2	8				
	1	1	2	9				
	2	1	2	4				
	3	1	2	9				
	4	1	2	9				Coopered
	5	1	2	10				
	6	1	2	10				Coopered
	7	1	2	7				
	8	1	2	11				
	9	1	2	7				
	50	1	2	7				
		50	1	10				
Nos.	25, 35, & 47	Opened for Customs						

I hereby certify that the above goods were in the condition stated when received.

_________*J. Johnson*,______

Warehousekeeper.

An importer who frequently imports goods would have periodic settlements with the dock authority, and a floating policy, renewed yearly, as regards the insurance.

Rent for warehousing is charged at so much per case, bag, etc., per week or fraction of a week, a week being, say, from Monday to Sunday; so that if goods were warehoused on a Saturday and taken out on the following Monday, two weeks' rent would be charged.

When goods are not removed within a given time, they are placed in one of the dock warehouses, particulars being entered on a form called the **Landing Account**, and a **Dock Warrant** is issued to the owner of the goods. This latter document must be produced before the goods can be removed from the warehouse.

If the goods are sold or a broker be employed to effect a sale while they are still in the hands of the dock authorities, the **Delivery Order** or the Dock Warrant, if issued, is transferred to the buyer or broker, so that both Delivery Order and Dock Warrant are negotiable.

EXAMPLE (i)—

Tabulate the costs incurred in transferring 55 bags of a certain commodity from a ship to a buyer's warehouse at the following rates: Landing rate, 2s. 1½d. per bag; delivery, 2s. 3d. per bag; warehousing, 2½d. per bag per week, Monday–Sunday—from Friday, 14th December, 19.., to Monday, 31st December, 19..; 6 bags opened for Customs inspection at 1s. 3d. per bag plus 7½ per cent. plus 12½ per cent. again, insurance for 1 month at 3s. 9d. per cent. per annum on £2,200; cartage, 1s. 10½d. per bag.

				£	s.	d.
Cost of landing	$= 55 \times 2\frac{1}{8}$ shillings	=		5	16	11
,, delivery	$= 55 \times 2\frac{1}{4}$,,	=		6	3	9
,, warehousing	$= 55 \times 2\frac{1}{2} \times 4$ pence	=		2	5	10
,, opening	= 7/6 + 7d. + 1/-	=			9	1
,, insurance	$= 22 \times 3\frac{3}{4} \times \frac{1}{12}$ shillings	=			6	11
,, cartage	$= 55 \times 1\frac{7}{8}$ shillings	=		5	3	2
		Total cost	=	20	5	8

NOTE 1.—14th December to 31st December includes two complete weeks and two fractions of a week, so that a charge for four weeks is made. In the case of the opening of the bags by the Customs authorities, the 12½ per cent. increase is on that amount after the 7½ per cent. increase.

131. ACCOUNT SALES.

When goods are imported on consignment and a broker is employed by the consignee to sell the goods, there would be two account sales: the broker's account sales to the consignee; and the latter's account sales to the consignor.

EXAMPLE (ii)—

100 Bales Cotton marked "T.S. 1/100" were shipped on Consignment

by T. Smith to W. Ross. The gross weight was 322 cwt. 2 qr. 12 lb. and tare 13 cwt. 1 qr. 16 lb., and freight was at £5 16s. 8d. per ton plus 10 per cent. The consignee instructed a firm of brokers (Bailey & Co.) to sell the cotton: bales 1/50 gross weight 161 cwt. 1 qr. 2 lb., tare 6 cwt. 2 qr. 22 lb., were sold at $7\frac{1}{2}$d. per lb.; and bales 51/100, gross weight 161 cwt. 0 qr. 26 lb., tare 6 cwt. 2 qr. 22 lb., were sold at $7\frac{5}{8}$d. per lb.; and the expenses were 100 samples at 6d. less 20 per cent. and sale expenses 3d. per bale; while the brokerage charged was $\frac{5}{8}$ per cent. The expenses to the consignee, who charged $2\frac{1}{2}$ per cent. commission, were: Freight as above, insurance at 2s. 3d. per cent. on £1,200, dock dues at 6d. per bale, warehousing at 1s. 4d. per ton, cartage at 9d. per bale and brokerage, etc., as stated. Make out the account sales of Bailey & Co. and W. Ross.

$$\begin{aligned} \text{Selling price of bales } 1/50 &= 17312 \times \tfrac{5}{8} \text{ shillings} \\ &= £541 \\ \text{,, ,, ,, } 51/100 &= 17308 \times 7\tfrac{5}{8} \text{ pence} \\ &= £549\ 17s.\ 10d. \end{aligned}$$

BROKER'S ACCOUNT SALES.

Account Sales of 100 bales Cotton, sold by auction on 17th October, 19.. for account of W. Ross.

Lot.	Bales.	Weight.	Price.	£	s.	d.
23	50 T S. 1/50	161 1 2 gross 6 2 22 tare				
		154 2 8 net	$7\frac{1}{2}$d. per lb.	541	–	–
24	50 T.S. 51/100	161 0 26 gross 6 2 22 tare				
		154 2 4 net	$7\frac{5}{8}$d. per lb.	549	17	10
				1,090	17	10

	£	s.	d.	£	s.	d.
100 samples @ 6d. less 20%	2	–	–			
Sale Expenses, 3d. per bale	1	5	–			
Brokerage, $\frac{5}{8}$% of £1,090 17s. 10d. . .	6	16	4	10	1	4
Net proceeds – – £				1,080	16	6

Liverpool.
20th October, 19..
Bailey & Co.

Freight on 16 tons @ £5 16s. 8d. per ton = £93 6s. 8d.

$$\text{,, } 292 \text{ lb. @ £5 16s. 8d. ,, } = \frac{292}{112 \times 20} \times \frac{35}{6} \times 20 \text{ shillings}$$

$$= 15s.\ 3d.$$

$$\therefore \text{ Total freight} = £94\ 1s.\ 11d. + £9\ 8s.\ 2d.$$

$$= £103\ 10s.\ 1d.$$

CONSIGNEE'S ACCOUNT SALES.

ACCOUNT SALES of 100 bales Cotton, sold by auction on 17th October, 19.. for account of T. Smith.

Lot.	Bales.	Weight.	Price.	£	s.	d.
23	50 T.S. 1/50	161 1 2 gross 6 2 22 tare				
		154 2 8 net	7½d. per lb.	541	–	–
24	50 T.S. 51/100	161 0 26 gross 6 2 22 tare				
		154 2 4 net	7⅝d. per lb.	549	17	10
				1,090	17	10

	£	s.	d.	£	s.	d.
Freight on 322 2 12 @ £5 16s. 8d. plus 10%	103	10	1			
Insurance, £1,200 @ 2/3 %	1	7	–			
Dock Dues @ 6d. per bale	2	10	–			
Warehousing, 322 2 0 @ 1/4 per ton	1	1	6			
Cartage @ 9d. per bale	3	15	–			
100 samples @ 6d. less 20%	2	–	–			
Sale Expenses @ 3d. per bale	1	5	–			
Brokerage, ⅝%	6	16	4			
Commission, 2½%	27	5	5			
				149	10	4
Net proceeds . . . . . £				941	7	6

E. & O.E.
Liverpool.
24th October, 19..
W. Ross.

NOTE 2.—The loss of weight of the goods during the voyage, in this case 12 lb. is known as the draft.

When goods are sent on consignment, the documents would be forwarded direct to the consignee, not negotiated by the consignor.

The net proceeds, £941 7s. 6d., would be paid into the Liverpool branch of the consignor's bank, or perhaps remitted by cable.

132. CUSTOMS AND EXCISE.

Import duties have to be paid on most commodities (unless Empire products) entering this country for consumption. The importer or agent can either pay the duty as soon as the ship arrives, or allow the goods to be placed in a warehouse and pay the duty whenever he wishes the goods to be released.

In the first case, the importer or the dock authority acting for him states exact particulars concerning the goods on a document known as "**Entry for Home Use *ex* Ship,**" which is then handed in at the Customs office at the dock, and the duty paid. When

the examination of the goods has been carried out, and if the result agrees with the particulars stated, the goods are released as far as the Customs officials are concerned.

In the second case, particulars of the goods are entered on a "**Warehouse Entry**" and passed at the Customs House. An officer of Customs examines the goods, which are then kept in the control of the Government and sent to a warehouse. The goods are then said to be **In Bond.** When the importer or agent wishes to obtain certain of the goods, he must fill in a "**warrant**" and pay the amount of the duty on the goods he wishes to be released.

Excise Duties.—Certain commodities on which import duties are payable are produced in this country. In order that the home producer should not have a preference over his foreign competitor and in order to obtain revenue, duties equivalent to the corresponding import duties are imposed.

Drawback.—Neither Customs nor Excise duties are payable on goods which are not consumed within the United Kingdom. In many cases, however, commodities are imported, duties paid; but after manufacture, etc., they are exported. On exportation, a sum called a "Drawback" is paid by the Customs authorities as repayment of the import duties.

On some commodities the Customs duties are *specific* (*i.e.*, based on quantity, net value) and on others the duties are "*ad valorem*" (*i.e*, a percentage of the value of the commodities). For Government statistical purposes and import duty *ad valorem*, the "c.i.f." value or the latest sale value of such goods is stated on the Customs "Entries."

In March, 1932, a 10 per cent. import duty was placed on nearly all kinds of merchandise which previously had entered the country free of duty. This tariff was not put on commodities coming from the Colonies and Mandate Territories.

Some of the exceptions were gold and silver bullion and coin; wheat; meat; fish of British taking; tea; raw cotton, wool; hides and skins; wood pulp.; raw rubber; iron ore; flax and hemp; uncut precious stones; radium.

The Customs Tariff of the United Kingdom (1934–35) in the case of some important commodities was as follows—

Commodities.	Rates of Duty.			
		£	s.	d.
Coffee	cwt.		14	0
Coffee or **Coffee** and **Chicory:** Roasted and ground	lb.			2
Cocoa	cwt.		14	0
Fruit, Dried: Preserved without sugar, Currants	cwt.		2	0
Figs, Fig Cake, Prunes, and Raisins	cwt.		7	0
Silk: Articles made of silk or artificial silk	33⅓% of value			
Sugar: Not exceeding 76 degrees of polarization	cwt.		4	6
Exceeding 98 ,, ,,	cwt.		11	8
(The duties of sugar of polarization between 76 and 98 degrees vary between 4s. 6d. and 11s. 8d cwt.)				
Molasses and invert Sugar, and all other extracts of Sugar which cannot be completely determined by the Polariscope and on which Duty is not otherwise charged— .				
If containing 70% or more of sweetening matter .	cwt.		7	5
,, between 70% and 50% sweetening matter	cwt.		5	4
,, 50% or less sweetening matter	cwt.		2	7
Glucose: Solid	cwt.		7	5
Liquid	cwt.		5	4
Saccharin and Mixtures containing Saccharin, or other substances of like nature and use	oz.		3	9
Spirits and Strong Waters:				
For every gallon computed at hydrometer proof of Spirits of any description (except perfumed Spirits), including Naphtha or Methylic Alcohol purified so as to be potable[1]; and mixtures and preparations containing Spirits—				
Brandy, Rum	proof gall.	3	15	4
Additional on Spirits imported in bottles	gall.		1	0
Liqueurs, Cordials, Mixtures, and other preparations containing Spirits in bottle, entered in such a way as to indicate that the strength is not to be tested	gall.	5	2	5
Perfumed Spirits	gall.	6	0	0
(Additional charges are made if the Spirits are immature.)				
Wine: Not exceeding 25 degrees of Proof Spirit	gall.		3	0
Exceeding 25 degrees but not exceeding 42 degrees of Proof Spirit	gall.		8	0
For every degree or part of a degree beyond 42 degrees, an additional duty of 8d. per gallon is charged.				
Additional: On Still Wine imported in bottles	gall.		2	0
On Sparkling Wine imported in bottles	gall.		12	6
Beer: Where the worts thereof were, before fermentation of a specific gravity of 1027 degrees or less	36 gall	1	5	3
Exceeding 1027 degrees—				
For the first 1027 degrees	36 gall.	1	5	3
For every degree in excess of 1027 degrees	36 gall.		2	0
Tobacco, Manufactured: Cigars	lb.		18	1
Cavendish or Negrohead	lb.		13	9
Cavendish or Negrohead manufactured in Bond	lb.		12	0
Cigarettes	lb.		14	7
Other manufactured Tobacco	lb.		12	0
Tobacco, Unmanufactured: If Stripped or Stemmed, containing 10 lb. or more of moisture in every 100 lb. weight thereof	lb.		9	6½
Containing less than 10 lb. of moisture in every 100 lb. weight thereof	lb.		10	6½
Duty ½d. per lb. less than above for unstripped or unstemmed tobacco.				

[1] Drinkable.

Commodities.	Rates of Duty.	
Clocks, Watches, and component parts thereof	33⅓% of value	
Musical Instruments, including Gramophones, Pianolas, and accessories and component parts	33⅓% of value	
Motor-Cars, including Motor Bicycles and Tricycles, and accessories and component parts thereof, other than tyres, except cars for use, as omnibuses, ambulances, or for trade purposes	33⅓% of value	
Motor Spirit	gall.	8d.
Matches: Boxes containing 20 to 50 matches	144 boxes	4s. 9d.
Cinematograph Films (negatives)	ft.	5d.

Commodities in this list which are British Empire products are liable to reduced rates or are totally exempt.

Customs Drawbacks may be allowed on commodities when they are in the same condition on being exported as they were when previously imported. The drawback is an amount equal to the duty paid.

Excise Duties on certain commodities are as follows: Beer per barrel (36 gall.) of specific gravity of 1027°, £1 4s. Matches, 4s. 2d. per 144 boxes (20–50 matches). Tobacco grown in the United Kingdom, manufactured in bond, 9s. 4¾d. per lb.; unmanufactured, if containing 10 per cent. moisture, 7s. 3½d. per lb.; and if containing less than 10 per cent. moisture, 8s. 0¾d. per lb. Spirits, home-made, per proof gallon, £3 12s. 6d.

Example (iii)—

Calculate the import duties (at the above rates) on each of the following items: 5 ton 13 cwt. 2 qr 21 lb. sugar, polarization 82·3° (duty 5s. 6·3d. per cwt.); 1 ton 7 cwt. 1 qr. 15 lb. molasses containing 72 per cent. sweetening matter; 7 ton 11 cwt. 14 lb. coffee; 12 cwt. of prepared cocoa-butter (duty 1½d. lb.).

				£	s.	d.
Import duty on	113 cwt.		of sugar =	31	4	3·9
,, ,,	2 qr.		,, =		2	9·15
,, ,,		14 lb.	,, =			8·29
,, ,,		7 lb.	,, =			4·14
∴ ,, ,,	113 cwt. 2 qr. 21 lb.		,, =	£ 31	8	1

				£	s.	d.
Import duty on	27 cwt.		of molasses =	10	0	3
,, ,,	1 qr.		,, =		1	10·25
,, ,,		14 lb.	,, =			11·12
,, ,,		1 lb.	,, =			·79
∴ ,, ,,	27 cwt. 1 qr.	15 lb.	,, =	£ 10	3	1

				£	s.	d.
Import duty on	151 cwt.		of coffee =	105	14	0
,, ,,		14 lb.	,, =		1	9
∴ ,, ,,	151 cwt.	14 lb.	,, =	£105	15	9

Import duty on 12 cwt. of cocoa-butter $= £\dfrac{12 \times 112}{8 \times 20}$

$= £8\ 8s.$

Example (iv)—

A & Co. imported tobacco at a net "c.i.f." price of 11d. per lb. The net weight of tobacco was 75 cwt. 18 lb. and the charges were as follows: Customs

Entry, 3s. 6d.; Import Duty at 13s. 9d. per lb.; Insurance on £400 at 2s. 1½d. per cent.; Dock Charges and Rent, £13 7s. 9d.; Cartage, £7 12s. 6d.: this being the cost of conveying the goods from Bond to the warehouse of B & Co., to whom the tobacco was sold. What was the total cost of tobacco per lb. to A & Co.? and what must the latter's selling price per lb. to B & Co. be in order for them to gain at least 7 per cent. on the total cost, neglecting credit in each case?

Cost of tobacco at 11d. per lb. = £385 16s. 6d.

Charges.

	£	s.	d.
Customs Entry		3	6
Import Duty, 8,418 lb. @ 13/9 lb.	5787	7	6
Insurance, £400 @ 2/1½%		8	6
Dock Charges and Rent	13	7	9
Cartage	7	12	6
Total charges =	£5808	19	9

∴ Total cost of 8,418 lb. tobacco to A & Co. = £6194 16 3

∴ „ „ 1 lb. „ „ $= £\frac{6194 \cdot 8125}{8418}$

= £·7359
= 14s. 8½d. to nearest ½d.

A & Co.'s selling price per lb. to gain 7% on cost = £(·7359 + ·0515)
= £·7874
= 15s. 8·98d.

∴ A & Co.'s selling price per lb. to gain at least 7% on cost = 15s. 9d.

TEST EXERCISES III, 2.

(1) Tabulate the expenses incurred in importing 75 cases of a certain commodity, each case measuring 4′ 3″ × 3′ 4″ × 2′ 6″, from the time the cases are placed on board at the foreign port to the arrival at their destination:

Freight, 27s. 6d. per ton of 40 cub. ft. plus 10% primage.
Marine Insurance on £750 @ 8s. 6d. % plus 15%.
Customs Entries, 3s. 6d.
Landing Dues, 1s. 1½d. per case.
Opening for inspection by Customs office, 7 cases @ 1s. 7½d. per case plus 20%.
Delivery, 1s. 8d. per case.
Rent for warehousing, 5 weeks @ 3d. per case per week.
Insurance, 2 months on £850 @ 4s. 6d. % per annum.
Cartage { 1s. 6d. per case, 30 cases.
2s. 3d. „ 45 „

(2) The goods in Question (1) were sold to the importer for £625, the price to include cost up to the placing on board ship. By what percentage is the cost of the goods to the importer at the time of delivery to his customer's warehouse greater than the former price?

(3) The importer sold 30 cases at £12 15s. 6d. per case and the remainder at £13 3s. 6d. per case. What was his gain, neglecting credit in every case?

(4) An importer purchased 36 barrels coffee at a "f.o.b." price of

82s. 6d. per cwt. The Gross weight was 69 cwt. 3 qr. 18 lb.; tare and draft 9 cwt. 1 qr. 4 lb. The expenses of importation were as follows—

Freight, 62s. 6d. per ton of 20 cwt.
Marine Insurance on £300 @ 13s. 6d. per cent.
Entries, Landing Dues, Dock Charges, £4 7s. 4d.
Import Duty on net weight @ 14s. per cwt.
Cartage and Porterage, £1 1s. 9d.

What is the total cost of the coffee by the time it has been delivered at his warehouse?

(5) The above importer added $4\frac{1}{2}$ per cent. to this total cost to cover management expenses, and wished to fix his list price such that by allowing a trade discount of 10 per cent. he would gain a net profit of 5 per cent. on his total outlay, including establishment charges. At what price per cwt. should the coffee be priced?

(6) A foreign agent bought for A. Smith & Co., of Bristol, 25 bales of a certain commodity at £22 15s. per bale. His expenses were as follows: Sale Expenses, $3\frac{1}{2}$d. per bale; Dock Charges, £2 11s. 9d.; Freight at 43s. 9d. per ton of 20 cwt. and 10 per cent. gross weight 27 cwt. 3 qr. 20 lb., tare and draft 2 cwt. 1 qr. 8 lb.; Insurance at 4s. 3d. per cent. on £700; Packing, £1 3s. 8d.; Commission, $2\frac{1}{2}$ per cent. on net buying price. A. Smith & Co employed a forwarding agent to receive the goods and forward them to his warehouse. The agent's expenses were: Landing Dues, 1s. 4d. per bale; Entries, Customs examination, and Dock Charges, £4 6s. 3d.; Import Duty on net weight at 7s. per lb.; Railway Charges, 1s. 6d. per bale; and $18\frac{1}{2}$ miles at 2·65d. per ton or fraction of a ton per mile; Commission, $2\frac{1}{4}$ per cent. on value of goods on board at the Bristol Channel Port. Calculate the total cost of the goods to A. Smith & Co., and find how much per cent. this was above the cost at the foreign market. Also calculate the cost per lb.

(7) Make out Account Sales for 120 bales Wool shipped at Sydney by G. Fenwood per ss. *Borneo* and sold in London by T. Hopper at $11\frac{1}{2}$d. per lb. Weight, 421 cwt. 2 qr. 15 lb.; tare, 14 cwt. 2 qr. 11 lb.

Charges payable at London: Freight, $\frac{3}{8}$d. per lb. on gross weight and 10 per cent. primage; Insurance on £2,400 at 3s. 2d. per cent.; Dock Dues, £11 4s. 2d.; Sale Expenses, £3 11s. 8d.; Brokerage, $\frac{1}{2}$ per cent.; Commission, $2\frac{1}{2}$ per cent.

(*N.B.*—T. Hopper is the broker as well as the importing agent: he charges $\frac{1}{2}$ per cent for effecting a sale and $2\frac{1}{2}$ per cent. for performing the work in connection with the importation. Base both percentages on the selling price.)

(8) Make out an Account Sales of 984 bags of Rice ex. *Arabian*, sold by F. Ford, London, by order and for account of Messrs. Robinson Bros., Calcutta.

676 bags, Gross 1,278 cwt. 1 qr. 4 lb., Tare and Draft 29 cwt. 3 qr. 21 lb., at 7s. 9d. per cwt.

308 bags, Gross 601 cwt. 0 qr. 26 lb., Tare and Draft 13 cwt. 1 qr. 10 lb., at 7s. 6d. per cwt.

Charges: Dock Dues, £7 4s. 6d.; Rent, £1 3s. 4d.; Insurance on £750 at 3s. 3d. per cent.; Sale Expenses, 3d. per bag plus 10 per cent.; Cartage, $7\frac{1}{2}$d. per bag; Commission, $2\frac{1}{2}$ per cent.

(9) Make out an Account Sales of 150 cases of a commodity ex. ss. *Johnstone*, sold by Messrs. Harland & Redford, for account of Messrs. Li Chauvre & Co.

55	cases	net weight	3,296 lb.	@ 2s. 1d.
40	,,	,,	2,397 lb.	@ 2s $1\frac{1}{2}$d.
30	,,	,,	1,804 lb.	@ 2s. $1\frac{3}{4}$d.
35	,,	,,	2,099 lb.	@ 2s. 2d.

less 3 m/s discount @ 5% per annum.

Charges: Entries, Dock Charges, and Rent, £16 13s. 8d.; Import Duty

on net weight at 1s. per lb.; Insurance on £1,150 at 4s. 2d. per cent.; Brokerage, ¼ per cent.; Commission, 2½ per cent.

(10) Make out an Account Sales for 72 barrels Rubber *ex.* ss. *Mexico* from Para, and sold by A. Wilson & Sons, Liverpool, on account of B. Lebouf & Co., Para.

46 barrels, Gross weight 62 cwt. 0 qr. 14 lb., Tare and Draft 11 cwt. 1 qr. 22 lb., sold at 3s. 9d. per lb.

26 barrels, Gross weight 35 cwt. 3 qr. 2 lb., Tare and Draft 6 cwt. 2 qr. 8 lb., sold at 3s. 10½d. per lb.

Charges: Dock and Town Dues, 14s. 6d.; Marine Insurance on £2,240 at 10s. 6d. per cent.; Insurance against War Risks on £2,240 at 42s. 6d. per cent.; Freight, 48s. 6d. per ton of 20 cwt.; Porterage and Petties, £7 2s. 6d.; Brokerage, ½ per cent, Commission, 2½ per cent.

(11) 90 barrels of a commodity marked F 1/90 were shipped per ss. *Mapleton* from Jamaica on consignment by W. Frantz & Co, the consignee being S. Young & Son, Liverpool The latter instructed a broker, B. Morton, to effect a sale.

50 barrels 1/50, Gross weight 97 cwt. 0 qr. 16 lb., net weight 94 cwt. 2 qr. 18 lb., sold at 165s. per cwt

40 barrels 51/90, Gross weight 78 cwt. 3 qr. 11 lb., net weight 76 cwt. 2 qr. 25 lb, sold at 166s. 6d per cwt

Charges paid by broker: 20 samples at 9d.; Sale Expenses, 4d. per barrel; Cartage, 9d. per barrel.

Charges paid by consignee in addition to the above: Brokerage, ⅝ per cent.; Freight, 47s. 9d. per ton of 20 cwt.; Marine Insurance on £1,750 at 9s. 4d. per cent.; Landing Dues, Entries, and Delivery, £14 3s. 7d.; Import Duty at £2 2s.; Rent, 3 weeks at 2d. per barrel per week; Insurance against War Risks at 39s 6d. per cent.

Make out the two Account Sales, the consignee's commission being 2½ per cent.

(12) The net proceeds remitted to W. Frantz & Co. were 6¾ per cent. greater than the price paid by them for the commodity. At what price per cwt. did the consignor purchase the commodity?

(13) W. Langton & Co., of Sydney, arranged to deliver 75 bales of wool to C. Mackay at 10½d. per lb., 30 bales to N. Anson at 10¾d. per lb., and 15 bales to R. Colland at 11d. per lb. A *del credere* [1] agent, D. Bonner, was instructed to arrange for the removal of the wool from ss. *Kingfisher* on her arrival at Liverpool, and to deliver the same to the premises of the buyers and remit the net proceeds to W. Langton & Co.

Weight: 75 bales Wool 1/75, gross 287 cwt. 1 qr. 10 lb., tare and draft 12 cwt. 0 qr. 4 lb., at 10½d. per lb.

30 bales Wool 76/105, gross 118 cwt. 0 qr. 11 lb., tare and draft 5 cwt. 1 qr. 9 lb., at 10¾d. per lb.

15 bales Wool 106/120, gross 59 cwt. 0 qr. 8 lb., tare and draft 2 cwt. 3 qr. 1 lb. at 11d. per lb.

Charges: Wharfage, Entries, and Delivery, £18 15s. 6d.; Insurance on £2,500 at 3s. 5d. per cent.; Rent, £1 1s. 6d.; Cartage, £4 11s. 4d.

D. Bonner received 2½ per cent. commission and *del credere* [2] ½ per cent. based on the above selling price.

Make out the Account Sales.

(14) Referring to Question (13), R. Colland was unable to make any payment. D. Bonner, therefore, instructed a broker, T. Elland, to sell the wool, the price obtained being 10¼d. per lb., and the charges were: 10 samples

[1] A *Del Credere* Agent is one who receives extra commission in consideration of his accepting responsibility of the payment for the goods should a merchant who has undertaken to purchase not carry out his share of the bargain.

[2] *Del credere* is the name given to this extra commission.

at 1s. 4d. each; Sale Expenses, 3d. per bale; Cartage, £1 7s. 9d.; Brokerage, ⅝ per cent. Make out the Account Sales. Also calculate how much D. Bonner gained in connection with the complete transactions.

(15) Make out D. Bonner's Account Sales had he not been a *del credere* agent, thus receiving 2½ per cent. commission on the total selling price actually obtained. How much did he gain or lose by being a *del credere* agent?

(16) Using the Customs Tariff on pages 264–265, calculate the total import duty on—

15 cwt. 2 qr. 18 lb. net, Coffee; 1 cwt. 3 qr. 7 lb. net, Coffee and Chicory, roasted and mixed.

(17) Calculate the total import duty on—

2 cwt. 1 qr. 18 lb. net, Sugar, polarization 81·3°, import duty at 5s. 4·5d. cwt.

1 cwt. 3 qr. 24 lb. net, Sugar, polarization 88·7 degrees, import duty at 6s 6·8d. cwt.

(18) Calculate the total import duty on—

3 gall. 1 qt. 1½ pt. Eau de Cologne at £6 per gall.

1 gall. 0 qt. ½ pt. Parma Violets (bottled) at £6 1s. per gall.

(19) Calculate the total import duty on—

25 proof gall. Brandy,

13 gall. 3 qt. 1 pt Still Wine (bottled), 40° of proof spirit;

7 gall. 1 qt 1½ pt Sparkling Wine (bottled), 42° of proof spirit;

703 gall. 2 qt Beer, sp gr 1220°.

(20) What are the import duties on the following?—

17¾ oz. Saccharin; 1 cwt. 1 qr. 10 lb Glucose (solid).

CHAPTER XVI.

EXPORT TRADE.

133. METHODS OF EXPORTING GOODS.

AN exporter might be—

I. An agent who has received an **Indent** from a foreign firm authorizing him to purchase certain commodities and arrange for the shipping of them to a certain port.

II. An agent commissioned by a wholesale trader, producer, or manufacturer to ship certain goods.

III. A wholesale trader, producer, or manufacturer.

The exporter's duties consist in attending to all arrangements for the placing of the goods on board; in the proper filling up of the Invoices, Bills of Lading, Insurance Certificates, etc.; and in settling matters connected with cartage, dock charges, Customs' formalities, and loading charges.

Within six days of the "clearance" of the vessel, the Customs' "**Specification**," in the case of "free goods," or the **Bond Note**, in case of bonded goods (that is, goods previously imported but not removed from bond), must be made out and sent to the Customs' House of the port of shipment. The filling up of these documents must be in accordance with the **Official Export List**, and must contain particulars concerning the quantity and value of the goods as well as the ultimate country of destination.

When goods are shipped to the United States, Portugal, or to most of the republics of South America, a **Consular Invoice** must be filled up and declared before the Consul of the district from which the goods are exported.

Although an invoice is a statement of goods bought, and goods shipped on consignment are not yet purchased, yet an invoice is written out in reference to the goods and answers two purposes: (1) It indicates to the consignee the approximate minimum price at which he should sell; and (2) it is required by the Customs authorities in connection with the Board of Trade statistics. An **Account Sales** is a statement of goods sold by the consignee, and is sent to the exporter of the goods after the sale has been effected. Comparatively few British traders export goods on

consignment, for it frequently happens that serious losses are incurred thereby.

In the case of goods exported in accordance with firm contracts, drafts drawn against them are sometimes in foreign currency and sometimes in sterling, and they are usually payable at 60 or 90 days' sight.

A **Letter of Hypothecation** authorizes the bank to sell the goods if the drawee fails to accept the bill or does not pay at maturity. Exporters frequently negotiate the bills they have drawn at the bank offering the best terms.

134. KINDS OF INVOICES.

The expenses incurred in removing goods from a warehouse in one country to the destination in another country are so considerable, that in fixing prices, consideration must be given to the question of how these expenses shall be divided among buyer and seller. The most important terms of sale are as follows—

Terms.	Seller Defrays:	Buyer Defrays:
"Loco" (Price at place of Purchase)	Nil	1. Packing 2. Transport to Dock 3. Dock Charges 4. Export Duty (if any) 5. Loading Charges 6. Freight 7. Marine Insurance 8. Insurance against War Risks (if any) 9. Landing Charges 10. Import Duty (if any) 11. Dock Charges 12. Rent for Warehousing 13. Fire Insurance (if any) 14. Cartage or other transport charge 15. Commissions (if any)
F.A.S. (Free alongside Ship)	Expenses 1 to 4	Expenses 5 to 15
F.O.B. (Free on Board)	Expenses 1 to 5	Expenses 6 to 15
C. & F. (Cost & Freight)	Expenses 1 to 6	Expenses 7 to 15
C.I.F. (Cost, Insurance, and Freight)	Expenses 1 to 8	Expenses 9 to 15
"Franco" (Free)	Expenses 1 to 15	Nil

Invoices on "**Loco**" and **F.O.B.** terms are now used chiefly by manufacturers who export their own goods. The expenses paid by these are added to the net cost of the goods on the invoices.

Invoices on **C. & F.** terms are very rare in the export trade of this country, but are common in the import trade.

C.I.F. invoices are frequently used in connection with export to the Far East.

"**Franco**" terms are becoming more and more common in connection with trade with the Continent. Goods are sold "franco Lyons," "franco Genoa," etc., so that the would-be buyer would know without inquiry and calculation the total amount the goods would cost him.

C.I.F. and "Franco" invoices are expressed in the units and currency of the country to which the goods are being exported and, whenever possible, in the language of the foreign country. The tendency is to do this more and more, and it is necessary for the maintenance and increase of the British export trade that British exporters should realize this, so as to cope with foreign competition. Thus it is seen that the methods of expressing quantities and prices in foreign units and currency should be known. As some importers buy goods in terms of foreign units and money, it is necessary to solve the converse problem of expressing these prices in terms of British weights, measures, and currency.

135. FOREIGN WEIGHTS AND MEASURES.

The Metric System has been adopted by the whole of Europe (except Great Britain) and the Republics of South America, although some of the old units are still in use in some countries. Several countries, however, designate the units by names different from those used in France, Belgium, Norway, Sweden, etc.

The following particulars of the weights and measures of certain foreign countries are given for reference.

Italy uses *Metro* for Metre; *Gramma* for Gram; *Litro* for Litre; *Ara* for Are; *Etto* for Hecto; *Chilo* for Kilo; *Miria* for Myria; *Ettara* for Hectare.

Holland uses *El* for Metre; *Streep* for Mm.; *Duim* for cm.; *Palm* for dm.; *Roede* for Dm.; *Mijle* for Km.; *Kan* for Litre; *Vingerhold* for cl.; *Maatje* for dl.; *Vat* for Hl.; *Wigtje* for Gram; *Korrel* for dgm.; *Lood* for Dgm.; *Onze* for Hgm.; *Pond* for Kgm.

Spain uses *Metro* for Metre; *Litro* for Litre; *Gramo* for Gram; *Area* for Are.

Austria and Germany use *Stab* for Metre; *Neuzoll* for cm.; *Strich* for mm.; *Kette* for Dm.; *Kanne* for Litre; *Schoppen* for Half-Litre; *Fass* for Hl.; *Neuloth* for Dgm.; *Centner* = 50 Kgm.; *Tonne* = 1,000 Kgm.

The names and the British equivalents of some of the principal units made use of in other countries are as follows—

United States.—The Imperial units *lb.* and *yard* are used; but instead of the units cwt. and ton, the units *Cental* (= 100 lb.) and *Short Ton* (= 20 centals or 2,000 lb.) are employed: 2,240 lb. (or 20 cwt.) is called a *Long Ton*. The *U.S. Bushel* = ·9694 Imperial bushels. The *U.S. Gallon* (*dry measure*) is $\frac{1}{8}$ U.S. bushel; but the *U.S. Gallon* (*liquid measure*) = 231 cubic inches approx., and is thus – of the Imperial gallon: $\frac{1}{8}$ of a U.S. gallon (liquid) is termed a pint.

Canada.—Same as British but with *short ton* of 2,000 lb.

Australia and **New Zealand** use the *Imperial weights and measures.*

India.—Imperial units are used in the case of trade between members of the white population. The units used by the natives are as follows: *Guz* = 33 in.; *Bigha* (Bengal) = ·625 acre; *Cawny* (Madras) = 1·33 acre; *Tola* (rupee-weight) = 180 grains; *Seer* (80 tolas) = 2 lb. 1 oz.; *Maund* (40 seers) = 82·28 lb.; *Maund* (*Madras*) = 24·68 lb.; *Candy* (*Madras*) = 500 lb.; *Visham* = 3 lb.; *Dangali* = 3 pints; *Parah* = 15 gallons; *Catty* (Singapore) = 1$\frac{1}{3}$ lb.; *Pikul* = 100 catties = 133 lb.

South Africa.—Imperial units are used, but some old Dutch measures are still employed. *Leaguer* = about 128 gall.; *Half Aum* = 15$\frac{1}{2}$ gall.; *Anker* = 7$\frac{1}{2}$ gal.; *Muid* = 3 bushels; *Morgen* = 2·11651 acres; *Cape foot* = 1·033 ft., *Short Ton* = 2,000 lb. By law, the use of the Metric System is permissive, but, with the exception of chemists who must use the Metric System, traders may use the Imperial standards.

Japan.—*Ri* = 2·4403 miles; *Square Ri* = 5·9553 *sq. miles*; *Cho* = 5·423 chains; *Square Cho* = 2·4507 acres, *Ken* = 1·9881 yards; *Tsubo* = 3·9538 sq. yards; *Liquid Koku* = 39·7033 gallons, *Dry Koku* = 4·9629 bushels, *Koku of capacity* = 1 ton, *Liquid Sho* = 1·5881 quarts, *Dry Sho* = ·1985 pecks; *Kwan* = 8·2673 lb.; *Kin* = 1·3228 lb.

China.—*Tael weight* = 1·33 oz.; *Catty weight* = 1·33 lb.; *Picul weight* = 133·33 lb.; *Ts'un* = 1·41 in., *Ch'ih* = 1·175 ft.; *Chang* = 11·75 ft.; *Li* = 2·115 ft.

136. In practice, **TABLES OF EQUIVALENT VALUES** are often used, examples of which are given below. It is left for the student to complete the tables up to 9 in each case, and to use the tables so constructed in a way similar to that shown in connection with simple and compound interest.

WEIGHT (*Avoirdupois*).

	Grains to Milli-grams	Milli-grams to Grains	Ozs. to Grams	Grams to Ounces.	Lbs. to Kilo-grams.	Kilo-grams to Lbs.	Cwts. to Quin-tals.	Quin-tals to Cwts.	Tons to Tonnes	Tonnes to Tons.
1	64·8	0·015	28·35	0·03527	0·45359	2·20462	0·508	1·968	1·0160	0·9842
2	129·6	0·030	56·70	0·07054	0·90719	4·40924	1·016	3·936	2·0320	1·9684

Troy Weight.

	Ounces Troy to Grams.	Grams to Ounces Troy.	Dwts. to Grams.	Grams to Dwts.
1	31·1035	0·03215	1·5552	0·643
2	62·2070	0·06430	3·1104	1·286

Apothecaries' Weight.

	Scruples to Grams.	Grams to Scruples.	Grains to Grams.	Grams to Grains.
1	1·296	0·7716	0·0648	15·432
2	2·592	1·5432	0·1296	30·864

Apothecaries' Measure.

	Fluid Drachms to Millilitres.	Millilitres to Fluid Drachms.
1	3·552	0·28153
2	7·104	0·56306

Linear Measure.

	Inches to Centimetres.	Centimetres to Inches.	Yards to Metres.	Metres to Yards.	Miles to Kilometres.	Kilometres to Miles.
1	2·5400	0·3937	0·91438	1·09363	1·60931	0·62138
2	5·0800	0·7874	1·82876	2·18726	3·21862	1·24276

Square Measure.

	Sq. Inches to Sq. Centimetres.	Sq. Centimetres to Sq. Inches.	Sq. Feet to Sq. Decimetres.	Sq. Decimetres to Sq. Feet.	Sq. Yards to Sq Metres.	Sq. M'tres to Sq. Yards.	Acres to Hectares	Hectares to Acres.
1	6·4516	0·155	9·2903	0·10764	0·8361	1·1960	0·40468	2·47114
2	12·9032	0·310	18·5806	0·21528	1 6722	2·3920	0·80936	4·94228

Cubic Measure.

	C. Inches to C. Centimetres.	C. Centimetres to C. Inches.	C. Feet to C. Metres.	C. Metres to C. Feet.	C. Yards to C. Metres.	C. Metres to C. Yards.
1	16·3871	0·061024	0·02832	35·3148	0·7645	1·3080
2	32·7742	0·122048	0·05664	70·6296	1·5290	2·6160

Capacity.

	Quarts to Litres.	Litres to Quarts.	Gallons to Litres.	Litres to Gallons.	Bushels to Dekalitres.	Dekalitres to Bushels.	Quarters to Kilolitres.	Kilolitres to Quarters.
1	1·1359	0·8803	4·5435	·2201	3·637	0·275	0·2909	3·4375
2	2·2718	1·7606	9·0870	·4402	7·274	0·550	0·5818	6·8750

EXAMPLE (i)

Given that £1 = 87·22 francs, find multipliers correct to 3 places of decimals to convert—

(1) Shillings per ton into francs per tonne.
(2) Francs per kilometre into pence per mile.
(3) Shillings per quarter into francs per kilolitre.

(1) 1s. per ton $= 4{\cdot}361$ f. per $1{\cdot}016$ tonne

$$= \frac{4{\cdot}361}{1{\cdot}016} \text{ francs per tonne}$$

$$= \underline{\underline{4{\cdot}292}} \text{ ,, ,,}$$

(2) 1 f. per Km. $= \dfrac{240}{87{\cdot}22}$ pence per $\cdot62138$ mile

$$= \frac{240}{87{\cdot}22 \times {\cdot}62138} \text{ pence per mile}$$

$$= \underline{\underline{4{\cdot}428}} \text{ ,, ,,}$$

(3) 1s per qr. $= 4{\cdot}361$ f. per $\cdot2909$ kilolitre

$$= \frac{4{\cdot}361}{{\cdot}2909} \text{ francs per kilolitre}$$

$$= \underline{\underline{14{\cdot}991}} \text{ ,, ,,}$$

EXAMPLE (ii)—

Using the multipliers above, express

(1) £1 12s. 4d per ton in francs per tonne to the nearest centime;
(2) ·65 f per Km in pence per mile to the nearest ¼d ;
(3) 81s 7d. per quarter in francs per kilolitre to the nearest 5-centimes.

(1) $32\frac{1}{3}$ shillings per ton $= 4{\cdot}292 \times 32\frac{1}{3}$ francs per tonne
$= \underline{\underline{138{\cdot}77}}$ francs per tonne

(2) ·65 franc per Km. $= 4{\cdot}428 \times {\cdot}65$ pence per mile
$= \underline{\underline{3\text{d}}}$ per mile.

(3) 81s. 7d. per qr. $= 14{\cdot}991 \times 81\frac{7}{12}$ francs per Kl.
$= \underline{\underline{1223{\cdot}02}}$ francs per Kl.

EXAMPLE (iii)—

Draw up a nine-multiple table converting bushels per acre into hectolitres per hectare and *vice versa*. Use the table to convert $37\frac{3}{8}$ bushels per acre into hectolitres per hectare and 31·94 hectolitres per hectare into bushels per acre

1 bushel per acre $= {\cdot}3637$ hl. per $\cdot40468$ hectare

$$= \frac{{\cdot}3637}{{\cdot}40468} \text{ hl. per hectare}$$

$$= \underline{\underline{{\cdot}8987}} \text{ ,, ,,}$$

1 hl. per hectare $= \dfrac{1}{{\cdot}8987}$ bush. per acre

$$= \underline{\underline{1{\cdot}1127}} \text{ ,, ,,}$$

Bushels per Acre.		Hectolitres per Hectare.
1·113	1	·899
2·225	2	1·797
3·338	3	2·696
4·451	4	3·595
5·564	5	4·494
6·676	6	5·392
7·789	7	6·291
8·902	8	7·190
10·014	9	8·088

30 bush. per acre	=	26·96 hl. per hectare
7 ,, ,,	=	6·291 ,, ,,
$\frac{3}{8}$,, ,,	=	·337 ,, ,,
∴ 37$\frac{3}{8}$,, ,,	=	33·59 ,, ,,

30 hl. per hectare	=	33·38 bush. per acre
1 ,, ,,	=	1·113 ,, ,,
·9 ,, ,,	=	1·001 ,, ,,
·04 ,, ,,	=	·045 ,, ,,
∴ 31·94 ,, ,,	=	35·54 ,, ,,

137. FOREIGN CURRENCY.

It has already been stated in Paragraph 62 that many European countries have a monetary standard equivalent to the Franc, and the names given to this standard in the various countries have been tabulated. Further particulars regarding the currency in foreign countries are as follows—

Australia, New Zealand, Union of South Africa, and **Jamaica** have the same monetary unit as the United Kingdom.

In **Canada** and the **United States,** the unit is the **Dollar** (= 100 Cents).

Germany uses the *Reichsmark* (= 100 Pfennige) and **Austria,** the *Schilling* (= 100 Groschen).

Denmark and **Norway** use the *Krone* (= 100 Ore); **Sweden** has the same monetary unit, which, however, is called the *Krona* (= 100 Ore).

Holland uses the *Florin* or *Guilder* (Dutch, *Gulden*) (= 100 Cents).

Portugal uses the *Escudo* (= 100 Centavos).

In India, Ceylon, Mauritius, the monetary unit is the *Rupee* (= 16 Annas), having an equivalent value of nominally 1s. 6d.

In Egypt, the *Pound* (= 100 Piastres) is the monetary unit.

The *Yuan* or *Dollar* (= 100 Cents) of China has an equivalent value of about 1s. 6d., and the *Yen* (= 100 Sen) of Japan has an equivalent value of about 2s. $0\frac{1}{2}$d.

EXAMPLE (iv)—

57 short tons 13 centals 15 lb. of a certain kind of grain was bought for \$10,000. Assuming that $\$3{\cdot}44\frac{1}{4} = £1$, calculate the equivalent price in £ *s. d.* per cwt.?

$$\text{Cost of } 1 \text{ lb.} = \frac{10000}{115315} \text{ dollars}$$

$$\therefore \quad ,, \quad 1 \text{ cwt.} = £\frac{10000 \times 112}{115315} \times \frac{100}{344\frac{1}{4}}$$

$$= £\frac{10000 \times 112 \times 400}{115315 \times 1377}$$

$$= £2{\cdot}821$$

$$= £2 \ 16\text{s.} \ 5\text{d.}$$

EXAMPLE (v)—

Find a multiplier for converting gulden per pond to pence per lb., and hence find the cost in English money of 876 lb. of butter at 1 gulden 15 cents per pond. (Assume £1 = 8·56 gulden.)

$$\text{If cost of } 1 \text{ pond} = 1 \text{ gulden}$$

$$i.e., \ ,, \quad ,, \quad 2{\cdot}2046 \text{ lb.} = \frac{240}{8{\cdot}56} \text{ pence}$$

$$\text{then} \quad ,, \quad 1 \text{ lb.} = \frac{240}{2{\cdot}2046 \times 8{\cdot}56}$$

$$\text{i.e., } 1 \textit{ gulden per pond} = 12{\cdot}7177 \text{ pence per lb.}$$

$$\therefore \text{ cost of } 876 \text{ lb. of butter} = £\frac{12{\cdot}7177 \times 1{\cdot}15 \times 876}{240}$$

$$= £53 \ 7\text{s.} \ 8\text{d.}$$

EXAMPLE (vi)—

Express in yen per ken, correct to the nearest sen, the price of cloth equivalent to 4s. $7\frac{1}{2}$d. per yard, given that 1 yen = 2s. $1\frac{1}{2}$d. Express 2,175 yds. in ken to the nearest $\frac{1}{4}$ ken, and calculate the cost of this quantity of cloth at the former price in yen per ken. Verify the result by first calculating the total cost in £ and then converting to yen.

$$4\tfrac{5}{8}\text{s. per yard} = \frac{37}{17} \text{ yen per } \frac{1}{1{\cdot}9884} \text{ ken}$$

$$= \frac{37 \times 1{\cdot}9884}{17} \text{ yen per ken}$$

$$= 4{\cdot}33 \text{ yen per ken.}$$

$$2{,}175 \text{ yd.} = \frac{2175}{1{\cdot}9884} \text{ ken} = 1{,}093\tfrac{3}{4} \text{ ken to the nearest } \tfrac{1}{4} \text{ ken.}$$

$$\therefore\ 1{,}093\tfrac{3}{4} \text{ ken at } 4{\cdot}33 \text{ yen per ken cost } \frac{4375 \times 4{\cdot}33}{4} \text{ yen}$$

$$i.e.,\quad 4735{\cdot}94 \text{ yen}$$

$$2{,}175 \text{ yd. at 4s. } 7\tfrac{1}{2}\text{d. per yard cost } £\frac{2175 \times 37}{8} \text{ shillings}$$

$$= \frac{2175 \times 37 \times 8}{8 \times 17} \text{ yen}$$

$$= 4733{\cdot}82 \text{ yen.}$$

NOTE 1.—The last result is correct to the nearest sen: the discrepancy in the former result is brought about by (1) the error in 4·33 increased 1,093¾ times on multiplication; and (2) the error in 1,093¾. This result, however, is what would appear on an invoice on which the length and price are expressed in the units used by the Japanese.

As regards the calculations involved in the illustrative examples given in this chapter, it should be remembered that the principles of approximation are involved. Much time is saved by the use of contracted methods, and in some cases, logarithms can be employed to advantage. Of course, the use of a slide rule would produce the results the most rapidly, and in most practical cases these results would be of a sufficient degree of accuracy.

138. A number of examples on **THE PREPARATION OF INVOICES** on "**Loco,**" **F.O.B.**, **C.I.F.**, and "**Franco**" terms are dealt with as follows—

EXAMPLE (vii)—

Cook, Russell & Co., of Halifax, received an indent from Messrs. Robinson & Co., Calcutta, by which 3 cases marked Calcutta R.&Co. 1/3 containing altogether 18 pieces, 54″ Navy Serge, were to be purchased and shipped to Calcutta. Particulars of goods bought and the charges were—

18 pcs. 54″ Navy Serge, 6/52½, 6/52, 4/51½, 2/51 at 3s. 4½d. per yard, less 2½ per cent. discount.

Charges: Packing at 7s. 6d. per case; Carriage to Liverpool, £1 7s. 9d.; Shipping Charges, Postage, and Bill Stamps, 8s. 3d.; Fire Insurance, 2s. 3d.; Freight at 43s. 6d. and 10 per cent. primage per 40 cub. ft., each case measuring 56″ × 24″ × 16″; Marine Insurance on £175 at 6s. 5d. per cent. plus 5 per cent. and 6d. stamp; Commission, 2½ per cent.

Make out a "Loco," a F.O.B., a C. & F., and a C.I.F. invoice.

"LOCO" INVOICE.

Invoice of 3 cases, Navy Serge, shipped by the undersigned per ss. *India*, Liverpool, for Calcutta, by order and for account and risk of Messrs. Robinson & Co.

INSURANCE EFFECTED HERE. *Indent No.* 793.

		£	s.	d.	£	s.	d.
Calcutta R.&Co. 1/3	3 cases containing— 18 pcs. 54" Navy Serge 6/52½, 6/52, 4/51½, 2/51 = 935 yds. @ 3s. 4½d.	157	15	8			
	2½%	3	18	11			
					153	16	9
	Packing 3 cases . @ 7s. 6d.	1	2	6			
	Carriage to Liverpool . .	1	7	9			
	Shipping Charges, Postage, Bill Stamps		8	3			
	Fire Insurance		2	3			
	Freight on 37½ cub. ft. @ 43s 6d. = £2 - 7 10% = 4 1	2	4	8			
	Marine Insurance on £175 @ 6s. 5d.% = 11 3 5% and stamp = 1 1		12	4			
					5	17	9
					159	14	6
	Commission 2½% .		.	.	3	19	10
	E. & O.E.				163	14	4
	HALIFAX. 28*th December*, 19.. COOK, RUSSELL & Co.						

On F.O.B. terms the cost to Cook, Russell & Co. to purchase the goods and have them placed on board at Liverpool is seen as follows—

	£	s.	d.
Net cost of goods	153	16	9
Packing 3 cases	1	2	6
Carriage to Liverpool	1	7	9
Dock Dues and Postage		5	6
Fire Insurance		2	3
∴ Total cost of goods on board =	156	14	9
Commission 2½% =	3	18	4
∴ Cost per yard on F.O.B. terms =	160	13	1 ÷ 935
= 3s. 5¼d. nearly.			

F.O.B. INVOICE.

INSURANCE NOT EFFECTED BY US. *Indent No.* 793.

		£	s.	d.
Calcutta R.&Co. 1/3	3 cases containing— 18 pcs. 54″ Navy Serge . . . . . 6/52½, 6/52, 4/51½, 2/51 = 935 yds. @ 3s. 5¼d. f.o.b. Liverpool .	160	14	1
	E. & O.E. Halifax. *28th December,* 19.. COOK, RUSSELL & CO.			

NOTE 2.—It is seen that the commission in the "Loco" terms exceeds that in the F.O.B. terms. This is rightly so, for, in the first case, Cook, Russell & Co. see to the freightage and insurance of the goods; whereas in the second case, they do not, so that the additional commission in the first case can be regarded as payment for the additional duties. When manufacturers export their goods direct, the "Loco" and F.O.B. invoices would be similar to those above, except that commission would not appear, as the manufacturers' net price would be such as to give him profit. If an agent of Messrs. Robinson & Co. in England be attending to the freightage and marine insurance, the F.O.B. invoice might possibly be made out to him.

On C. & F. terms, the total cost to Cook, Russell & Co. can easily be calculated, as all the charges of exporting, with the exception of Marine Insurance and Insurance against War Risks, are paid by this firm.

	£	s.	d.
Net cost of goods	153	16	9
Packing, 3 cases	1	2	6
Carriage to Liverpool	1	7	9
Shipping Charges, Postage, Bill Stamps .		8	3
Fire Insurance		2	3
Freight on 37⅓ cub. ft. @ 43s. 6d. and 10%.	2	4	8
	159	2	2
Commission 2½% .	3	19	7

∴ Cost per yard on C. & F. terms = 163 1 9 ÷ 935
= 3s. 5⅞d. nearly.

C. & F. INVOICE.

INSURANCE NOT EFFECTED BY US. *Indent No.* 793.

		£	s.	d.
Calcutta R &Co. 1/3	3 cases containing— 18 pcs. 54″ Navy Serge 6/52½, 6/52, 4/51½, 2/51 = 935 yds @ 3s.5⅞d C. & F. Calcutta	163	2	9
	E. & O.E. Halifax. *28th December,* 1917. COOK, RUSSELL & CO.			

On C.I.F. terms, all the charges involved in exporting, together with the Marine Insurance, are included in the price which is given in the currency of the country to which the goods are shipped.

From "the Loco" invoice it is seen that the total cost, including commission, is £163 14s. 4d. Assuming the rate of exchange to be 1 Rupee = 1s. 6d., this amount is 2182 rupees 14 annas. As European merchants in India mostly use the Imperial Weights and Measures, the length will remain expressed in yards.

$$\therefore \text{ Cost per yard } 2182\tfrac{7}{8} \div 935 \text{ rupees}$$
$$= 2 \text{ rupees } 5\tfrac{1}{4} \text{ annas.}$$

C.I.F. INVOICE.

INSURANCE EFFECTED BY US. *Indent No.* 793.

		Rupees.	Annas.
Calcutta R.&Co. 1/3	3 cases containing— 18 pcs. 54" Navy Serge 6/52½, 6/52, 4/51½, 2/51 = 935 yds. @ Rs.2 5¼ c.i.f. Calcutta E. & O.E. Halifax. 28*th December*, 19.. COOK, RUSSELL & CO.	2,176	13

NOTE 3.—The exporter loses Rs. 6 15 (*i.e.*, 10s. 5d.) by taking the price per yard at Rs. 2 5¼. Should he not have wished to lose this, he could have considered the price per yard as Rs. 2 5½d., in which case he would have had a small profit in addition to his commission.

NOTE 4.—On "Loco," F.O.B., C. & F., and C.I.F. terms, the exporter is not concerned with the landing and dock charges, duty (if any), and transport in the country to which the goods are exported. As regards "Franco" terms, however, before fixing his price the exporter must find out particulars of all these. Usually a shipping agent would be employed by the exporter to receive the goods at the foreign port, and to see to all matters concerning them until they are finally delivered. The exporter must estimate the total of all charges and base his price accordingly.

EXAMPLE (viii)—

Suppose that the goods of the previous example were to be sold on "Franco" terms by Cook, Russell & Co. to R. Chantier & Cie, Lyons, and that the charges were—

Packing, Carriage to Liverpool, Shipping Charges, Postage and Bill Stamps, Fire Insurance, the same as before; Freight at 21s. 9d. and 10 per cent. primage; Marine Insurance on £190 at 4s. 3d. per cent. plus 15 per cent. and Stamp 6d.; Landing Dues, Dock Charges, Duty, etc., and transport from Marseilles to Lyons, fcs. 1250, payment to M. Leblanc, shipping agent, Marseilles, fcs. 75; Commission, 2½ per cent. Also suppose that 2½ per cent. discount off quoted price to be allowed.

Make out the "Franco" invoice, given that the exchange is £1 = 87·60 fcs.

	£	s.	d.
Net cost of goods	153	16	9
Packing, Carriage to Liverpool, Shipping Charges, Postage, Bill Stamps, and Fire Insurance . .	3	–	9
Freight on 37½ cub. ft. @ 21s. 9d. and 10% . .	1	2	4
Marine Insurance on £190 @ 4s. 3d. % and 15% and 6d. stamp		9	9
	£158	9	7

£	s.	d.			
158	–	–	@ 87·60 exchange	fcs.	13840·8
	5	–	„ „ „ . . .		21·9
	4	–	„ „ „ . . .		17·52
		6	„ „ „ . . .		2·19
		1	„ „ „ . . .		·37
158	9	7	„ „ „ . . .		13882·78
Charges, Duty, and transport in France .					1250
Shipping Agent's Charge . . .					75
					15207·78
			Commission $2\frac{1}{2}$% . .		380·19
					15587·97
Adding $2\frac{1}{2}$% so as to allow $2\frac{1}{2}$ discount .					389·70
			Total . . .		15977·67

Again, $52\frac{1}{2}$ yd. $= 52\frac{1}{2} \times \cdot 9144$ metres $= 48\cdot 01$ metres,
and 52 yd $= 47\cdot 55$ m.; $51\frac{1}{2}$ yd. $= 47\cdot 09$ m.; 51 yd. $= 46\cdot 63$ m.
∴ Total length $= 288\cdot 06 + 285\cdot 30 + 188\cdot 36 + 93\cdot 26$ metres
$= 854\cdot 98$ metres.

$$\therefore \text{ Cost per metre } = \frac{15977\cdot 67}{854\cdot 98} \text{ francs} = \text{fcs. } 18\cdot 688$$

say, fcs. 18·69

Thus—

854·98 metres @ fcs. 18·69 per metre . . .	fcs.	15979·58
Discount $2\frac{1}{2}$% . . .		399·49
Net total price		15580·09

Lastly, 54″ $= 1\frac{1}{2} \times \cdot 9144$ m. $= 1\cdot 37$ m.

"FRANCO" INVOICE

INVOICE of 3 cases, Navy Serge, bought by Cook, Russell & Co., Halifax, by order and for account of Messrs. R. Chantier et Cie, Lyons; and forwarded through M. Leblanc, Marseilles.

R.C.&Cie 1/3	3 cases containing— 12 pcs 1·37 m. Navy Serge 6/48·01, 6/47·55, 4/47·09, 2/46·63 = 854·98 m. @ Fcs. 18·69 Discount $2\frac{1}{2}$%	15979·58 399·49 Fcs. 15580·10

NOTE 5.—Adding $2\frac{1}{2}$ per cent. to a quantity and then subtracting $2\frac{1}{2}$ per cent. of the increased quantity will, of course, not result in the original quantity. The difference, however, as $2\frac{1}{2}$ per cent. is a small percentage, is not large.

NOTE 6.—Most exporters would not calculate to so close a degree of accuracy as has been done in this example.

NOTE 7.—If the price per metre on "franco" terms had been merely asked

for, less calculations would have been necessary, for the lengths of the pieces in metres would not have been required.

NOTE 8.—The 2½ per cent. commission (*i.e.*, fcs. 380·19 approx.) represents the profit of Cook, Russell by the transaction. An importer quoting a price in order to receive an order should decide what percentage profit he ought to obtain on his total outlay by the transaction, and should add this on to the gross cost at the same place where the commission 2½ per cent. has been added on in the above example.

TEST EXERCISES III, 3.

(1) Express 315 yd. 2 ft. 3⅝ in. in metres to the nearest mm.

(2) A vessel is capable of holding 2 gall. 1¾ pt. How many litros would it hold (Italian)?

(3) Express 2 oz. (Tr.) 13 dwt. 7 gr. in wigtjes (Dutch).

(4) What is the length of 1 skein of cotton expressed in (i) metres, (ii) ch'ih (Chinese)?

(5) Express the weight of 1 barrel of butter in the Danish units, centners and punds. (1 centner = 100 punds = 110·231 lb.)

(6) Express 755 U.S. bushels in Imperial quarters and bushels.

(7) What weight in tons, cwt., qr. lb. is equivalent to 3 short tons 14 centals?

(8) Draw up a double conversion table from 1 to 9, giving equivalent values in the case of cwt. and centners (Danish) Use the table to express 5 cwt. 2 qr. 16 lb. in centners and punds, and 7·57 centners in cwt., qr , lb.

(9) Calculate multipliers to convert (i) bushels per acre to dry kokus per square cho (Japanese); and (ii) dry kokus per square cho to bushels per acre.

(10) Given that £1 = 66·33 lire, find multipliers correct to 3 places of decimals to convert (i) lire per chilometro into pence per mile, (ii) shillings per lb. into lire per chilogramma; (iii) shillings per bushel into lire per ettolitro.

(11) Using the results in Question (10), express—

(i) ·70 lire per chilometre into pence per mile to nearest ⅛d.
(ii) 3s. 7d. per lb. into lire per chilogramma.
(iii) 9s. 4d. per bushel into lire per ettolitro.

(12) At the rate of exchange, £1 = 3·45 dollars, calculate a multiplier to convert dollars per U.S. bushel into shillings per Imperial bushel. Hence express 1·84 dollars per U.S. bushel in *s. d.* per Imperial bushel.

(13) A vessel in the shape of a rectangular prism is 10¼ in long, 7¼ in. wide. What should be the depth of oil in the vessel if it contain 2 U.S. gallons? What is the cost in *£ s. d.* of 50 U.S. gallons of oil at 32 cents per U.S. gallon, the exchange being £1 = 3·64 dollars?

(14) Express 9·50 marks for 100 kgm. in £ per ton, and find the cost of 1 ton 14 cwt. of a commodity at this rate. (Assume that £1 = 14·60 marks.)

(15) Given that £1 = 3·64 dollars, find the value in English currency of the wheat contained by a bin 6 ft. long, 4 ft. 8 in. broad, and 3 ft. 6 in. deep at 1·64 dollars per U.S. bushel.

Express £9,475 per mile and £7 13s. 6d. per mile in roubles per verst.

(16) Taking 3 dollars 46 cents and 88 francs 10 centimes as each equivalent to £1, construct a table of 1 up to 9 francs in terms of dollars. Use the table in finding the equivalent of 7498·35 francs in dollars and cents to the nearest cent.

(17) Express 75 rupees 9 annas per bigha (Bengal) in *£ s. d.* per acre and £7 11s. 4d. per acre in rupees, annas per bigha. (Assume 1 rupee = 1s. 6d.)

(18) A case weighs 4 cwt. 2 qr. 11 lb What is the cost in *£ s. d.* to convey it 87 Km. at ·47 pesetas per 100 kg. per Km., assuming that £1 = 24·68 pesetas?

(19) If an ardeb of rice be bought for 265 piastres, find the equivalent price in pence per lb., given that £1 = 97½ piastres, and 1 ardeb = 418·21 lb.

(20) Construct a table of three columns giving the value of £1 to £9, 1s. to 9s., and 1d. to 9d. in francs, the value of £1 being 88·205 francs. Hence express £523 14s. 7d. and £295 7s. 11d. in francs.

(21) Calculate the following in English currency—

(i) Transport on 295 cwt. 3 qr. 26 lb. for 75 kilometres at 1·12 franc per tonne per kilometre.

(ii) Transport on 173 cwt. 18 lb. for 157 miles at $0·06 per short ton per mile.

(Assume £1 = 84·48 francs; £1 = $3·48¾.)

(22) The "loco" price at Bradford of a certain kind of cloth was 2s. 11⅝d. per yard. 1,057 yd. were exported to Delhi, the total of charges in England amounting to £4 11s. 7d. and in India 136 rupees 7 annas. Calculate the "franco" price at Delhi in rupees and annas per yard, if (i) the cloth were exported direct from the manufacturers at Bradford; and (ii) the cloth were purchased by a merchant who fixed his price so as to make a profit of 5 per cent. on his total outlay. (Assume that 1 rupee = 1s. 4d.)

(23) In the above question, assuming the sum of £4 11s 7d. included the payment for marine insurance and freight, calculate the price "c.i.f." Bombay in both cases.

(24) An exporter purchased machinery in London for £750. He exported it to Italy, the charges being as follows—

Packing, Cartage, Dock and Shipping Charges, £2 15s. 6d., Freight, £1 2s. 9d.; Marine Insurance and Stamp, £1 11s. 7d. The charges in Italy, comprising Landing Dues, Dock Charges, Transport, and Commission, altogether amounted to 271·30 lire.

Calculate the net "franco" price in lire, assuming that the exporter requires 3 per cent. on his total outlay as profit. Also calculate the "franco" price, if a trade discount of 2½ per cent. is to be allowed. [£1 = 67·20 lire.]

(25) A merchant is willing to sell a certain kind of blanket f.o b. Liverpool at 23s. 9d. per pair. He was asked to quote a c i.f price in supplying 648 pairs to a firm at Genoa. The freight was 27 cases, each 5′ × 4′ 3″ × 3′ 6″, at 20s. 6d. per 40 cub. ft., Marine insurance on £850 at 3s. 3d. per cent. Assuming the merchant requires as additional profit 2½ per cent. on the outlay on freight and insurance, what should his "c.i f." price be?

(26) Convert the "c.i.f." price in the above question to lire per pair, given that £1 = 68 lire.

(27) Draw up a foreign invoice for the following—

Leeds, 21st Dec., 19.. Shipped by R. Cooper & Co. to S. Brenta & Co., Singapore.

S B & Co Singapore.

4 cases each containing 24 pcs. Coloured Serges, each 48 yds. at 3s. 2½d.

Assortment of white, blue, grey, and black in each case. Deduct 1¾ per cent. discount. Each case, 4′ × 2′ 4″ × 4′, 4 cwt. 1 qr. 10 lb.

Extend and add the following charges—

Packing, 13s. 6d. each case, Making-up, 10d per piece; Fire Insurance on £800 at 3s. 1½d. per cent.; Carriage to Liverpool, 23s. 4d. per ton weight; Dues at 9d. per case; Freight, 42s. per ton of 40 cub. ft. and 10 per cent. primage; Marine Insurance on £850 at 7s. 8d. per cent. and 2s. 3d. stamp; Insurance against War Risks at 37s. 9d. per cent.

Show the total. Also draw up an invoice, f.o.b. Liverpool, and another invoice in rupees and annas, c.i.f. Singapore, taking 1 rupee = 1s. 4d.

(28) Make out in proper form an invoice for the following goods shipped.

by T. Longwood & Co., Kidderminster, to Messrs. Martago & Sons, Buenos Aires, per ss. *Swallow* from Liverpool—

2 cases containing 12 pcs. carpet, each 9′ wide and 18′ long;
6 No. A 735 at 3s. 8d. per sq. yard;
4 No. A 471 at 4s. 1½d. per sq. yard;
2 No. B 27 at 2s. 11½d. per sq. yard.

Deduct 1½ per cent. discount.

Charges: Packing and making-up, 14s. 8d. per case; Carriage to Liverpool, 11s. 3d.; Fire Insurance on £50 at 3s. 6d. per cent.; Freight (each case measuring 4′ 9″ × 4′ 9″ × 2′) at 37s. 6d. per ton of 40 cub. ft. and 10 per cent.; Bs/L., 2s. 6d.; Dock Dues, 2s. 4d.; Marine Insurance at 4s. 3d. per cent. and 3d. Stamp.

(29) Make out a "franco" invoice for this shipped by T. Ormond & Co., Erith, to Messrs. Tagaz & Sons, Lisbon, per *Mantua* from London—

1 case, 18 cwt. 3 qr. 10 lb. gross.

Charges. Packing, £1 5s., Cartage and Dock Charges, 19s. 4d.; Freight, at 29s. 4d. per ton weight, Marine Insurance at 2s 10d per cent. on £300 and 10 per cent. and 9d. stamp; Dock Charges, Cartage, etc., at Lisbon, 556 escudos (assume £1 as equivalent to 110 paper escudos).

Make out a "franco" invoice in the proper form, stating a price that will give T. Ormond & Co. a profit of 10 per cent. on their buying price, £270, as well as a 2½ per cent. commission on the total of the charges.

(30) T. Roberts & Co., Bradford, shipped goods on consignment to Adams, Roberts & Co., Colombo. An extract from the *pro forma* Invoice was—

	£	s.	d.
6 cases, each containing 24 pcs. 48″ Coloured Vicunas, each 48 yds. @ 1s. 1½d.	388	16	–
Freight and Insurance	13	7	4
	£402	3	4

Adams, Roberts & Co. sold all the Vicunas at 1 rupee $2\frac{5}{16}$ annas per yard net, and the charges were as follows—

Landing, Clearing, and Cartage, 14 annas per case; Warehouse Rent for 5 Months at 5 annas per case per month; Fire Insurance, 2½ annas per cent. per month on 8,000 rupees; Commission, 2½ per cent. on sale price.

Make out the Account Sale of Adams, Roberts & Co. (Assume 1 rupee = 1s. 4d.)

(*N.B.*—A *pro forma* Invoice is sent with goods on consignment as a guide for effecting a sale and to satisfy Customs regulations.)

(31) Suppose that the above goods were sold on 1st May at the price stated, the terms, however, being 60 days at 5 per cent.; and suppose that Adams, Roberts & Co. made out their Account Sale on 11th May. What amount should be credited to T. Roberts & Co on this date?

(*N B.*—The net proceeds are due to T. Roberts & Co. 60 days after 1st May, so that the amount to their credit on 11th May is the net proceeds less interest on the same for 50 days at 5 per cent. per annum.)

(32) Make out the Account Sale on 11th May, if the terms had been 1½ per cent. trade discount and 60 days at 5 per cent.

(*N.B.*—The commission is the same as in the case above, as it is based on the gross selling price. To obtain the net proceeds, the total of the charges is, of course, deducted from the gross selling price less the discount of 1½ per cent.)

(33) Assuming £1 = 15 rupees, calculate in each of the above three cases how much per cent. the amount credited to T. Roberts & Co. on the Account Sale is greater or less than the *pro forma* invoice price.

(34) A merchant bought 1,200 yds. of cloth at 2s. 4¾d. a yard and sold it all at 14·40 francs per metre. The expenses paid by the merchant amounted to £5 11s. 8d. plus 156·50 francs. Calculate his gain per cent. on the transaction, assuming that £1 = 88·20 francs.

CHAPTER XVII.

ACCOUNTS CURRENT—TURNOVER—FUTURES.

139. AN **ACCOUNT CURRENT** is a statement of transactions which have taken place between two parties acting as principal and agent during an accounting period, usually quarterly or half-yearly. In the case of goods sent on consignment, the consignee frequently renders an Account Current to the consignor by which payments made and amounts received by the former on account of the latter are debited and credited. Interest at some agreed rate, say, 5 per cent., is allowed on each item, in order to find the balance on the date which closes the accounting period. Three examples are given, and the points of difficulty are explained as they occur.

EXAMPLE (1)—

William Jones & Co., London, sold goods to S. Levy & Co., Shanghai, the amounts and dates when payable being: 5th January, 19.., £375; 2nd February, £550; 2nd March, £875; 1st April, £720; 27th April, £250; 25th May, £845; 16th June, £280. Remittances were received for £1,100 due on 26th February, £800 due on 31st March, and £2,000 due on 20th June. The balance in favour of William Jones & Co. at the beginning of the period 1st January to 30th June, 19.., was £127 11s. 7d. By means of an Account Current, find the balance at 30th June, allowing interest pro and con at 5 per cent. per annum.

Interest on £375 for 176 days $= £\frac{375 \times 176 \times 5}{36500} = £\ 9$ 0s. 10d.

„ £550 „ 148 „ $= £\frac{550 \times 148 \times 5}{36500} = £11$ 3s. 0d.

„ £875 „ 120 „ $= £\frac{875 \times 120 \times 5}{36500} = £14$ 7s. 8d.

„ £720 „ 90 „ $= £\frac{720 \times 90 \times 5}{36500} = £\ 8$ 17s. 6d.

„ £250 „ 64 „ $= £\frac{250 \times 64 \times 5}{36500} = £\ 2$ 3s. 10d.

„ £845 „ 36 „ $= £\frac{845 \times 36 \times 5}{36500} = £\ 4$ 3s. 4d.

„ £280 „ 14 „ $= £\frac{280 \times 14 \times 5}{36500} = £\ 0$ 10s. 9d.

„ £127·579 „ 180 „ $= £\frac{127{\cdot}579 \times 180 \times 5}{36500} = £\ 3$ 2s. 11d.

„ £1,100 „ 124 „ $= £\frac{1100 \times 124 \times 5}{36500} = £18$ 13s. 8d.

$$\text{Interest on } £800 \text{ for 91 days} = £\frac{800 \times 91 \times 5}{36500} = £9 \ 19s. \ 5d.$$

$$\text{,, } £2{,}000 \text{ ,, } 10 \text{ ,, } = £\frac{2000 \times 10 \times 5}{36500} = £2 \ 14s. \ 10d.$$

The Account Current is as shown on page 288.

It is noticed in the Account Current that the interest on every item is put down. This, however, would only be done if the interest amounts could readily be obtained from a set of tables, for the final balance can be obtained with much less calculation as follows—

$$\text{Balance of interest on 30th June} = £\frac{5}{36500}\begin{bmatrix}127{\cdot}579 \times 180 + \ldots + 280 \times 14 \\ -\ (1100 \times 124 + \ldots + 2000 \times 10)\end{bmatrix}$$

$$= £\frac{5}{36500}\left(390504 - 229200\right)$$

$$= £\frac{161304 \times 5}{36500} = £22 \ 1s. \ 11d.$$

Thus the process of division has only to be performed once. It is seen that the quantity in the square bracket consists of the sum of the products of the number of pounds with the number of days on the *Dr.* side minus the sum of similar products on the *Cr.* side. Assuming, therefore, that the balance is to be obtained purely by calculation, the Account Current should be put down as shown on page 289.

NOTE 1.—Until the item "Balance of Interest" is entered, the figures put down are the same, no matter what rate of interest is allowed. It is clear that the Balance of Interest is obtained by multiplying the Balance of Products by the rate per cent and dividing by 36,500. Many firms divide by 36,000 instead of 36,500.

NOTE 2.—Assuming that £127 11s. 7d. was the balance carried forward at the end of the previous 31st December, the number of days for this item to the end of June is really 181, although in practice it would often be taken as 180.

NOTE 3.—Letting $n \equiv$ the number of days *before* the last day of the period when the balance of the amounts apart from interest should be paid to completely clear the account, it can be easily proved that

$$n = \frac{(\text{sum of } Dr. \text{ products}) - (\text{sum of } Cr. \text{ products})}{(\text{sum of } Dr. \text{ amounts}) - (\text{sum of } Cr. \text{ amounts})}$$

for if $P_1, P_2, \ldots$ be the *Dr.* amounts; $n_1, n_2, \ldots$ the number of days when payable to the end of the period; and $r \equiv$ rate per cent. per annum;

then total of *Dr.* amounts on last day of period $= P_1 + P_2 + \ldots + \frac{r}{36500}\left(P_1n_1 + P_2n_2 + \ldots\right)$

Similarly, denoting the amounts and number of days in the *Cr.* side by $Q_1, Q_2 \ldots$ and $s_1, s_2 \ldots$,

then total of *Cr.* amounts on last day of period $= Q_1 + Q_2 \ldots + \frac{r}{36500}\left(Q_1s_1 + Q_2s_2 + \ldots\right)$

Messrs. S. Levy & Co.,
Shanghai.

London.
30th *June*. 19..

In Account Current with

Wm. JONES & Co.

Dr.

Date		Particulars	Days	£	s.	d.	£	s.	d.
Jan.	1	To Balance of last a/c	180	3	2	11	127	11	7
,,	5	,, Goods	176	9	–	10	375	–	–
Feb.	2	,, ,,	148	11	3	–	550	–	–
Mar.	2	,, ,,	120	14	7	8	875	–	–
Apr.	1	,, ,,	90	8	17	6	720	–	–
,,	27	,, ,,	64	2	3	10	250	–	–
May	25	,, ,,	36	4	3	4	845	–	–
June	16	,, ,,	14		10	9	280	–	–
				53	9	10			
,,	30	,, Balance of Int.					22	1	11
							4,044	13	6
July	1	,, Balance					144	13	6

Cr.

Date		Particulars	Days	£	s.	d.	£	s.	d.
Feb.	26	By Remittance	124	18	13	8	1,100	–	–
Mar.	31	,, ,,	91	9	19	5	800	–	–
June	20	,, ,,	10	2	14	10	2,000	–	–
,,	30	,, Balance of Int.		22	1	11			
				53	9	10			
		,, Balance car. fwd.					144	13	6
							4,044	13	6

E. & O. E,
London,
30th *June*, 19..
Wm. Jones & Co.

Dr. *Cr.*

Date	Particulars	Days.	Products.	Amounts. £	s.	d.	Date	Particulars	Days.	Products.	Amounts. £	s.	d.
19..							19..						
Jan. 1	To Bal. of last a/c	180	22,964	127	11	7	Feb. 26	By Remittance	124	136,400	1,100	–	–
„ 5	„ Goods	176	66,000	375	–	–	Mar. 31	„ „	91	72,800	800	–	–
Feb. 2	„ „	148	81,400	550	–	–	June 20	„ „	10	20,000	2,000	–	–
Mar. 2	„ „	120	105,000	875	–	–	„ 30	„ Bal. of Products		161,304			
Apr. 1	„ „	90	64,800	720	–	–							
„ 27	„ „	64	16,000	250	–	–							
May 25	„ „	36	30,420	845	–	–							
June 16	„ „	14	3,920	280	–	–							
			390,504							390,504			
„ 30	„ Bal. of Interest at 5%		161,304	22	1	11		„ Bal. car. fwd.			144	13	6
				4,044	13	6					4,044	13	6
July 1	„ Balance			144	13	6							

$$\therefore P_1 + P_2 + \dots + \frac{r}{36500}\left(P_1 n_1 + P_2 n_2 + \dots\right) - \left[Q_1 + Q_2 + \dots + \frac{r}{36500}\left(Q_1 s_1 + Q_2 s_2 + \dots\right)\right]$$

$$= \left(P_1 + P_2 + \dots - Q_1 - Q_2 - \dots\right) + \frac{rn}{36500}\left(P_1 + P_2 + - Q_1 - Q_2 \dots\right)$$

$$\therefore \quad n = \frac{(P_1 n_1 + P_2 n_2 + \dots) - (Q_1 s_1 + Q_2 s_2 + \dots)}{(P_1 + P_2 + \dots) \quad - (Q_1 + Q_2 + \dots)}$$

It would not do to use this formula taking n, n_1 . . . and s_1 as the number of days from a zero date as was done in Paragraph 126, for the payments in the present case are not paid before they are due, and r is the rate of *interest* charged on the items *after* they are due and not the rate of discount as was the case in Paragraph 126.

Applying the formula to the above example, $n = \dfrac{161304}{122{\cdot}579}$ (= 1,316 days)

and interest on £122·579 for $\dfrac{161304}{122{\cdot}579}$ days at 5% per annum

$$= £\frac{122{\cdot}579 \times \dfrac{161304}{122{\cdot}579} \times 5}{36500} = £\frac{161304 \times 5}{36500} = £22 \text{ 1s. 11d.}$$

$\therefore$ Balance amounts on 30th June to £122 11s. 7d. + £22 1s. 11d. = £144 13s. 6d.

NOTE 4.—If Messrs. S. Levy & Co. had made out the Account Current, it would be as the above with the *Dr.* and *Cr.* sides reversed. In some cases, firms or agents in foreign countries forward to the firms at home Accounts Current on which the amounts are entered in the currency of the foreign country in question.

It happens sometimes that one or more items are due for payment after the last day of the accounting period. Theoretically, the balance to be carried forward should be calculated by first finding the balance on the last of the days when payments are due by the method shown in the last example of Accounts Current, and then calculating the sum that amounts to this balance in the number of days from the end of the accounting period to the last day previously mentioned. For example, if the balance on 28th July were calculated to be £95, then the balance on 30th June at 5 per cent. per annum would be £95 × $\dfrac{100}{100\frac{28}{73}}$. In practice, however, it has already been pointed out that when a payment is made before it is due, the discount is based as the interest on the nominal sum at the stated rate per cent. and for the number of days' payment is made before the due date. Thus in the case of those items due after the end of the accounting period, discount calculated in the way just stated is deducted. To distinguish the discounts from the interests, the former are put down in red ink (in thick type as shown in the example on page 292) and, of course, added on to the interests on the other side of the account (or deducted from interests on the same side of the account).

The following example will make the method clear—

Example (ii)—

Robert Mills & Co., New York, sent goods on consignment to Alfred Higgs & Co., Liverpool, the Account Sales being as follows: 5th July, net proceeds £1,557 10s., due 8th September; 30th August, net proceeds £2,159 5s., due 2nd October; 5th October, net proceeds £1,907 12s. 6d., due 5th January; 7th November, net proceeds £2,338, due 10th February. The balance in favour of Robert Mills & Co. was £175 10s. 3d. on 1st July, and this firm drew drafts against the shipments as follows: 6th July, £1,600 due 9th October; 30th August, £2,000 due 2nd December; 8th November, £4,200 due 11th February.

Make out the Account Current of Alfred Higgs & Co. to Robert Mills & Co. (See next page.)

Note 5.—The products and red ink products are totalled and balanced separately. In the case of the ordinary products, the excess of 214,499 on the *Cr.* side means interest to be added that side; but in the case of the red ink products, the excess of 71,050 on the *Dr.* side means interest to be subtracted that side (*i.e.*, added to the *Cr.* side).

A simpler method would be to obtain the net total products on each side by adding the ordinary products and subtracting the sum of the red ink products. The net total products on *Dr* side is 22,200 and that on the *Cr.* side 307,749, giving a *Cr.* balance of 285,549 and $\frac{£285549 \times 6}{36500} = £46\ 18s.\ 9d.$

140. TURNOVER.

The amount of profit gained in any business depends mainly on: (1) difference between cost and selling prices; (2) quantity of goods bought and sold; (3) the expenses incurred in carrying on the business. The success of a business cannot be completely measured by the profits obtained, but by the ratio of the total yearly, half-yearly, or quarterly net profit to the total capital involved. The following example illustrates this—

A and B each buy stock during a year to the value of £5,000, and each sell an equal quantity of similar goods for £7,000. A keeps a large stock: his capital is £8,000 and his expenses £1,000. B keeps a small stock: his capital is £4,000 and his expenses £500.

The ratios of net profit to capital are $\frac{1}{8}$ and $\frac{3}{8}$ respectively, so that B's business could be said to be three times as prosperous as A's.

To gain large profits, a trader might decide to make his selling price greatly exceed his buying price; but by doing this he would not sell rapidly, and his volume of business would be small. On the other hand, he might seek by selling large quantities of goods frequently to obtain large profits, but probably he would have to make his selling price just above cost price to bring about a large volume of business. It will be shown that it is better

MESSRS. ROBERT MILLS & CO., NEW YORK, in Account Current

With ALFRED HIGGS & CO., LIVERPOOL.

Interest at 6% per annum to 1st January, 1932.

Dr.

		Due.	Days	Products.	Amounts. £	s.	d.
1931.							
July 6	To Draft No. 731	Oct. 9	84	134,400	1,600	–	–
Aug. 30	,, ,, No. 784	Dec. 2	30	60,000	2,000	–	–
Nov. 8	,, ,, No. 812	Feb. 11	41	172,200	4,200	–	–
1932.							
Jan. 1	,, Balance of Products .			214,499			
				408,899			
				172,200			
,, ,,	,, Bal. car. fwd.				384	16	6
					8,184	16	6

Cr.

		Due.	Days	Products.	Amounts. £	s.	d.
1931.							
July 1	By Balance of last a/c .	July 1	184	33,294	175	10	3
,, 5	,, Acct. Sales	Sept. 8	115	179,113	1,557	10	–
Aug. 30	,, ,,	Oct. 2	91	196,492	2,159	5	–
Oct. 5	,, ,,	Jan. 5	4	7,630	1,907	12	6
Nov. 7	,, ,,	Feb. 10	40	93,520	2,338	–	–
				408,899			
1932.							
Jan. 1	,, Balance of Red Ink Products .			71,050			
				172,200			
Jan. 1	,, Bal. of Int.			214,499			
,, ,,	,, Bal. of Red Interest .			71,050	46	18	9
					8,184	16	6
,, 1	,, Balance .				384	16	6

E. & O. E.
Liverpool.
1st *Jan.*, 1932.
Alfred Higgs & Co.

for the trader and his customers for the former to aim at increasing his sales, even though his selling price is such that per article the profit obtained is not high.

The quantity of goods bought and sold during a certain period of time is known as the **turnover**, and the time taken to buy and sell a quantity of goods of a certain value is known as the **period of turnover**. Owing to the credit system of payment, four times are involved in every transaction, namely: (1) the time of the receipt of goods; (2) that for the payment for goods; (3) the date of the dispatch of goods; and (4) that of the receipt of money for the goods. These times are not necessarily in the order named above, for the goods might be sold by a trader before the time comes for him to pay for them. The period (2) to (4) is the period of turnover for the trader's capital; and the shorter these periods are, the greater will be the number of transactions during the trading period. As in particular branches of trade, the credit allowed is more or less uniform: the number of transactions per trading period depends very largely on the period (1) to (3) which is known as the period of turnover for the goods in stock, which should, therefore, be as short as possible if the number of transactions is to be as great as possible.

A part of the expenses incurred in business is the warehousing of goods; and the greater the quantity of goods warehoused, the greater the expense will be. Moreover, goods in the warehouse represent capital which bears no interest and also is liable to depreciate. Thus there exist four good reasons why a trader should aim at reducing the time his goods are in stock to a minimum, namely: (1) the turnover during a certain period will be greater; (2) the expense of warehousing will be less; (3) less capital is idle; and (4) less depreciation takes place in the goods.

The advantage of a speedy turnover might be gained in a variety of ways; for example: by skilful advertising; by careful choice of locality; by prompt and honest dealing, etc. This subject is one admitting of interesting theoretical development, but this is beyond the scope of the present book. The following examples illustrate the principles involved—

EXAMPLE (i)—

A and B start business at the beginning of a year by spending £2,400 in buying equal quantities of flour at 75s. per quarter. A sells at 78s. per

quarter and B at 80s. per quarter. A's monthly turnover is 600 qr. and B's, 250 qr.; and each renews his stock at the above cost price at the end of each month. Find (1) their profits for the year; (2) the average values of stock (cost) during the year respectively; (3) the average periods of turnover for the flour in stock respectively.

(1) A's and B's stocks at the beginning of each month are each 640 qr.

$$\text{A's monthly profit} = £\frac{600 \times 3}{20} = £90$$

$$\text{B's} \quad ,, \quad ,, \quad = £\frac{250 \times 5}{20} = £62\ 10s.$$

$\therefore$ Their yearly profits are $\underline{£1,080}$ and $\underline{£750}$ respectively.

(2) Average quantity of flour A's stock $= \dfrac{640 + 40}{2}$ qr. $= 340$ qr.

,, ,, ,, B's ,, $= \dfrac{640 + 390}{2}$ qr. $= 515$ qr.

$\therefore$,, value of A's stock $= £\dfrac{340 \times 75}{20} = \underline{£1,275,}$

and ,, ,, B's ,, $= £\dfrac{515 \times 75}{20} = \underline{£1,931\ 5s.}$

(3) By the end of the year, A has purchased $640 + 11 \times 600$ qr. of flour and has 40 qr. left in stock.

$\therefore$ Quantity of flour bought and sold by A $= 640 + 6,600 - 40$ qr.
$= 7,200$ qr.

Let x days $\equiv$ average period of turnover for A's flour in stock,

$$\text{then } \frac{x}{365} = \frac{340}{7,200}$$

$$\therefore x = \frac{340 \times 365}{7200} = \underline{17} \text{ to nearest day.}$$

Also, Quantity of flour bought and sold by B $= 3,000$ qr.
Let y days $\equiv$ average period of turnover for B's flour in stock,

$$\text{then } \frac{y}{365} = \frac{515}{3,000}$$

$$\therefore y = \frac{515 \times 365}{3,000} = \underline{63} \text{ to nearest day.}$$

Ansr.—£1,080, £750; £1,275, £1,931 5s.; 17 days, 63 days.

EXAMPLE (ii)—

A merchant had stock to the value of £7,350 at the beginning of a year: during the year he purchased goods to the value of £25,200, and at the end of the year his stock was to the value of £8,450. Calculate the average period of warehousing and the number of times the average stock is turned over during the year. Assuming interest is charged on goods warehoused at the rate of 5 per cent. per annum, what percentage of the average value of stock would the interest charged per period of warehousing be?

Value of stock at beginning of the year = £ 7,350
,, purchases during the year = £25,200

Total = £32,550
Stock at the end of the year = £ 8,450

∴ Turnover during the year = £24,100

Average value of goods in stock $= £\frac{7,350 + 8,450}{2} = £7,900.$

Let x days ≡ average period of warehousing,

$$\text{then } \frac{x}{365} = \frac{7,900}{24,100}$$

$$\therefore x = \frac{79 \times 365}{241} = 120 \text{ to the nearest day.}$$

∴ Number of times the average stock is turned over during the year $= \frac{24,100}{7,900} = 3{\cdot}05.$

Interest per cent. per warehousing period on average value of goods in stock $= \frac{120}{365} \times 5 = 1{\cdot}64.$

Ansr—120 days; 3·05 times; the average amount charged for interest per average warehousing period is 1·64% of the average value of the stock.

NOTE 1.—The loss of interest due to keeping goods in stock amounts in the above case to £1·64 for every £100 worth of stock brought in and taken out of the warehouse. The average period of goods in stock is

$$\frac{\text{quantity (or value) of average stock}}{\text{quantity (or value) turned over annually}} \times 365 \text{ days,}$$

and the number of times the average stock is turned over in a year $= \frac{\text{annual turnover}}{\text{average stock.}}$

EXAMPLE (iii)—

The capital of a retailer of boots and shoes was £1,200, his average stock (cost price) £845, his yearly expenses £180, and his average weekly turnover, 48 pairs. He sold his business for £1,200, borrowed £800 at 6 per cent. per annum interest, and with the £2,000 purchased a similar business in a busier neighbourhood. His average stock was now £1,050; his yearly expenses, £320, and the average weekly turnover, 110 pairs. What additional yearly income did he obtain by making the change, assuming that in each case the average cost per pair was 15s. 6d. and the selling price 15 per cent. above cost price. Also find the average period of turnover for goods in stock in each case.

Annual income, 1st case $= £\left(48 \times \frac{31 \times 15}{40 \times 100} \times 52 - 180\right)$

,, ,, 2nd ,, $= £\left(110 \times \frac{31 \times 15}{40 \times 100} \times 52 - 48 - 320\right)$

∴ Increase of income $= £\left(\frac{31 \times 15 \times 52}{40 \times 100} \times 62 - 368 + 180\right)$

$= £(374{\cdot}79 - 188)$

$= £186 \ 15s. \ 10d.$

Average period of turnover, 1st case $= \dfrac{845}{48 \times \frac{31}{40} \times \cdot 52} \times 365$ days

$= \underline{159 \text{ days}}$ to the nearest day

" " " 2nd " $= \dfrac{1050}{110 \times \frac{31}{40} \times 52} \times 365$

$= \underline{86 \text{ days}}$ to the nearest day.

141. "FUTURE" CONTRACTS.

Many manufacturers, in order to secure their profits, undertake to buy raw material at a certain price to be delivered during some future periods of time and then, knowing precisely the cost of manufacture, they are in a position to state a price for the finished articles before they are manufactured. Thus, commodities are arranged to be sold, even though at the time of contract they may not exist. **"Future"** dealings are those which do not immediately come into effect, whereas in **"spot"** dealings the goods are ready for delivery.

"Future" dealings are essential in business. For example: Suppose a contractor on a large scale has fixed his price for the work undertaken, and suppose that after commencing work the price of the necessary materials rises considerably, then he may incur a great loss. If, however, he buys all his materials at stated prices to be delivered during periods of time in the future when he requires them, he is safeguarded from loss against possible increase of prices.

Many brokers speculate in "future" dealings at the great exchanges by buying and selling to gain profit, without coming into possession of the goods and without performing any function necessary to the community. This, of course, does not apply to all brokers; for example: a broker in America might be commissioned in April to purchase 100 qr. of wheat to be delivered at Liverpool in September–October: by carrying out this commission, even though he may not even see the wheat, he is performing a necessary function in trade.

A **"bear"** is a person or firm who sells goods, at a certain price, to be delivered, say, any time in July–August, without as yet possessing the goods himself, with the hope that in the meantime the price will fall so that he can buy at a price considerably less than that at which he has undertaken to sell. He has the option of delivering the goods at any time between 1st July and 31st August, and thus the element of time is in his favour.

A **"bull"** is one who buys with the idea that by the time or before the time the goods are to be delivered, the price will rise so that by selling again

he will gain a profit. In some exchanges it has been known for goods not yet in existence (*e.g*, wheat not yet ripe) to be bought and sold several times in one day. Payment in money is not made by the buyer to the seller: the latter issues a "ticket" representing the goods to the buyer, who passes it on when he comes to sell, and so on.

By means of a Clearing House, the first seller is brought into contact with the last buyer, to whom the goods are delivered, without passing through the hands of the intermediate buyers and sellers, whose only part in the transaction is to draw their profits from or pay in their losses to the Clearing House.

A merchant who has goods which he cannot sell immediately (*e.g.*, the goods may be in a ship on the way to a distant port) might decide to ***hedge*** in order to safeguard himself from a possible fall in prices by the time his goods are offered for sale. To do this he sells an equivalent quantity of goods as "futures." Should prices fall, he loses by selling the original goods at a low price; but in order to fulfil his "future" contract, he is able to purchase at the low price and so gains on this transaction an amount approximately compensating him for his loss in the sale of his goods. This buying in against a sale is called "*covering*" his sale; and although by doing this the merchant loses the chance of making a high profit on the sale of his goods, yet it enables him, if he anticipates a fall in price, to protect himself from heavy loss.

EXAMPLE (IV)—

A man forwarded 400 qr. of wheat from Chicago to Liverpool, and by selling in September at 61s. 4d. per quarter he would neither gain nor lose. Fearing a fall in price, he covered his sale by making a contract to sell 360 qr. at 64s. 9d. per quarter in September–October. On 29th September his wheat was sold in Liverpool at 60s. 3d per quarter; and he purchased 360 qr. at 58s. 10d. per quarter and fulfilled his contract on 10th October, incurring expenses over the latter transaction amounting to £37 15s. Find the extent of his gain or loss, neglecting credit and interest on capital.

$$\text{Amt. lost by the first sale} = £\frac{400 \times 13}{240}$$

$$= £21\ 13\text{s. } 4\text{d.}$$

$$\text{,, gained by "future" contract} = £\frac{360 \times 71}{240} - £37\ 15\text{s.}$$

$$= £68\ 15\text{s.}$$

$$\therefore \text{Gain by the double transaction} = \underline{£47\ 1\text{s. } 8\text{d.}}$$

TEST EXERCISES III, 4.

(1) From the following particulars, make out an Account Current to 30th June, allowing interest pro and con. at the rate of 5 per cent. per annum—

Debit items: Invoices of goods shipped, 20th January, £417 8s. 6d.; 17th February, £703 15s.; 10th March, £455 13s. 4d.; 12th May, £180; 10th June, £623 4s. 6d.

Credit items: Remittances, 20th February, £500; 2nd April, £950; 24th May, £875.

(2) James Roskin & Co., London, sold goods to R. Smith, of Bombay, as follows: 24th July, £452 12s.; 10th September, £387 10s.; 15th November, £575 17s. 6d. Remittances were received for £500 due 20th September, and for £850 due 5th December. The balance in favour of James Roskin & Co. on 1st July was £117 11s. 9d. Make out the Account Current of James Roskin & Co. with R. Smith, allowing interest pro and con. at 6 per cent. per annum, so as to show the balance on 31st December. Also make out the Account Current of R. Smith with James Roskin & Co.

(3) Make out an Account Current to 30th June for the following transactions between Jackson & Co., London, and Egerton & Co., Calcutta, debiting and crediting interest (including "red" interest) at 5 per cent. per annum and showing the balance—

		£	s.	d.
Debit Items:				
Jan. 21.	To Invoice per *Nigel*	257	4	6
Mar. 4.	,, ,, ,, *India*.	512	18	–
Apr. 6.	,, 3 m/s Draft on us due 9th July .	700	–	–
June 14.	,, Invoice per *Slavonia*	385	11	10
Credit Items:				
Feb. 9.	By A/S Wheat ex *Rangoon*	455	17	4
Apr. 10.	,, ,, Shellac ex *Hector*	304	13	8
,, 25.	,, Remittance 3 m/s due 28th July . .	500	–	–
May 10.	,, A/S Jute ex *Wanderer*	103	14	9
June 4.	,, Remittance 3 m/s due 7th September .	350	–	–

*(4) Calculate the date on which the balance of the following sums of money should be paid so as to make a complete settlement—

Dr.				*Cr*	
Mar. 4.	Goods, £122 10s.;	credit, 2 months.	Apr. 10	By Cash	£100
Apr. 20.	,, £ 47 5s.;	,, 60 days.	May 14.	,, ,,	£ 65
May 15.	,, £216 5s.;	,, 2 months.	June 21.	,, ,,	£ 75

Also calculate the sum (without drawing up an Account Current) that should be paid on 30th June, in order to settle the accounts, reckoning interest at 4½ per cent. per annum.

(*N.B.*—Use the formula given in Note 3, Paragraph 139.)

*(5) Calculate the date on which the balance of the following amounts should be paid, so as to make a complete settlement, and hence find the balance on 31st December, reckoning interest at 5 per cent. per annum.

Dr.			*Cr.*	
July 10.	Goods, £545;	credit. 3 months.	By Draft due Oct. 14,	£500
Sept. 15.	,, £712;	,, 2 ,,	,, ,, ,, Nov. 20,	£650
Nov. 6.	,, £840;	,, 3 ,,	,, ,, ,, Feb. 9,	£875

Verify the result by drawing up an Account Current.

(6) Make out an Account Current to 31st December for the following transactions between Messrs. R. Taplow & Co., Leeds, and S. M. Turner & Co., New York. Reckon interest at 6 per cent. per annum.

Dr.

July 1.	To Balance . . .	\$503·15
Aug. 10.	,, £600 90 d/s ex 3·45, due Aug. 10.	
Oct. 4.	,, £450 90 d/s ex 3·44, ,, Oct. 4.	
Nov. 14.	,, £500 90 d/s ex 3·42, ,, Nov. 14.	
Dec. 1.	,, Duty, Freight, and Insurance, \$873·25.	

Cr.

July 10.	By Account Sales, due Aug. 11.	\$ 703·14
Aug. 15.	,, ,, ,, ,, Sept. 18.	\$1,079·73
Oct. 24.	,, ,, ,, ,, Dec. 5.	\$3,142·56
Dec. 14.	,, ,, ,, ,, Feb. 16.	\$1,440·20

(7) Make out an Account Current between N. Lambert et Cie, Paris, with S. Naylor & Co., London, so as to give the balance on 30th June, reckoning interest at 5 per cent. per annum.

Dr.

Jan. 1.	To Balance	. . .	£238 13s. 1d.
Feb. 10.	„ Goods; credit 2 months,		£ 425
Mar. 26.	„ „ „ 3 „		£1,106
Apr. 20.	„ „ „ 3 „		£ 734
June 6.	„ „ „ 3 „		£ 527

Cr

By 28,500·00 fcs.	due	Apr.	4.	ex	82·02
„ 39,750·00	„	May	20,	„	81·54
„ 66,000·00	„	July	5,	„	81·42
„ 88,000·000	„	Aug.	20,	„	86·88

(8) A dealer's stock is turned over 8·14 times annually. What is the average time his goods are kept in stock ? If this time were reduced by 10 days, how many times would he turn over his stock annually ?

(9) A tea merchant at the beginning of a half-year has 835 lb. of tea in stock, which he valued at 2s. 2d. per lb. His average weekly sales amounted to 428 lb. of tea at 2s. 6d per lb., and he purchased on an average 432 lb. at 2s. 2d. at the end of each week. Calculate (i) his profit for the half-year; (ii) his stock at the beginning of the next half-year; (iii) the average value (cost) of the tea in stock; (iv) the average period of turnover for the tea in stock; (v) the number of times he turned over his stock during the half-year.

(10) In the above question, calculate the loss due to the loss of interest on the capital represented by the tea in stock, reckoning interest at 5 per cent. per annum.

(11) A wholesaler had stock to the value of £4,280 at the beginning of a year: during the year he purchased goods to the value of £28,540, and at the end of the year his stock was to the value of £5,860. Apart from loss of interest on capital represented by stock, the cost of warehousing stock to the value of £1,000 was £1 3s. 9d. per week. Calculate the total cost of warehousing, including interest on capital at 5 per cent. per annum for the entire year.

(12) Referring to the data of the previous question, calculate the total average cost of warehousing each £1,000 worth of stock during the period of time it is in the warehouse.

(13) The average amount charged for interest per average warehousing period is 2·13 per cent. of the average value of the goods in stock. Reckoning interest at 6 per cent. per annum, calculate the length of the average warehousing period and the number of times the stock is turned over annually.

(14) The capital of a wholesale dealer in a certain commodity was £3,000; his average stock was £1,140 (cost price); his weekly turnover was 315 cwt., the average cost price being at £1 17s. 3d. per cwt. and the average selling price at £2 0s. 2d. per cwt. His yearly expenses amounted to £995. He borrowed £1,500 at 6 per cent. interest and entered larger premises. During the first year after the change, his average stock was £1,950 (cost price); his weekly turnover was 740 cwt., the average selling price being at £1 19s. 6d. per cwt.; while the cost price was the same as before. His yearly expenses, apart from interest on the loan, amounted to £1,105. Calculate the change in his income, and in each case calculate the average period of turnover for the commodity in stock.

(15) The annual cost of warehousing, apart from loss of interest, is £270; and the average value of stock is £8,400. The average stock is turned over 2·65 times during the year. Calculate the total cost, including loss of

interest at 5 per cent. per annum, of warehousing each £100 worth of stock.

(16) A "bear" undertook to supply 340 tons of metal in March–April at £39 13s. 9d. per ton. On 2nd March he purchased 150 tons at £38 1s. 11d. per ton, and on 16th March he purchased the remaining quantity at £38 19s. 10d. per ton. If his expenses amounted to £88 11s. 6d., what did he gain?

(17) A sold 750 bush. of wheat to B at \$2·80 per bushel: B sold to C at \$2·88 per bushel, and C sold to D at \$3·12 per bushel. The wheat is sent by A direct to D. What should D pay into the Clearing House, and what should A, B, and C each receive?

(18) A sold 413 centals of cotton at \$0·29 per lb. to B, who sold 355 centals at \$0·31 to C and the remainder at \$0·31½ to D. C sold 260 centals at \$0·30 to D and 95 at \$0·29½ to E. The cotton is delivered direct to D and E. What should each pay into or receive from the Clearing House?

(19) A manufacturer entered into a contract with a merchant, who undertook to supply him monthly during four months with 348 tons of a certain raw material at £28 11s. 4d. per ton, the first delivery to take place a month hence. The market prices at the times of delivery were £28 3s. 11d., £28 10s. 9d., £29 2s. 4d., and £29 3s. 8d. per ton respectively. To what extent did the manufacturer gain or lose by the contract each month?

(20) A man undertook to deliver to a miller 450 qr. of wheat at 54s. 4d. per quarter during October. In July, certain events happened which caused the market price of wheat to steadily rise. The man purchased 450 qr. of wheat at 58s. 8d. per quarter in August and also contracted to buy 415 qr. at 61s. 3d. per quarter, delivering during October. He fulfilled both contracts in October; and in December he sold 235 qr. at 63s. 4d. per quarter, and in February he sold the remainder at 63s. 10d. per quarter. Neglecting interest, find his net gain or loss.

(21) Referring to the initial contract in the above question, suppose that the events would cause a fall in price. Find the man's gain if he delivered to the miller, wheat he had purchased in the last week of October at 47s. 4d. per quarter.

(22) A "bull" anticipating a certain event purchased "futures" to the value of £37,000, and he calculated that if the event happened he would be able to sell again for £45,000. He paid a 37½ per cent. premium to a company insuring him to the extent of £5,000 if the event did not take place. Find his net gain under the following circumstances: (i) if the event took place and he sold at £43,800; (ii) if the event did not take place and he sold at £35,750.

(23) A paid 1 per cent. option money to B to deliver him 24 tons of a certain commodity at £3 14s. 10½d. per cwt. during September. A also paid 1 per cent. option money to C to purchase from him 28 tons of the commodity at £3 13s. 9d. per cwt. during August. At the beginning of August the market price was £3 15s. 2d. per cwt., and A claimed his right to purchase from C. The market price continued to rise slightly, and A allowed his option with B to run out. If he sold 28 tons of the commodity in October at £3 16s. 5d. per cwt., find his net gain. Had the market price fallen to £3 13s. 10d. in September, A would have claimed his option to B: find, in this case, the extent of his net gain or loss over the double transaction, assuming that he sold the remaining 4 tons at £3 13s. 10d. per cwt. (*N.B.*—Include option money in calculating gains and losses.)

SECTION IV.

FINANCE.

INTRODUCTION.

MONEY has been defined as "a third commodity, chosen by common consent to be a *means of exchange* and a *measure of value* between every other two commodities." From the point of view of Arithmetic, however, it is best regarded as a means by which the value or desirability of any commodity can be measured. Just as the length of a piece of cloth can be measured in yards, feet, and inches, so its value can be measured in pounds, shillings, and pence. The unit "yard" never varies; but even if a sovereign represents a definite amount of gold, yet should gold be plentiful and the supply of a commodity, say, wheat, be normal, a greater quantity of gold would have to be exchanged for a cwt. of wheat than would be the case if gold were scarce and the supply of wheat the same. This has the effect of raising the price of wheat; but the supply of wheat being the same, its value or desirability to the community is approximately the same, so that the alteration of price is due not to an increase in the value of wheat, but to the fact that the scale of measurement of value has been altered.

As an illustration, if a yard were changed to a length of 2 feet, then a piece of cloth which formerly measured 12 yards would be said to be of length 18 yards.

As a means of measuring the *relative* values or the ratio of the values of any two commodities, however, the employment of gold has in the past been considered satisfactory, as the ratio in question is independent of the supply of gold.

Money, in its broad sense, does not only refer to coins and bullion, but also to any instrument for the transference of wealth. Money is issued as follows—

Post Office. Stamps, Postal Orders, Money Orders (Ordinary and Telegraphic).

Banks. Notes, Cheques, Bills of Exchange, Telegraphic Transfers.

Mint. Bullion, Gold, Silver, and Copper Coins.
In addition, certain negotiable instruments (*e.g.*, a Bill of Lading) can in a way be regarded as money.

The Arithmetic problems, with one or two exceptions, dealt with in the first three chapters are fairly easy; but Chapter XXI is difficult, and should not be read until the student knows thoroughly how to use the tables of seven-figure logarithms.

CHAPTER XVIII.

COINAGE SYSTEMS.

142. BRITISH COINS.

In the Middle Ages, in England, the standard measure of value was the pound-weight (Troy) of silver, the medium of exchange consisting of silver pennies, each of weight $\frac{1}{240}$ lb. (Troy): hence the term "pennyweight" (240 dwt. = 1 lb. Troy). By the **Currency Law of 1816**, the gold standard of Great Britain was fixed as the **Pound Sterling.** In the British Isles, coin is made at the London Mint; but coins of the British system of currency are also minted at Ottawa, in Canada; and at Sydney, Melbourne, and Perth, in Australia.

During the Great War, the "Gold Standard" was suspended, but in 1925 it was restored. While the "Gold Standard" is in operation a Pound Note represents a definite amount of gold and the Bank of England will give gold in exchange for notes. No gold coins have been struck in this country since 1915.

In September, 1931, so much gold had been used in making payments to France and the United States that insufficient was left in the Bank of England for the maintenance of the "Gold Standard," which was therefore suspended. As a result of this, the Bank of England discontinued giving gold in exchange for notes, merchants could not forward bullion to foreign countries in discharge of their debts, the minting of sovereigns was discontinued, and the price of gold as measured by Bank of England notes greatly appreciated.

At the time of writing it seems unlikely that the original "Gold Standard" can be restored, but it may happen that a new standard will be introduced by which the Pound Sterling will again represent a definite amount of gold, less, however, than that under the original standard.

Standard gold is $\frac{11}{12}$ fine (or 22 carat), that is, it contains 11 parts of pure gold to 1 part of alloy, the value of which compared with that of gold is negligible. Before 1915, 480 oz. Troy of standard gold was

used in the manufacture of 1,869 sovereigns or twice the number of half-sovereigns. Thus the sovereign weighs $\frac{480 \times 480}{1869}$ grains (*i.e.*, 123·27447 grains), and the weight of pure gold it contains is $\frac{11}{12} \times 123{\cdot}27447$ grains (*i.e.*, 113·0016 grains).

As it was found impossible to make a sovereign having this exact weight, a margin of 0·2 grains (called the "**remedy**") was allowed. Thus sovereigns weighing over 123·474 grains or under 123·074 grains would not be issued from the Mint, but would be melted down for the manufacture of fresh coins. The "remedy" in the case of a half-sovereign was 0·15 grain, so that the maximum and minimum weights when issued from the Mint were 61·787 grains and 61·487 grains respectively. The **least current weights** of a sovereign and a half-sovereign were 122·500 grains and 61·125 grains respectively.

The **Mint Price of Gold** under the original "Gold Standard" was £3 17s. 10½d. per ounce Troy of standard gold; and the **Bank of England Price of Gold** was, by Act of Parliament, £3 17s. 9d. This difference of 1½d. per ounce Troy represented compensation for the loss of interest sustained by the Bank during the period of about three weeks before the gold could be coined.

Standard silver formerly consisted of $\frac{37}{40}$ of fine silver and $\frac{3}{40}$ of alloy. By an Amending Act passed during the Great War, silver for coinage consists of ½ oz. fine silver and ½ oz. alloy. 12 oz. Troy of this silver (fineness 500) are coined into 66 shillings or the equivalent numbers of other silver coins. The weight and the "remedy" in the case of each silver coin are as follows—

Coin.	*Standard Weight.*	*Remedy.*	*Coin.*	*Standard Weight.*	*Remedy.*
	Grains.	*Grains.*		*Grains.*	*Grains.*
Half-Crown . .	218·18182	1·264	Sixpence . .	43·63636	0·346
Florin . . .	174·54545	0·997	Threepenny-Piece . .	21·81818	0·212
Shilling . .	87·27273	0·578			

Silver is bought not at a fixed price, as in the case of gold, but at the market price, which, of course, varies from time to time. In 1930 the average price per ounce Troy of standard silver was 17¾d., whereas in 1925 it was 32⅛d.

Bronze is an alloy of copper 95 parts; tin, 4 parts, and zinc, 1 part. Particulars of the bronze coins are as follows—

Coin.	*Standard Weight.*	*Remedy.*
	Grains.	*Grains.*
Penny	145·83333	2·91666
Halfpenny	87·50000	1·75000
Farthing	43·75000	0·87500

By a study of the above figures it can be seen that a penny weighs $\frac{1}{3}$ oz. Av.; a halfpenny, $\frac{1}{5}$ oz. Av.; and a farthing, $\frac{1}{10}$ oz. Av. The diameter of a halfpenny is 1 inch.

In **New Zealand**, **Falkland Islands**, **Fiji**, and **Gibraltar**, Imperial sterling coins are the sole legal metallic currency. In **Australia**, certain silver (2s., 1s., 6d., 3d.) and bronze coins have designs different from the Imperial sterling coins, but the weight and composition are the same as those of the Imperial coins having the same denomination. In **South Africa**, the bronze and silver coins correspond to Imperial coins in denomination, weight, and composition, except that the fineness of the silver is 800 (*i.e.*, $\frac{4}{5}$ fine silver).

EXAMPLE (i)—

Calculate the former Mint Price of pure gold, and verify that a sovereign contained £1 worth of pure gold.

Price per oz. (Troy) of pure gold = £3 17s. 10½d. × $\frac{12}{11}$
= £4 4s. 11½d. to nearest ½d.

Value of pure gold in a sovereign of standard weight $= \dfrac{£113·0016 \times 4·248}{480}$

= £1·000 correct to 3 places of decs.

EXAMPLE (ii)—

Find the intrinsic value of 20 shillings of standard weight when the market price of standard silver is $17\frac{9}{16}$d. per ounce (Troy).

Price of fine silver = $17\frac{9}{16} \times \frac{40}{37}$ pence per oz. tr.

∴ Value of silver in

20 shillings $= 20 \times \frac{1}{2} \times \frac{12}{66} \times \frac{281}{16} \times \frac{40}{37}$ pence
= 2s. 10½d.

EXAMPLE (iii)—

If a penny and a halfpenny were of the same thickness, what should be the diameter of a penny?

Let x in. $\equiv$ diameter of a penny,

then $\frac{x^2}{1} = \frac{\frac{1}{3}}{\frac{1}{5}}$ $\quad \therefore x = \sqrt{1{\cdot}666667}$

$= 1{\cdot}29$ in. correct to nearest $\cdot 01''$

EXAMPLE (iv)—

What number of sovereigns each having the least current weight has a total intrinsic value as near as possible £1 less than the nominal value?

Let $n \equiv$ number of sovereigns having the least current value,

then $122{\cdot}5n = 123{\cdot}27447\ (n - 1)$

$\therefore \cdot 77447n = 123{\cdot}27447$

$\therefore n = 159$

159 sovereigns each having the least current weight contain as much gold as 158 sovereigns of standard weight.

143. **THE MINT PAR OF EXCHANGE** is the rate of exchange existing between the pound sterling and the monetary units of any foreign country as obtained by comparison of the amount of pure gold in the former with that in the latter. For example: the weight of pure gold in a gold dollar (which weighs 25·8 gr. and is composed of gold $\frac{9}{10}$ fine) is $\frac{9}{10} \times 25{\cdot}8$ grains. $\therefore$ £1 $= \frac{113{\cdot}0016 \times 10}{9 \times 25{\cdot}8}$ dollars = £4·867. In England, £1 is taken as the unit of exchange with most foreign European countries; but as regards Eastern countries, the foreign monetary unit is generally expressed in British money. Thus, £1 = 124·21 fc. and 1 Rupee = 1s. 6d.

The table on pages 307, 308 gives particulars of the coinage systems of the most important countries; and, for purposes of calculation, the par rate of exchange is given in both ways.

144. SPECIE POINT.

In normal times a merchant could remit money to a foreign country by means of foreign bills of exchange, drafts, or by sending specie or bullion. The latter would usually be done through the agency of a banker, and the total cost would be the amount remitted plus the cost of insurance and freight. If the merchant could purchase a bill of exchange enabling him to remit the amount, and if this would cost him less than if he were to send specie or bullion, naturally he would prefer to discharge his debt

Country.	Monetary Unit.	Par Exchange			Gold Coins.	Silver Coins.
		s.	d.	NUMBER TO £1		
America—						
Argentina	Peso (= 100 centavos)	3	11·58	5·04	5, 2½ pesos	1 peso; 50, 20, 10, 5 cents
Bolivia	Boliviano (= 100 centavos)	1	6	13·33	—	1 bol.; 50, 20, 10 cent
Brazil	Milreis (= 1000 reis)		5·899	40·7	—	—
Chile	Peso (= 100 centavos)		6	40	100, 50, 20 pesos	5, 2, 1 pesos
Colombia	Peso (= 10 centavos)	4	0	5	5, 2½ pesos	1 peso; 50, 20, 10 cent
Costa Rica	Colon (= centesimos)	1	10·9	10·45	—	1 colon; 50, 25, 10, 5 cent
Ecuador	Sucré (= 100 centavos)		9·9	24·33	50, 20 sucrés	2, 1, ½ sucrés
Honduras	Lempira (= 100 centavos)	2	0·66	9·73	—	1 lempira; 50, 20, 1 cent
Liberia	U.S. Dollar (= 100 cents)	4	1·32	4·867	—	50, 25, 10 cents
Mexico	Peso (= 100 centavos)	2	0½	9·76	—	2, 1 pesos; 50, 20, 10 cent
Nicaragua	Cordoba (= 100 centavos)	4	1·32	4·867	—	50, 20, 10, 5 cent
Panama	Balboa (= 100 cents)	4	2	4·8	20, 10, 5, 2½, 1 bal.	50, 20, 10, 5 cents
Paraguay	Peso (= 100 centavos)	4	0	5	—	2, 1, ½ pesos
Peru	Sol (= 100 centavos)	1	1¾	17·45	—	1, ½, ⅕ sol.
Salvador	Colon (= 100 centavos)	2	0·6	9·73	50, 25, 10, 5 colon	1 colon; 50, 20, 12½ cent
United States	Dollar (= 100 cents)	4	1·32	4·867	20, 10, 5, 2½, 1 dollars	$1; 50, 25, 10 cents
Uruguay	Peso (= 100 centesimos)	4	3	4·7	10 pesos	1 peso; 50, 20, 10 cent
Venezuela	Bolivar (= 100 centavos)		9½	25·25	100, 25, 20 bol.	5, 2, 1 bol.; 50, 25 cent
Asia—						
China	Dollar or Yuan (= 100 cents)	1	6	13⅓	—	1 dollar
Japan	Yen (= 100 sen)	2	0½	9·8	20, 10, 5 yen	50, 20, 10 sen
Korea	Won (= 100 chon)	2	0½	9·8	20, 10, 5 won	50, 20, 10 chon
Persia	Pahlavi (= £1 = 20 riyals = 2000 dinars	20	0	1	1, ½ pahlavi	5, 2, 1, ½ riyals
Siam	Baht (= 100 satangs)	1	9·82	11·0	—	1, ½, ¼ baht
Turkey	Lira (= 100 piastres)	18		110 piastres	500, 250, 100, 50, 25 piastres	20, 10, 5, 2, 1, ½ piastres
Africa—						
Algeria, Tunis, Madagascar, Morocco	Same as France					
Egypt	Pound (= 100 piastres)	20	6½	97½ piastres	1, ½ pounds	20, 10, 5, 2 piastres
Tripoli	Same as Italy					

Country.	Monetary Unit.	Par Exchange. s.	Par Exchange. d.	Par Exchange. Number to £1	Gold Coins.	Silver Coins.
British Empire—						
Canada	Dollar (= 100 cents)	4	1⅓	4·867	$10, $5 (Canadian & U.S.) British gold	$1; 50, 25, 10, 5 cents
Ceylon	Indian Rupee (= 100 cents)	1	6	13⅓	British	50, 25, 10 cents
India	Rupee (= 16 annas = 64 pice)	1	6	13⅓	British and 15-rupee piece	1 rupee; 8, 4, 2 annas
Iraq	Dinar (= 1,000 fils)	20	0	1	—	50, 20 fils
Irish Free State	Saorstat Pound	20	0	1	—	2s. 6d., 2s., 1s
Kenya, Tanganyika and Uganda	Shilling (= 100 cents)	1	0	20	—	1s., 50 cents
Palestine	Palestine Pound (= 1,000 milliemes)	20	0	1	—	100, 50 mils.
Sudan	Pound (= 100 piastres)	20	6¼	·975	1 pound, 50 piastres	20, 10, 5, 2 piastres
Europe—						
Austria	Schilling (= 100 groschen)		6·94	34·585	100, 25 schilling	2, 1, ½ schilling
Belgium	Belga (= 5 francs = 50 centimes)		6 6/7	35	20 francs	Nickel 5, 2, 1, ½ fc.
Bulgaria	Lev (= 100 stotinki)		0·35	673·66	—	100, 50, 10 leva
Czechoslovakia	Crown (= 100 heller)		1·22	196·7	—	10, 5 crowns
Denmark	Krone (= 100 ore)	1	1¼	18·159	20, 10 kroner	2, 1 kr.; 25, 10 ore
Finland	Markka (= 100 penni)		1·24	193·23	—	Alum.-bronze 20, 10, 5 markka
France	Franc (= 100 centimes)		1·932	124·21	100 francs	20, 10 francs
German States	Reichsmark (= 100 pfennige)		11·748	20·43	20, 10 mark	5, 3, 2, 1, ½ mark
Greece	Drachma (= 100 lepta)		0·64	375	—	20, 10, 5, 1 dr; 50, 20 lepta
Hungary	Pengo (= 100 fillér)		8·62	27·82	—	1 Pengo; 50, 10 fillér
Italy	Lira (= 100 centesimi)		2·60	92·46	100, 50, 20, 10 lire	20, 10, 5, ½ lire
Netherlands	Florin or Gulden (= 100 cents)	1	7·824	12·107	10, 5 florins	2½, 1 fl.; 50, 25, 10 cents
Norway	Krone (= 100 ore)	1	1¼	18·159	20, 10, 5 kroner	2, 1 kr.; 50, 25, 10 ore
Poland	Zloty (= 100 grosz)		5·5	43·38	100, 50, 20, 10 zloty	10, 5, 1, ½ zloty
Portugal	Escudo (= 100 centavos)	4	5¼	4·50	10, 5, 2, 1 escudo	10, 5, 2½ escudo
Rumania	Leu (= 100 bani)		0·3	813·6	—	100, 20, 10, 5, 2, 1 lei
Russia	Tchernovetz (= 10 roubles of 100 kopecks	21	1⅓	—	—	—
Spain	Peseta (= 100 centimos)		9·516	25·225	—	5, 2, 1 pes.; 50, 20 cents
Sweden	Krona (= 100 ore)	1	1¼	18·159	20, 10, 5 kronor	2, 1 kr.; 50, 25, 10 ore
Switzerland	Franc (= 100 centimes)		9·516	25·225	20, 10 francs	5, 2, 1, ½ francs
Yugoslavia	Dinar (= 100 paras)		·89	276·32	—	20, 10 dinars

by means of the bill of exchange. If, however, the latter cost more, he would prefer to send specie or bullion. Thus brokers and bankers in possession of foreign bills of exchange could not expect to sell them at a higher figure than the cost to send gold. On the other hand, if a merchant had a bill payable in a foreign country, he could sell the bill or wait until maturity. The amount received in specie or bullion would be the amount of the debt less the cost of insurance and freight from the foreign country. Thus he would not be disposed to sell his bill for less than he would receive by adopting the latter course. Theoretically, therefore, the rate of exchange in buying and selling foreign bills ought to vary between **par rate of exchange plus cost of insurance and freight**, and **par rate of exchange less cost of insurance and freight.** These latter are known as the gold **exporting** and **importing points** respectively.

During the suspension of the "Gold Standard," restrictions are placed on the exportation of gold, with the result that the rates of exchange have, with few exceptions, increased or diminished far outside the range limited by the specie points.

EXAMPLE (v)—

From the data given in the table, calculate the weight in grams of pure gold in a 20-mark piece. (1 Kg = 2·205 lb Av.)

Value of gold in a 20-mark piece $= £\frac{20 \times 11·748}{240}$

$\therefore$ Weight of pure gold " " $= \frac{20 \times 11·748 \times 113·0016}{240}$ grains

$= \frac{20 \times 11·748 \times 113·0016 \times 1000}{240 \times 7000 \times 2·205}$ gm

$= \underline{7·17}$ grams, correct to 2 places of dec

EXAMPLE (vi)—

Calculate the par exchange between lire and dollars, expressing 1 dollar in lire and 1 lire in dollars.

4·867 dollars = 92·46 lire

$\therefore$ 1 dollar $= \frac{92·46}{4·867}$ lire $= \underline{19·00 \text{ lire}}$

and 1 lire $= \frac{4·867}{92·46}$ dollar $= \underline{·053 \text{ dollar.}}$

TEST EXERCISES IV, 1.

(1) How many sovereigns of standard weight would weigh 1 ton?

(2) A light sovereign weighing 121·95 gr. is clipped. What is its value compared with a sovereign of full weight?

(3) What is the value of 2 oz. (Tr.) 17 dwt. 11·95 gr. of pure gold when pure gold is worth 119s. 2d. per oz. tr.?

(4) What is the intrinsic value of a half-crown weighing 217·37 gr., if the price of standard silver be 19¼d. per ounce (Troy)?

(5) How many shillings together contain 1 cwt. of pure silver?

(6) What quantities of copper, tin, and zinc does a penny weighing 139·13 gr. contain?

(7) What is the value of metal contained by 20 pennies each of standard weight, when the price of bronze is 2s. 5½d. per lb. (Av.)?

(8) Standard silver is quoted at so many pence per ounce (Troy). Obtain a multiplier to convert this price so as to be quoted at so many cents per ounce Troy of fine silver. When standard silver is 23 3/16d per ounce Troy, what is the price of fine silver in cents per ounce Troy? Also what is the price in pence of an ounce Troy of standard silver when pure silver is quoted as 53½ cents per ounce Troy?

(9) During a certain year the London Mint issued gold coins to the value of £21,301,000. Find the approximate weight of pure gold used in their manufacture.

(10) What is the value of a cubic inch of pure gold at £4 4s. 11½d. per oz. tr. given that the density of gold is 19? (1 gall. of water weighs 10 lb. Av.; 1 gall. = 277·274 cub. in.)

(11) One silver dollar contains 412·5 gr of silver $\frac{9}{10}$ fine. Calculate the fixed U.S. price of fine silver in cents per ounce Troy. Also calculate the equivalent price of standard silver in pence per ounce Troy.

In the following questions use, where necessary, the figures given in the table of coinage systems—

(12) What is the weight in grams of fine gold in a 20-franc piece?

(13) Calculate the least number of 10-gulden pieces which contains more fine gold than 30 sovereigns.

(14) At the Mint par exchange, express (i) £27 10s. in dollars; (ii) 758·45 pesos (Argentina) in £ *s. d.*; (iii) £485 14s. in yen; and (iv) 78·75 kronor in £ *s. d.*

(15) At the Mint par exchange, express (i) 84 dollars 35 cents in francs and centimes; (ii) 135 Swiss francs in U.S. dollars.

(16) A French banker gave an American 25 francs for 1 dollar. What was his percentage gain?

(17) At the par rate of exchange, how many yen are equivalent to 100 Dutch florins?

CHAPTER XIX.

BANKING AND EXCHANGE

145. Some particulars concerning the means whereby payments may be made through the medium of the **Post Office** are as follows—

Postage Stamps can be used for payment up to 1s.

Postal Orders are issued and paid in nearly all parts of the British Empire. They are issued for 6d. and for various other amounts up to 21s. They must be presented for payment within three months of the date of issue, otherwise a fresh commission is charged.

To obtain a **Money Order**, a form has to be filled up on which must be stated the name of the payee, the name of the office at which payment is to be made, the amount to be paid, and the name and address of the sender. For inland money orders, the poundage charged is 4d. up to £3; 6d. up to £10; 8d. up to £20; 10d. up to £30; 1s. up to £40, which is the maximum amount. As regards foreign money orders, the poundage as well as the rates of exchange can be found in the Post Office Guide.

The poundage, in the case of **Inland Telegraph Money Orders,** is the same as for inland money orders plus a fee of 2d., together with the cost of the official Telegram of Advice. Particulars as regards charges for **Foreign Telegraph Money Orders** can also be found in the Post Office Guide.

146. TREASURY.

Treasury Bills are issued by the Government in return for money lent, and they bear a stated rate of interest.

The issue of **Exchequer Bonds** by the Treasury has been discontinued.

147. Through the medium of **BANKS**, payments up to any amounts can be met with extreme convenience and rapidity.

Bank Notes are issued by several banks; but those commonly

known as bank notes are issued by the Bank of England and are legal tender up to any amount from 10s. upwards.

A Cheque is an order written by a person having a current account at a bank to his banker, and it orders the latter to pay a specified sum of money either to a person named or to his "order" or to the "bearer." To be valid, a cheque must bear a 2d. stamp. The **drawer** is the person who draws the cheque, the **drawee** is the bank on whom it is drawn, and the **payee** is the person to whom it is payable. Cheques are payable at the time of presentation to the drawee.

A cheque to **bearer** is paid to anyone presenting the cheque, but a cheque to **order** will only be paid if the payee has indorsed the cheque. The **indorser** may simply sign his name on the back of the cheque, and this constitutes a blank indorsement. Indorsed in this way, a cheque may be passed from hand to hand without further indorsement, and any holder of the cheque can bank or cash the same. If, however, the original payee (whose name, say, is A. Roberts) wishes to ensure that the cheque is payable to a person, B. Williams, and no one else, he would make an **indorsement in full** by writing on the back of the cheque: "Pay B. Williams or order. A. Roberts." Having done this, A. Roberts is no longer the payee, but is known as the indorser, while B. Williams is now the indorsee. The latter may make a blank indorsement or an indorsement in full; for example: "Pay Messrs. C. Robinson & Co. or order. B. Williams." In the latter case, Messrs. C. Robinson & Co. becomes the indorsee. This process can be repeated any number of times, and thus there may be several indorsers who are liable to be called upon for payment should the drawer suspend payment before the cheque is presented. Each indorsee before indorsing the cheque is the payee, and after indorsing the cheque he becomes an indorser.

If a cheque be **crossed**, the drawee will pay the amount only to a bank so that the payee who is not a banker cannot receive cash in return for the cheque should he present it to the drawee. He can, however, pay it into his own bank, and the amount will be added to his account and the bank will receive payment from the drawee at the time settlement is made at the **Bankers' Clearing House.** The rules concerning the indorsement of cheques apply to crossed cheques as well as to those which are not crossed.

Although a cheque is not legal tender, a man who holds a crossed cheque and has no banking account would, after indorsing it, if necessary, have little difficulty in having it accepted in payment, provided, of course, the drawer is a reputable firm or person. Cheques can be crossed in various ways, particulars of which can be found in a text-book on *Commerce.*

Banks send representatives to the Bankers' Clearing House, where the cheques are balanced and then cancelled For example, if a bank A holds cheques payable at a bank B, and B has cheques payable at A, the aggregates would be calculated (by the use of calculating machines) and the difference would be credited to the bank whose aggregate preponderates. The amounts dealt with at the Bankers' Clearing House, as the following table shows, are extremely vast, but the balances in general are comparatively small—

1935	*Town Clearing.*	*Metropolitan Clearing*	*Country Cheque Clearing*	*Total.*
Sept 12 .	£111,485,000	£4,900,000	£8,454,000	£124,839,000
,, 13 .	89,708,000	4,956,000	9,685,000	104,349,000
,, 14 .	54,939,000	5,039 000	10,621,000	70,599,000
,, 16 .	113,050,000	5,723 000	8,738,000	127,511,000
,, 17 .	78,987,000	4,916,000	9,586,000	93,489,000
,, 18 .	82,574,000	5,238,000	10,373,000	98,185,000
Week .	£530,743,000	£30,772,000	£57,457,000	£618,972,000

148. A BILL OF EXCHANGE is legally defined as: "An unconditional order in writing, addressed by one person to another, signed by the person giving it, requiring the person to whom it is addressed to pay on demand, or at a fixed or determinable future time, a sum certain in money to or to the order of a specified person, or to bearer."

If a bill is both drawn and payable in the British Isles, or drawn in the British Isles upon some person resident therein, it is an **Inland Bill.** Any other bill is a foreign bill. All bills should be dated at the time they are drawn.

The **drawer** is the person who writes out and signs the bill; the **drawee** is the person on whom it is drawn; and the **payee** (who may be the drawer himself) is the person to whom, or to whose order, the money is payable. In practice, nearly all bills are payable to the payee or **order.**

Bills for any amount payable on demand or not exceeding three days after date or sight must bear a 2d stamp.

Bills of exchange drawn in the United Kingdom otherwise than on demand must bear a stamp to a value which can readily be estimated from the following table—

Inland.

	£	s.	d.
Not exceeding £10	–	–	2
Exc. £10 and not exc. £25	–	–	3
,, £25 ,, £50	–	–	6
,, £50 ,, £75	–	–	9
,, £75 ,, £100	–	1	–
Every £100 or fract. part	–	1	–

Bills not drawn on demand are made payable either at so many days or months after date or sight. These bills are sent to the drawee for **acceptance.** In case of possible loss, foreign bills are usually made out in duplicate or triplicate, and are numbered. The drawee accepts one of the bills by writing across it the word "**accepted**" and the name of the bank where payment can be received. The unaccepted copies of the bill have no value. In the case of bills made payable a certain time after sight, the date of sight must be stated in the acceptance. After accepting the bill, the drawee is called the **acceptor** and the bill is called an **acceptance.** If the bill in the meantime has not been negotiated, the acceptance is sent to the payee. Acceptances payable to bearer can be transferred from one person to another without indorsement, but those payable to "order" must be indorsed by the payee before they become transferable. These indorsements are similar to those in the case of cheques payable to "order," and thus bills can be transferred from person to person. An indorser, unless he indorses "*without recourse to me,*" is liable to be called upon to pay the amount to any subsequent indorser in the event of the acceptor failing to pay the bill at maturity.

Usance is the name given to the customary period of time at which bills are drawn at or on certain places. The usance in the case of some important foreign towns is as follows—

14 days' sight (14 d/s).	30 days' date (30 d/d).	1 month's date (1 m/d).	2 months' sight (2 m/s).	3 months' date (3 m/d).
Berlin	Bordeaux	Amsterdam	Gibraltar	Genoa
Dantzic	Geneva	Antwerp	Madrid	Leghorn
Dresden	Lisbon	Bremen		Milan
Frankfort	Malta		60 days' date	Naples
Leipzig	Oporto	Rotterdam		Palermo
Trieste	Paris		Barcelona	Venice
Vienna		60 days' sight	Cadiz	
		New York		

A bill is payable on demand—

(*a*) When expressed to be payable on demand or at sight or on presentation; or

(*b*) When no time of payment is expressed; or

(*c*) When overdue after being accepted and indorsed.

A bill is payable at a determinable future time within the meaning of the Act when it is expressed to be payable at a fixed period after date, after sight or after a specified occurrence.

Three Days of Grace are, where the bill does not otherwise provide, added to the time of payment fixed by the bill and the bill is payable on the last day of grace unless—

(*a*) The last day of grace falls on Sunday, Christmas Day, Good Friday or a public fast or thanksgiving day on which occasions (with the following exception) the bill is payable on the preceding business day.

(*b*) The last day of grace is (i) a bank holiday (other than Christmas Day or Good Friday), or (ii) a Sunday and the second day of grace a bank holiday, on which occasions the bill is payable on the succeeding business day.

No days of grace are allowed on Government bills and Bank Post bills The numbers of days of grace differ in foreign countries: in New York and in many European countries, none is allowed.

There are no days of grace allowed in the case of bills or promissory notes which are payable on demand or at sight, or when the allowance of days of grace has been specially negatived on the face of the bill or promissory note in question

A study of the following example shows how the last days in which bills are payable can be computed—

EXAMPLE (i)—

A bill was drawn on London and dated 25th February, 1932 Calculate the last day on which payment must be made, the term being (1) on demand, (2) 30 d/d., (3) 1 m/d. If the acceptor had first seen the bill and accepted it on Feb 29th, what would have been the last days had the term been (4) 30 d/s., (5) 3 m/s.?

25th February, 1932, was a Thursday.

(1) As the bill is payable on demand, payment must be made whenever it is presented.

(2) The due date was Tuesday, 29th March, which was the last day for payment.

(3) The due date was Monday, 28th March, which was a Bank Holiday, so that Tuesday, 29th March, was the last day for payment.

(4) The last day for payment was Saturday, 2nd April.

(5) ,, ,, ,, Wednesday, 1st June.

To **negotiate** a bill is simply to transfer it to another party by indorsement for its equivalent in money or goods. When a copy of a bill accompanied by documents for security is transferred to another person before the bill has been accepted, the bill is also said to be negotiated (*cf.* Chap. XV).

To **retire** a bill is to pay it at maturity, that is, when it becomes due. When a bill is paid before the day of maturity, a discount is allowed and the bill is **retired under discount.**

To **renew** a bill is to accept in place of a previous one which was not paid when due.

The discounting and the buying and selling of bills is dealt with later on in the chapter.

149. By means of **TELEGRAPHIC TRANSFERS**, money can be transferred by cable from one country to another. To remit money in this way, a person would go to a bank having a

branch or agent at the place to which the amount is to be remitted, pay the equivalent amount at the current rate of exchange plus the cost of telegraphing, and give the necessary instructions.

In addition to the above, a **PROMISSORY NOTE,** which is a promise in writing made by one person to another, signed by the maker, engaging to pay on demand or at a specified time a sum of money to or to the order of a specified person (the payee) or to bearer, can, if properly stamped and indorsed, if necessary, be used as a negotiable instrument. A promissory note, whether payable on demand or not, is subject to the same stamp duty as an inland bill of exchange not payable on demand. The rules of indorsement are, with slight modification as to liability, the same as in the case of bills of exchange, and the three days' grace are allowed by law.

An **I O U** is not a negotiable instrument.

150. BANKS.

There are three kinds of banks: Chartered Banks, Joint Stock Banks, and Private Banks.

The chartered banks are the Bank of England, the Bank of Scotland, and the Bank of Ireland.

The **Bank of England** consists of the Issue Department and the Banking Department. The issue of notes on documentary security (not gold) is known as the *fiduciary* issue. The extent of this issue has in recent years been increased considerably. The Banking Department to a large extent manages the financial affairs and keeps the accounts of the Government. It also acts as the banker of all the chief joint stock banks and for brokers in a large way of business, and regulates the price paid for short loans and loans of a longer period. The **Bank Rate** is the official minimum rate per cent. charged by the Bank of England in discounting bills of exchange having first-class security. The rate of interest charged by other banks for short loans is usually somewhat less than the Bank Rate, but it rises and falls as the Bank Rate rises and falls. The Bank of England is by law bound to issue publicly each week a statement known as the **Bank Return,** which shows the position of each department. The following is a specimen Bank Return—

Issue Department.

Notes issued—		Government debt .	£11,015,100
In circulation . .	£398,149,572	Other Government securities . . .	246,415,705
In banking department . . .	55,328,136	Other securities . .	1,029,844
		Silver coin . .	1,539,351
		Amount of fiduciary issue . . .	260,000,000
		Gold coin and bullion .	193,477,708
	£453,477,708		£453,477,708

Banking Department.

Capital . . .	£14,553,000	Government securities.	£83,159,999
Rest	3,720,564	Other securities—	
Public deposits[1] . .	17,464,679	Discounts and advances . .	11,852,658
Other deposits—		Securities . .	14,159,650
Bankers . .	92,018,504	Notes . . .	55,328,136
Other accounts .	37,581,219	Gold and silver coin .	837,523
	£165,337,966		£165,337,966

—	Amount.	Inc or dec. on last week.	Inc. or dec. on last year.
	£	£	£
Rest	3,720,564	+ 16,393	+ 16,923
Public deposits . .	17,464,679	+ 1,428,969	— 3,034,380
Other deposits—			
Bankers . . .	92,018,504	+ 982,390	— 7,818,715
Other accounts . .	37,581,219	— 261,547	+ 80,261
Government securities .	83,159,999	— 1,390,000	+ 1,480,835
Other securities—			
Discounts and advances .	11,852,658	— 566,176	+ 5,714,820
Securities . . .	14,159,650	+ 1,927,536	+ 1,931,685
Reserve	56,165,659	+ 2,194,845	— 19,888,311
Note circulation . .	398,149,572	— 2,106,710	+ 21,769,826
Coin and bullion . .	194,315,231	+ 88,135	+ 1,881,515
Proportion . . .	$38\frac{3}{16}$%	+ $\frac{15}{16}$	— 10

[1] Including Exchequer, Savings Banks, Commissioners of National Debt, and Dividend Accounts.

Joint Stock Banks are managed by a Board of Directors, which meets usually once a week, thus enabling the Manager to consult them over matters other than the ordinary routine of business. Their capital has largely been subscribed by the public, and a balance sheet must be exhibited at periodic times, wherein is given the rate of dividend out of realized profits. Many of the joint stock banks have several branches in London, in provincial and in foreign towns.

A **Private Bank** is managed by a few persons who compose the firm. It conducts business in the same way as a joint stock bank; but, being a private concern, is not obliged to publish a balance sheet.

151. BANKING.

The business carried on by banks consists of: (1) Receiving deposits at interest; (2) advancing money on securities; (3) keeping current accounts of customers; (4) discounting and negotiating bills of exchange; (5) remitting money for customers; (6) storing valuable articles and documents for customers.

Money can be deposited for a fixed term, say, six months, or on the condition that a certain number of days' notice be given. The longer the notice, the higher, as a rule, is the rate of interest allowed. As a rule, banks will advance money only on security; for example: deeds, valuable articles, bills of exchange; and the rate of interest charged is somewhat higher than that allowed on money deposited. Most loans are made to brokers and stock-brokers, and the rate of interest charged is greater for monthly loans than for daily or weekly loans. The rates of interest quoted recently were as follows: Day-to-day money, 3½ per cent; seven-day loans, 4-5 per cent.; bankers' deposits, 4 per cent (2 per cent less than bank rate). These rates are approximate, and it is seen that banks will charge a slightly less rate of interest on loans for an indefinite period, when they reserve to themselves the power to call for the return of the money at any time, than they would if they had to give some days' notice.

The **Post Office Savings Bank** pays interest at the rate of 2½ per cent. per annum, payable at the end of each year, on completed £'s, but only if the money has stood to the credit of the depositor during a complete calendar month. Thus the interest is ½d. per complete £ per complete calendar month; for example: if £10 be deposited on 1st May and withdrawn on 30th June, no interest is payable; but if deposited a day before and withdrawn a day later, the interest would be 10d. Bills of exchange are not accepted; but postal orders and cheques, if not crossed to a particular bank, are accepted.

EXAMPLE (ii)—

A man borrowed £1,500 on 7th March at $4\frac{1}{2}$ per cent. per annum. On 21st March it was renewed at 5 per cent. per annum. on 30th March it was again renewed at 6 per cent.; and the money, together with interest, was repaid on 9th April. What was the amount repaid ?

$$\text{Interest from 7th Mar. to 21st Mar.} = £\frac{1500 \times 14 \times 4\frac{1}{2}}{365 \times 100}$$

$$\text{,, ,, 21st Mar. ,, 30th Mar.} = £\frac{1500 \times 9 \times 5}{365 \times 100}$$

$$\text{,, ,, 30th Mar. ,, 9th Apr.} = £\frac{1500 \times 10 \times 6}{365 \times 100}$$

$$\therefore \text{ Total interest} = £\frac{15}{365} \times (63 + 45 + 60)$$

$$= £\frac{3 \times 168}{73} = £6 \text{ 18s 1d.}$$

$$\therefore \text{ Amount repaid} = £1,506 \text{ 18s 1d.}$$

EXAMPLE (iii)—

A person on 31st December had £35 14s 8d. deposited in the Post Office Savings Bank. During the following year his deposits were: 10th March, £2 5s, 8th May, £5 10s, 8th July, £4 12s., 12th November, £4 his withdrawals were 23rd May, £8, 15th August, £10. To what interest is he entitled by the end of the year ? Find the actual rate per cent, had fractions of £'s borne interest and had the interest been based on the number of days.

The minimum numbers of complete £'s standing to the depositor's credit each month were 35, 35, 35, 37, 35, 35, 35, 30, 30, 30, 30, 34 respectively.

$$\therefore \text{ Total interest} = 401 \times \tfrac{1}{2} \text{ pence}$$
$$= 16\text{s } 8\text{d.}$$

Let $r \equiv$ actual rate per cent.

$$\text{then } £\cdot35733r + £\frac{r}{36500} \times \begin{pmatrix} 2\frac{1}{4} \times 296 + 5\frac{1}{2} \times 237 + 4\frac{3}{5} \times 176 + 4 \times 49 - \\ 8 \times 222 - 10 \times 138 \end{pmatrix}$$

$$= \text{total interest for the year.}$$

$$\text{i.e., } \cdot35733r - \frac{180\cdot9r}{36500} = \frac{2}{3}$$

$$\text{i.e., } \cdot35733r - \cdot00496r = \tfrac{2}{3}$$

$$\therefore r = \frac{2}{3 \times \cdot35237}$$

$$= 1\cdot892.$$

EXAMPLE (iv)—

A man deposited money in a bank paying interest at $2\frac{1}{2}$ per cent. per annum half-yearly to enable him to have £500 for the education of his son

sixteen years from the date the money was deposited. After six years, the bank increased the rate of interest to 3 per cent. per annum. What sum could the man have withdrawn at this date, so that at the end of the additional ten years his deposit would amount to £500 ?

Sum that amounts to £500 in 10 yr. @ 2½%, payable half-yearly $= £\dfrac{500}{(1{\cdot}0125)^{20}}$

Sum that amounts to £500 in 10 yr. @ 3 %, payable half-yearly $= £\dfrac{500}{(1{\cdot}015)^{20}}$

$$\therefore \text{ Amount he could withdraw } = £500 \times \left[\frac{1}{(1{\cdot}0125)^{20}} - \frac{1}{(1{\cdot}015)^{20}}\right]$$

$$= £500 \times \left({\cdot}7800097 - {\cdot}7424719\right)$$

$$= £500 \times {\cdot}0375378$$

$$= £18{\cdot}7689$$

$$= £18 \ 15s. \ 5d.$$

152. INTEREST TABLES.

In practice, to find the simple interest or the compound interest on a sum of money for a given time at a given rate of interest, tables are used. Space will not permit of the complete tables being given, but the following parts are sufficient to enable the methods of using the tables to be understood.

INTEREST ON £100 AT 2%, 3%, 4%, AND 5% FOR GIVEN NUMBERS OF DAYS.

Days	2%	3%	4%	5%	Days	2%	3%	4%	5%
	s. d.	*s. d.*	*s. d.*	*s. d.*		*s. d.*	*s. d.*	*s. d.*	*s. d.*
1	0 1¼	0 2	0 2¾	0 3¼	11	1 2½	1 9¾	2 5	3 0¼
2	0 2¾	0 4	0 5¼	0 6½	12	1 3¾	1 11¾	2 7½	3 3½
3	0 4	0 6	0 8	0 10	13	1 5	2 1¾	2 10¼	3 6¾
4	0 5¼	0 8	0 10½	1 1¼	14	1 6½	2 3¾	3 0¾	3 10
5	0 6½	0 10	1 1¼	1 4½	15	1 7¾	2 5½	3 3½	4 1¼
6	0 8	0 11¾	1 3¾	1 7¾	16	1 9	2 7½	3 6	4 4¾
7	0 9¼	1 1¾	1 6½	1 11	17	1 10½	2 9½	3 8¾	4 8
8	0 10½	1 3¾	1 9	2 2¼	18	1 11¾	2 11½	3 11¼	4 11¼
9	0 11¾	1 5¾	1 11½	2 5½	19	2 1	3 1½	4 2	5 2½
10	1 1¼	1 7¾	2 2¼	2 9	20	2 2¼	3 3½	4 4¾	5 5¾

The interest at 1 per cent., ½ per cent., ¼ per cent., ⅛ per cent. can be obtained by dividing the sums in the 2 per cent. column by 2, 4, 8, 16 respectively. The solution of Example (ii) by the use of the above table is obtained as follows—

		£	s.	d.
Interest on £100 for 14 days @ 4½%	=		3	0¾
				4½
,, ,, 9 ,, ,, 5 %	=		2	5½
,, ,, 10 ,, ,, 6%	=		3	3½
Total interest on £100	=		9	2¼
∴ ,, ,, £1,500	=	6	17	9¾
∴ Amount repaid	=	£1,506	17s.	10d.

Note 1.—The small discrepancy is due to the fact that the sums of money in the table are correct to the nearest ¼d., so that on multiplication by 15 the error is magnified 15 times. It is clear that the interest for 10 days at 6 per cent. is the same as that for 20 days at 3 per cent. Had the loan been, say, £1,566 18s., the interest would have been £6 17s. 9¾d. + £·669 × ·459 = £6 17s. 9¾d. + £·304 = £7 3s. 10¾d.

Amount of 1 Unit of Money at 2½%, 3%, 3½%, 4%, 4½%, 5%,
PAYABLE YEARLY, FOR GIVEN NUMBER OF YEARS.

Years.	2½%	3%	3½%	4%	4½%	5%
1 .	1·025000	1·030000	1·035000	1·040000	1·045000	1·050000
2 .	1·050625	1·060900	1·071225	1·081600	1·092025	1·102500
3 .	1·076890	1·092727	1·108718	1·124864	1·141166	1·157625
4 .	1·103812	1·125509	1·147524	1·169859	1·192518	1·215506
5 .	1·131407	1·159273	1·187688	1·216653	1·246181	1·276281
6 .	1·159692	1·194051	1·229256	1·265319	1·302259	1·340095
7 .	1·183684	1·229873	1·272280	1·315932	1·360860	1·407100
8 .	1·218401	1·266771	1·316809	1·368569	1·422098	1·477455
9 .	1·248861	1·304774	1·362897	1·423311	1·486092	1·551328
10 .	1·280083	1·343913	1·410598	1·480243	1·552966	1·628894
11 .	1·312085	1·384231	1·459969	1·539453	1·622850	1·710339
12 .	1·344889	1·425758	1·511068	1·601031	1·695878	1·795856
13 .	1·378511	1·468531	1·563955	1·665072	1·772192	1·885649
14 .	1·412974	1·512587	1·618694	1·731675	1·851941	1·979931
15 .	1·448298	1·557965	1·675348	1·800942	1·935279	2·078928
16 .	1·484505	1·604708	1·733985	1·872980	2·022366	2·182874
17 .	1·521618	1·652849	1·794674	1·947899	2·113372	2·292018
18 .	1·559658	1·702434	1·857487	2·025815	2·208474	2·406619
19 .	1·598649	1·753507	1·922498	2·106847	2·307855	2·526950
20 .	1·638615	1·806113	1·989785	2·191121	2·411708	2·653298

The above table applies equally well if the unit of money be a pound sterling, shilling, franc, dollar, etc. To find the amount when interest is payable half-yearly, by halving the rate per cent. and doubling the time, the result can be obtained from the table (assuming, of course, it be complete). For example, in 8 years at 5 per cent. per annum payable half-yearly, £1 amounts to £1·484505; in 5 years' time at 6 per cent. per annum, payable half-yearly, £$\frac{1}{1{\cdot}343913}$ amounts to £1.

The amount at 2½ per cent. per annum, interest payable half-yearly, cannot be obtained from the above table; but had the

interest in Example (iv) been payable yearly, the solution could have been obtained as follows—

Sum that amounts to £500 in 10 yr. @ $2\frac{1}{2}\%$ int. payable yearly $= £\frac{500}{1\cdot280083}$

Sum that amounts to £500 in 10 yr. @ 3% int. payable yearly $= £\frac{500}{1\cdot343913}$

$$\therefore \text{ Amount he could withdraw} = £500 \times \left(\frac{1}{1\cdot280083} - \frac{1}{1\cdot343913}\right)$$
$$= £500 \times (\cdot781199 - \cdot744095)$$
$$= £500 \times \cdot037104$$
$$= £18 \text{ 11s. 0d.}$$

NOTE 2.—It should be remembered that when the need arises to make a considerable number of similar calculations and no table is available, a table of nines should first be constructed.

EXAMPLE (v)—

What is the compound interest on £714 12s. for 8 yr. 10 mon. at $3\frac{1}{2}$ per cent. per annum?

$$\begin{aligned}
\text{Amount at the end of 8 yr.} &= £714\cdot6 \times 1\cdot316809 \\
&= £940\cdot9917 \\
\text{Interest for next 10 mon.} &= £9\cdot409917 \times \tfrac{5}{6} \times \tfrac{7}{2} \\
&= £9\cdot409917 \times (3 - \tfrac{1}{12}) \\
&= £27\cdot4456 \\
\therefore \text{ Amount at the end of 8 yr. 10 mon} &= £968\cdot4373 \\
\therefore \text{ Interest} &= £253 \text{ 16s. 9d.}
\end{aligned}$$

153. A CURRENT ACCOUNT is an account with a bank, and is such that it can be withdrawn in whole or in part by cheque. In general, banks require that a sum not less than £50 should stand to the credit of a customer having a current account with them; but in the case of well-known and reliable customers, bankers sometimes honour cheques even though by doing so the account is overdrawn. As a rule, accounts can only be overdrawn if security can be given. Most London banks do not pay interest on current accounts; but, on the other hand, they do not charge their customers for the services rendered to them by (1) keeping their accounts; (2) giving them the means whereby they can remit by cheques; (3) collecting their cheques and bills of exchange which are payable at other banks; (4) discounting bills of exchange for them when required; and (5) giving them information concerning financial affairs and the standing of customers. Certain provincial banks, especially those in the North of England, allow a small rate of interest on current accounts; but for

the services rendered to a customer, a commission proportionate to the extent of those services is charged.

EXAMPLE (vi)—

The amount standing to the credit of a man's current account on 1st January was £245 7s. 3d. The bank allowed interest at $2\frac{1}{2}$ per cent. per annum payable on 30th June and 31st December, and charged $\frac{1}{4}$ per cent. commission for collecting and paying cheques. At the end of each month the bank collected cheques to the value of £60 for the man and paid out a total amount of £50. What sum stood to the credit of the man's account on 30th June the same year?

$$\text{Commission charged each month} = £\tfrac{1}{400} \times 110 = £\tfrac{11}{40}$$

$$\therefore \text{ Amount added to account at the end of each month} = £9\tfrac{29}{40}$$

$$= £9 \text{ 14s. 6d.}$$

$$\text{Interest on £245 7s. 3d. for six months} = £2{\cdot}453625 \times 1\tfrac{1}{4} = £3{\cdot}0670$$

$$\text{Additional interest} = £9\tfrac{29}{40} \times \tfrac{1}{40} \times (\tfrac{5}{12} + \tfrac{4}{12} + \tfrac{3}{12} + \tfrac{2}{12} + \tfrac{1}{12})$$

$$= £\frac{389 \times 15}{40 \times 40 \times 12} = £{\cdot}3039$$

$$\therefore \text{ Total interest} = £3{\cdot}3709$$

$$= £3 \text{ 7s. 5d.}$$

$$\therefore \text{ Total of current account} = £245 \text{ 7s. 3d.} + £58 \text{ 7s. 0d.} + £3 \text{ 7s. 5d.}$$

$$= £307 \text{ 1s. 8d.}$$

154. DISCOUNTING AND RETIRING OF BILLS.

Banks are always willing to *discount* bills of exchange for their customers. If the sum named on the bill be in British currency (inland bill or foreign bill drawn on a place where British currency is used), the amount the banker will pay is the nominal sum less discount reckoned at so much per cent. per annum. For example, if the nominal sum be £P, the rate r per cent., and the number of days from the date of discounting the bill to that of maturity plus the three days of grace n, then the discount would be £$\frac{Prn.}{36500}$ The rate of discount depends partly on the Bank Rate, partly on the length of time before maturity, and partly on the financial standing of the acceptor of the bill.

Bank bills and trade bills accepted by firms of high repute can be discounted at a lower rate than those accepted by those who are not well known or whose financial position is unsound. Should the banker be doubtful as to the receiving of the money at maturity, he would only allow the bill to be discounted *with*

recourse, so that in the event of the acceptor failing to pay, he could call upon the customer for the payment of the money.

Should the bill be in the currency of a foreign country, the banker would quote his customer a rate of exchange which would be somewhat higher (assuming £1 be expressed in the foreign unit) than the rate for bills payable on demand. The difference calculated from the two rates represents the discount, but in practice it is never spoken of as such.

Bankers, after discounting bills for their customers, usually wait for the bills to mature; and in some cases they might use certain foreign bills for the remitting of money abroad for their customers.

It frequently happens that after a bill has been discounted by a bank, the acceptor is in a position to make payment before the date of maturity. The bank would allow him to **retire the bill under discount** at a rate possibly slightly less than the rate at which the bill was discounted.

Example (vii)—

A bill for £713 15s. was drawn on 3rd April and made payable three months after date. It was discounted on 15th April at $4\frac{1}{4}$ per cent. What was the discounted value of the bill?

The time the bill has to run is from 15th April to 6th July.

$$\therefore \text{ Number of days} = 82$$

$$\therefore \text{ Discount} = £\frac{2855 \times 17 \times 82}{4 \times 4 \times 36500}$$

$$= £6 \text{ 16s. 4d. to nearest 1d.}$$

$$\therefore \text{ Discounted value of bill} = \underline{£706 \text{ 18s. 8d.}}$$

Example (viii)—

A bill for £950 payable 4 months after sight was first seen and accepted on 5th August. It was retired by the acceptor under discount at $3\frac{3}{4}$ per cent. on 18th September. What sum did the latter pay?

The time the bill has to run is from 18th September to 8th December.

$$\therefore \text{ Number of days} = 81$$

$$\therefore \text{ Rebate} = £\frac{950 \times 15 \times 81}{4 \times 36500}$$

$$= £7 \text{ 18s. 1d.}$$

$$\therefore \text{ The acceptor paid} = \underline{£942 \text{ 1s. 11d.}}$$

Example (ix)—

A man wishes to retire a bill for $8,000 payable to the London branch of an American bank in 28 days' time. If the rate of exchange for New York

cheques is 3·45 and the bank ask for a payment of £2311 14s. 6d., at what rate per cent. has the rebate been calculated?

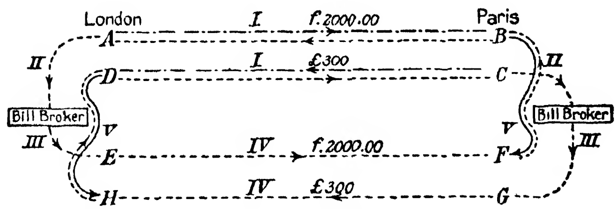
$$\text{Nominal value of bill at 3·45 exchange} = \pounds\frac{8000}{3·45}$$
$$= \pounds 2318·841$$
$$\therefore \text{ Rebate} = \pounds 7·116$$

Let $r \equiv$ rate per cent. of discount,

$$\text{then } \frac{2318·841 \times 28r}{36500} = 7·116$$
$$\therefore r = \frac{711·6 \times 365}{2318·841 \times 28}$$
$$= 4, \text{ approx.}$$

155. FUNCTION OF A BILL OF EXCHANGE.

Bill Brokers are those who buy and sell bills of exchange. They perform an extremely valuable service in the carrying on of international trade: for merchants wishing to remit to foreign countries are able to buy from bill brokers, bills of exchange drawn on the places in question. Thus the transfer of bullion and specie between foreign countries is reduced to a minimum. A simple illustration is given as follows—



I. A, of London, sells goods to B, of Paris. A draws a bill of exchange for f. 2000·00 on B, and sends it to B, who accepts it and returns it to A. C, of Paris, sells goods to D, of London. C draws a bill for £300 on D and sends it to D, who accepts it and returns it to C.

II. A and C sell their bills to bill brokers at London and Paris respectively. Thus A and C receive payment on account of the goods they have sold to B and D respectively.

III. E, of London, buys goods from F, of Paris, and wishes to remit f. 2000·00 to the latter. E buys from the London bill broker the bill for f. 2000·00 formerly sold by A. Similarly, G, of Paris, who wishes to remit £300 to H, of London, in payment for goods received, buys from the Paris bill broker the bill for £300 formerly sold by C. Thus both bill brokers receive a return of their money (plus a profit for their selling prices to E and G respectively are most likely higher than their buying prices).

IV. E and G send the bills to F and H respectively. Thus E and G make payment for the goods received from F and H respectively.

V. At the date when the bills mature, F and H present them to B and D

(or their bankers) respectively, and the latter retire the bills. Thus, F and H receive payment on account of the goods sold to E and G respectively, and also B and D make payment on account of the goods received from A and C respectively.

It is seen from the above illustration that four quantities of goods have been bought and paid for without the transference of cash between the two cities. The brokers act as a connecting link between the merchants who hold bills drawn on the foreign city and require cash, and those who wish to remit money to the foreign city and require bills drawn on that city, so that the remitting may be easily performed.

Of course, many modifications might take place; for example: (1) A might have negotiated the bill or his broker might have discounted it; (2) the bill might have been negotiated (and indorsed) several times before the broker buys it; (3) E and A might be customers at the same bank, so that if the banker had discounted the bill, he might have used it to remit to Paris for E; (4) F might have discounted the bill at his bank and B, being notified of this, might have retired the bill before maturity.

Supposing E had wished to remit f. 3000·00: he might have bought an additional bill for f. 1000·00, but if one was unobtainable, he could have paid the money at his bank, who would remit for him by telegraphic transfer or a banker's draft; and F could obtain the money at the bank's Paris branch or agent.

156. FOREIGN EXCHANGES.

In all parts of the world there are, in normal times, merchants who, having purchased goods from England, desire to pay for them by means of bills drawn on London. The acceptance of an English banker or first-class business house has become to be an absolute guarantee that the bill will be met at maturity, and so the price of English bills depends mostly on the supply at the town in question. Should there be, say, at Amsterdam a supply of English bills far in excess of the demand, prices will tend to fall; while if there are insufficient to meet the demand, prices will rise. It has already been pointed out that in normal times the fluctuation of prices is kept within the limits defined by the specie points; but when the free exporting and importing of gold is suspended, these restrictions are removed, and prices are liable to rise and fall as those of commodities.

The term "Short Exchange" denotes cheques and bills payable at sight, while "Long Exchange" is applied to bills which have a term to run before maturity. The difference between the rates is due to (1) cost of bill stamps; (2) discount for the time between the dates of quotation and maturity; and (3) the risk that the bill will not be paid at maturity.

Each day the approximate rates at which bills on London can be bought and sold at some of the most important towns of the

world are tabulated under the heading "Foreign Exchanges." The following list, including telegraphic transfers, will serve as an example. Bank rates are indicated by the figures in parentheses.

FOREIGN EXCHANGES.

Place.	%	*Method of Quoting*	*Par of Exchange.*	*Current*	*Previous.*
New York	$(1\frac{1}{2})$	$ to £	$4\ 86\frac{2}{3}$	$4.91\frac{1}{4}$–$92\frac{3}{8}$	$4.90\frac{7}{8}$–$91\frac{7}{8}$
Montreal .	.	$ to £	$4.86\frac{2}{3}$	4 97–$4.98\frac{1}{2}$	4 98–$4\ 99\frac{3}{4}$
Paris .	(3)	Fr. to £	124.21	$74\frac{1}{2}$–$74\frac{1}{16}$	$74\frac{1}{2}$–$74\frac{11}{16}$
Brussels .	(2)	Bel. to £	35.00	29 06–29 13	29.05–29.13
Milan .	(5)	Lire to £	92.46	$60\frac{1}{8}$–$60\frac{1}{2}$	$60\frac{1}{8}$–$60\frac{1}{2}$
Switzerland	$(2\frac{1}{2})$	Fr. to £	25 22	15.10–15.15	15 09–15.14
Athens .	(7)	Dr. to £	375 00	516‡	517‡
Helsingfors	(4)	M to £	193 23	$226\frac{3}{8}$–$227\frac{3}{8}$	$226\frac{3}{8}$–$227\frac{3}{8}$
Madrid .	$(5\frac{1}{2})$	Pts. to £	25 22	$35\frac{7}{8}$–$36\frac{1}{16}$	$35\frac{15}{16}$–$36\frac{1}{16}$
Lisbon .	(5)	Escu. to £	110.00	$109\frac{7}{8}$–$110\frac{3}{8}$	$109\frac{7}{8}$–$110\frac{3}{8}$
Amsterdam	(6)	Fl. to £	12·11	$7{\cdot}26\frac{1}{2}$–$29\frac{1}{2}$	7·26–7·28
Berlin .	(4)	M. to £	20.43	12.19–12.23	12.18–12 23
Vienna .	(4)	Sch. to £	34.59	25–27	25–27
Budapest .	(4)	Pen. to £	27.82	$16\frac{1}{2}$*‡	$16\frac{1}{2}$*‡
Prague .	$(3\frac{1}{2})$	Kc. to £	164.25	$118\frac{1}{2}$–119	$118\frac{1}{2}$–119
Danzig .	(6)	Gul. to £	25.00	26–$26\frac{1}{2}$	26–$26\frac{1}{2}$
Warsaw .	(5)	Zloty to £	43.38	$25\frac{7}{8}$–$26\frac{3}{8}$	$25\frac{7}{8}$–$26\frac{3}{8}$
Riga .	(6)	Lats to £	25.22	$14\frac{1}{2}$–$15\frac{1}{2}$	$14\frac{1}{2}$–$15\frac{1}{2}$
Bucharest	$(4\frac{1}{2})$	Lei to £	813.60	623‡	624‡
Constantinople .		Pst. to £	110.00	613‡	613‡
Belgrade .	(5)	Din. to £	276.32	210–220	211–221
Kovno .	(6)	Lit to £	48.66	29–30	29–30
Sofia .	(6)	Lev. to £	673 66	385–415	385–415
Oslo .	$(3\frac{1}{2})$	Kr. to £	18.16	19.85–19.95	19.85–19.95
Stockholm	$(2\frac{1}{2})$	Kr. to £	18.16	19 35–19.45	19.35–19.45
Copenhagen	$(3\frac{1}{2})$	Kr. to £	18.16	22.35–22 45	22 35–22.45
Alexandria	.	Pst. to £	97.50	$97\frac{3}{8}$–$97\frac{5}{8}$	$97\frac{3}{8}$–$97\frac{5}{8}$
Bombay .	$(3\frac{1}{2})$	Per rup.	1s. 6d.	$1/6\frac{3}{32}$–$\frac{5}{32}$	$1/6\frac{3}{32}$–$\frac{5}{32}$
Calcutta .	$(3\frac{1}{2})$	Per rup.	1s. 6d.	$1/6\frac{3}{32}$–$\frac{5}{32}$	$1/6\frac{3}{32}$–$\frac{5}{32}$
Madras .	$(3\frac{1}{2})$	Per rup.	1s. 6d.	$1/6\frac{3}{32}$–$\frac{5}{32}$	$1/6\frac{3}{32}$–$\frac{5}{32}$
Hong-Kong	.	Per dol.	—	$2/0\frac{1}{8}$–$2/0\frac{3}{4}$	$2/0\frac{1}{8}$–$2/0\frac{7}{8}$
Kobe .	(3·65)	Per yen	24.58d.	1/2–$1/2\frac{1}{8}$	1/2–$1/2\frac{1}{8}$
Shanghai .	.	Per dol.	—	1/6–$1/6\frac{1}{2}$	1/6–$1/6\frac{5}{8}$
Singapore .	.	Per dol.	2s. 4d.	2/4–$2/4\frac{3}{16}$	2/4–$2/4\frac{3}{16}$
Batavia .	(4)	Fl. to £	12.11	$7.25\frac{1}{2}$–7.28	$7.24\frac{1}{2}$–$26\frac{1}{2}$
Rio de Janeiro .		Per mil.	5.90d.	$2\frac{9}{16}$–$2\frac{3}{4}$d.	$2\frac{11}{16}$–$2\frac{13}{16}$d.
Buenos Aires .		Paper pes. to £	11.45	17.50–18.50	17.50–17.90
Valparaiso† .		Pesos to £	40.00	118	119
Montevideo .		Per peso	4s. 3d.	20–$20\frac{1}{2}$d.	20–$20\frac{1}{2}$d.
Lima† .	$(6\frac{1}{2})$	Soles to £	17.38	20.00	20.00
Mexico .	$(6\frac{1}{2})$	Pesos to £	9.76	$17\frac{1}{2}$–$18\frac{1}{2}$	$17\frac{1}{2}$–$18\frac{1}{2}$
Manila .	.	per peso	24.66d.	$2/0\frac{3}{8}$–$2/0\frac{3}{4}$	$2/0\frac{3}{8}$–$2/0\frac{5}{8}$

* Official rate. † 90 days ‡ Sellers.

EMPIRE EXCHANGE RATES*

BUYING RATES PER £100

—	Demand.	30 *days' sight.*	60 *days' sight.*	90 *days' sight.*
On—	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Australia . .	126 7 6	126 17 6	127 7 6	127 17 6
New Zealand .	125 10 0	126 0 0	126 10 0	127 0 0

SELLING RATES

—	*Cable Transfers.*	*Demand.*
	£ s. d.	£ s. d.
Australia	125 0 0	125 1 3
New Zealand	124 0 0	124 1 3

—	*Buying rate.*	*Selling rate.*
	£ s. d.	£ s. d.
S.A. Union Territory (T.T.'s)	100 17 6	100 2 6
Do. (sight drafts)	101 7 6	100 2 6
Rhodesia (T.T's)	100 5 0	99 15 0
Do. (sight drafts)	100 17 6	99 15 0

* Per 100 London pounds

157. **INDIRECT EXCHANGE** is the name given to an exchange between two countries through the medium of one or more other countries. For example, a person wishing to remit to Stockholm might find it to his advantage to buy a bill drawn on Copenhagen, and send it to an agent at the latter town with instructions to sell and with the proceeds buy a bill on Stockholm. The equivalent rate of exchange in the case of indirect exchange operations is called the **arbitrated rate of exchange.**

EXAMPLE (x)—

A broker wishing to buy a bill for 20,000 fl. drawn on Amsterdam, borrowed £2,500 from a bank for a week at $3\frac{3}{4}$ per cent. He bought the bill at $8\frac{5}{8}$ exchange, sold it a week later at $8\frac{9}{16}$, and repaid his loan to the bank. What did he gain?

$$\text{Interest paid to bank} = £25 \times 3\tfrac{3}{4} \times \tfrac{7}{365}$$
$$= £1\ 16s.$$

$$\text{Sum paid for bill} = £\frac{20000 \times 8}{69}$$

$$\text{,, received for bill} = £\frac{20000 \times 16}{137}$$

$\therefore$ Amount gained by buying and selling the bill

$$= £20000 \times 8 \times \left[\frac{2}{137} - \frac{1}{69}\right]$$
$$= £\frac{160000}{137 \times 69}$$
$$= £16\ 18s.\ 7d.$$
$$\therefore \text{ Net gain} = £15\ 2s.\ 7d.$$

EXAMPLE (xi)—

If the rate of exchange in London on Paris be 88·75 for 3-months' bills and 87·60 for cheques, what is the equivalent rate per cent. of discount?

$$\text{Price of a 3-month's bill for 1 franc} = £\frac{1}{88{\cdot}75}$$
$$\text{,, ,, cheque ,,} = £\frac{1}{87{\cdot}6}$$
$$\therefore \text{ Discount} = £\left[\frac{1}{87{\cdot}6} - \frac{1}{88{\cdot}75}\right]$$

Let $d \equiv$ rate per cent. per annum of discount,

$$\text{then } \frac{1}{87{\cdot}6} - \frac{1}{88{\cdot}75} = \frac{1}{88{\cdot}75} \times \frac{1}{4} \times \frac{d}{100}$$
$$\therefore d = 400 \times 88{\cdot}75 \times \left[\frac{1}{87{\cdot}6} - \frac{1}{88{\cdot}75}\right]$$
$$= 400 \times \left[\frac{88{\cdot}75}{87{\cdot}6} - 1\right]$$
$$= 400 \times 0{\cdot}013128$$
$$= 5{\cdot}25$$

EXAMPLE (xii)—

A bill broker bought a bill for 4,000 kroner on Copenhagen at 18·25 and sold it at 18·12. What did he gain?

$$\text{Gain} = £4{,}000 \times \left[\frac{1}{18{\cdot}12} - \frac{1}{18{\cdot}25}\right]$$
$$= £\frac{4000 \times {\cdot}13}{18{\cdot}12 \times 18{\cdot}25}$$
$$= £1\ 11s.\ 5d.$$

TEST EXERCISES IV, 2

(1) Find the interest on £37,450 for 10 days at $3\frac{1}{4}$ per cent. per annum.

(2) (*a*) Find the interest on \$8746·00 for 21 days at $3\frac{1}{2}$ per cent. per annum.

(*b*) Find the interest on fcs. 30750·00 for 30 days at $3\frac{3}{4}$ per cent. per annum.

(3) A man deposited £500 on 20th April in a bank paying interest at 2 per cent. per annum. He withdrew £300 on 15th May and deposited £400 on 6th June. How much interest was due to him on 30th June?

(4) A firm borrowed £15,000 for 14 days from a bank and agreed to pay interest at $3\frac{1}{4}$ per cent. per annum. At the end of the time, the firm being unable to repay, the loan was renewed for another week at $3\frac{1}{2}$ per cent. per annum. Calculate the interest to be paid at the end of the three weeks?

(5) A man borrowed £750 on 10th July at $4\frac{1}{2}$ per cent. per annum, £375 on 12th September at $4\frac{1}{4}$ per cent. per annum, and £425 on 7th November at 4 per cent. per annum. The total sum borrowed was repaid with interest on 1st December of the same year. What was the amount repaid?

(6) A person started an account in the Post Office Savings Bank by depositing £12 10s. on 20th February. During the year his deposits were: 2nd April, £5; 25th June, £6 5s.; 10th August, £3 14s.; and 14th November, £5 10s. 6d.; his withdrawals were: 27th July, £15; 10th September, £5. What interest was due to him at the end of the year ?

(7) A and B started accounts in the Post Office Savings Bank on 31st December, 1930, and 1st January, 1931, respectively. Each deposited £5. A added £5 on the last weekday of each following month and B added £5 in the first weekday of each month. Their books are to be made up in January, 1932. What would be their respective accounts on 20th January, 1932?

(8) Calculate the actual rate of interest paid on B's deposit account during 1931.

(9) A man had £85 13s. 7d. deposited in the Post Office Savings Bank on 31st December. During the following year he deposited £8 10s. on 5th May and £13 6s. 6d. on 20th October, and he withdrew £46 4s. on 25th August. What interest is due to him at the end of the year ?

(10) A bank pays interest on deposit accounts at 2 per cent. per annum half-yearly on 31st March and 30th September. A man deposited £450 on 10th January and withdrew the whole amount on 18th October. What was the amount withdrawn ?

(11) If the interest on £4,800 for 30 days be £11 16s. 9d., what is the rate per cent. per annum ?

(12) What sum deposited on 2nd April in a bank which pays interest at the rate of 2¼ per cent. per annum will amount to £500 by 30th October of the same year ?

(13) Interest is paid quarterly by a bank at a nominal rate of 2½ per cent per annum. What is the effective rate of interest ?

(14) Calculate the compound interest on £100 for 2 years at 2½ per cent per annum, interest payable half-yearly. Use the result in finding the interest on £85 from 1st September, 1929, to 3rd April, 1932, assuming that interest at 2½ per cent. per annum is payable on 30th June and 31st December

(15) A man deposited a sum of money in a bank which paid interest at 2¼ per cent. per annum and calculated it would amount to £1,000 in 12 years after the time the money was deposited. After 4 years, however, the rate of interest was reduced to [illegible] per cent. By how much would the final amount fall short of £1,000 ?

(16) What sum would amount to £100 in 3 years at 2¾ per cent. per annum, interest payable (i) yearly, (ii) half-yearly ?

(In working questions 17–24, the interest tables should be used.)

(17) £420 was borrowed on 8th May, interest at 3 per cent. per annum; it was renewed on 29th May for two weeks' interest at 3½ per cent. per annum. What interest was due at the end of the period ?

(18) Calculate the interest on £73 11s. 7d. for 19 days at 3¼ per cent. per annum.

(19) Calculate the total interest on £107 14s. 8d., the rates of interest being as follows: 5th July to 19th July, 3 per cent. per annum; 20th July to 1st August, 3¼ per cent. per annum; 2nd August to 1st September, 3¾ per cent. per annum.

(20) What is the interest on £745 2s. 6d. for 7 years at 3½ per cent. per annum ?

(21) To what does £437 4s. 4d. amount in 4½ years at 6 per cent. per annum, interest payable half-yearly ?

(22) What is the compound interest which is payable yearly on (i) $475·40 for 13 years at 2¼ per cent. per annum ? (ii) fcs. 1475·75 for 9 years at 3 per cent. per annum ? (iii) 285 kronor for 10 years at 3½ per cent. per annum ?

(23) What sum of money will amount to (i) £1,000 in 6 years at 4 per cent. per annum, interest payable yearly ? (ii) $550·00 in 9 years at 5 per

cent. per annum, interest payable half-yearly? (iii) fcs. 2400·00 in 7 years at 6 per cent. per annum, interest payable half-yearly?

(24) A sum of money accumulating at 4 per cent. per annum becomes doubled after a period of time lying between 17 and 18 years. Assuming that the money was deposited on 31st December, 1914, what is the first day during 1932 when a sum of money not less than double the original sum could be withdrawn?

(25) Draw up a table of nines giving the interest on 1 unit of money at 1 per cent. per annum for 1 to 9 days, correct to 11 decimal places. Hence calculate the interest on—

(i) fcs. 1745·00 for 21 days at $3\frac{1}{4}$ per cent. per annum.
(ii) \$2193·50 for 30 days at $3\frac{1}{2}$ per cent. per annum.
(iii) £1,128 12s. 3d. for 60 days at 4 per cent. per annum.

(26) Using the table of the previous question and also the compound interest table, calculate the interest on—

(i) £750 from 10th March, 1925, to 1st September, 1930, interest payable yearly on 30th April at $2\frac{1}{2}$ per cent. per annum.

(ii) \$4000·00 from 20th October, 1921, to 8th November, 1930, interest payable half-yearly on 31st January and 31st July at 5 per cent. per annum.

(27) On what principal would the interest for 30 days at $3\frac{1}{2}$ per cent. per annum be the same as the interest on £840 for 42 days at $4\frac{1}{4}$ per cent per annum?

(28) On 1st July a man had £500 in his current account at a bank which paid on 30th June and 31st December interest at 2 per cent. per annum, the interest being based monthly on the least sum of money in the account each calendar month. The bank charged $\frac{1}{4}$ per cent. commission for paying cheques drawn by the man. The latter added £240 in August and £300 in September to his account; and during the six consecutive months July–December, cheques for totals of £110, £105, £85, £90, £75, and £75 respectively were paid. What was the amount in his current account on 1st January of the following year?

(29) A bill for £640 was drawn on 15th February, 1932, at 4 months after date, and discounted on 12th April at $3\frac{3}{8}$ per cent. Calculate the discount deducted by the banker.

(30) A bill for £12,750 was payable 60 days after sight. It was accepted on 4th March, and discounted on 25th March. What was the discounted value if the rate of discount be $3\frac{3}{4}$ per cent.?

(31) If the bill of the previous question be retired on 2nd April at $3\frac{1}{4}$ per cent., what would be the rebate? Also, if the banker who discounted the bill received payment from the acceptor, what would be his gain and his percentage gain during the period?

(32) A bill drawn on 10th July and payable two months after date was discounted on 25th July at 4 per cent., and the sum of money paid was £452 13s. 10d. For what amount was the bill drawn?

(33) A man holds a Paris draft for 103,500 francs, which was drawn on 19th May at 30 days after date. If the rate of exchange for Paris cheques be 88·50 what should the man receive for the bill if he discounted it on 25th May, reckoning discount at 3 per cent per annum? (In calculating discount, do not allow any days of grace and reckon 1 day as $\frac{1}{360}$ of a year.)

(34) An agent of a London firm in Bombay sold goods and received a bill for 228,000 rupees accepted on 10th June at 60 days after sight. He discounted it on 13th June at $5\frac{1}{2}$ per cent., and the value of the rupee at Bombay at the time was 1s. $6\frac{1}{4}$d. What amount in British currency should the agent put down in his account sales as having received on 13th June?

(35) Given that the rate of exchange between New York and London be 3·46, make a table showing in three columns the values in dollars and cents of 1 up to 9 pence; 1 up to 9 shillings; and 1 up to 9 £'s. Use the

table to write down the values of £175 7s. 9d., £237 13s. 10d., and £516 11s. 7d. in dollars and cents each to the nearest cent.

(36) If the rate of exchange in London on Madrid be 41·55 and the rate of discount be 4¾ per cent. per annum for a 3 months' bill, what debt could a man holding a 3 months' bill for £750 discharge in Madrid?

(37) A broker on 10th May bought a bill on Sydney for £12,000 at 5 per cent. per annum discount, the date of maturity being 5th July. On 30th June he sold the bill at a premium of ¾ per cent. per annum. How much did he gain? Also if he borrowed £12,000 on 10th May at 3½ per cent. per annum and discharged his debt on 6th July, what did he gain?

(38) A bill broker bought a bill for $750 on Montreal at 4·02 and sold it at 3·98. What did he gain?

(39) Calculate the prices of a bill for £8,570 drawn on London under the following conditions,—

(i) Bought at	Lyons	@ 87·85 francs	per £1.
(ii) ,,	Madrid	,, 39·10 pesetas	,,
(iii) ,,	Rome	,, 69·26 lire	,,
(iv) ,,	Shanghai	,, 1s. 10½d. per	tael.
(v) ,,	Kobe	,, 2s. 0½d. ,,	yen.

(40) A bill broker bought a bill for 10,000 fl. on Holland at 10·15¾ and sold it 4 days later at 10·14. What was his gain? Also find his net gain if he borrowed sufficient money to buy the bill and discharged the debt 5 days later, the rate of interest being 3¼ per cent. per annum.

(41) A broker bought a cheque on Melbourne at £126 10s. per £100 sterling and sold it at £125. What was the percentage gain on his outlay?

(42) A cheque on Copenhagen was bought at 18·24 and sold at 18·15. What was the percentage gain on the outlay?

(43) If the rate of exchange in London on Amsterdam be 10·14 for 3 months' bills and 10·30 for cheques, what is the equivalent rate per cent. per annum of discount?

(44) The rate of exchange between London and Oslo was 18·40 kr. per £1, between Oslo and Stockholm 0·98 kr. per kroner, and between Stockholm and London 17·80. What would it cost a London merchant to discharge a debt of 3,000 kr. in Stockholm (i) by remitting direct to Stockholm? and (ii) by remitting through Oslo?

(45) Find the arbitrated rate of exchange in the latter case of the previous question.

(46) A London merchant sold to a merchant at Paris 1,800 metres of cloth at fcs. 15·75 per metre. For what amount in British currency to the nearest £ should the former draw a bill of exchange on the latter, reckoning the rate of exchange as 87·85?

(47) The rate of exchange between London and Paris was 87·80 fcs. per £, between Paris and Madrid 2·05 fcs. per peseta. Calculate the arbitrated rate of exchange between London and Madrid.

CHAPTER XX.

THE STOCK EXCHANGE.

158. **A STOCK EXCHANGE** is "the market for stocks and shares." There are stock exchanges at many of the principal commercial towns of England; but what is usually referred to as "The Stock Exchange" is the **London Stock Exchange,** situated in Throgmorton Street. At the capitals and other large cities of foreign countries there are stock exchanges: that at Paris is known as "The Bourse."

The **London Stock Exchange** building belongs to **shareholders**, who are nearly all members, for, since 1904, every new member has had to become a proprietor by taking up a certain number of shares. The **Trustees,** nine in number, are appointed to look after the interest of the proprietors, who draw their income from entrance fees, annual subscriptions of members and clerks, and rent of offices. The **Committee of Management,** consisting of thirty members, re-elected annually, fixes the days of settlement, prepares the Official Price List, and possesses the power to examine the books of members and expel them if found guilty of unprofessional conduct. The **Members,** a few thousand in number, comprise **Stock-brokers** and **Stock-jobbers.** Each member can employ one **authorized clerk,** two **unauthorized clerks,** and two **Settling-room clerks.** With the exception of clerks of four years' service, a candidate for admission as a member must obtain the nomination of a member willing to retire in his favour, or of a former member.

An Authorized Clerk can transact business in the same way as his employer, who is bound by the contracts made by the former on his behalf.

Unauthorized Clerks and **Settling-room Clerks** have no authority to deal in securities. All these clerks have admission to the House.

A firm consisting of two or more members may employ two authorized, three unauthorized, and six Settling-room clerks.

Members, of course, may employ any number of clerks for work in their own private offices, which are, without exception, established near the House.

159. A **STOCK-BROKER** is a member who acts as agent for others in connection with the purchase or sale of shares. Usually his duty is to act for a client, who is a member of the public, by selling to a jobber securities belonging to his client; or to purchase from a jobber securities on behalf of his client. A broker is permitted to act either for another broker or for a jobber. To cover payment for his services, a broker charges commission or brokerage, which is based on the number of shares or amount of stock and varies according to the class of business.

The customary scale of brokerage is 5s. per £100 stock, but there are exceptions: for example, the rate is 10s. per £100 on British Railway stocks. On shares not exceeding 15s. the brokerage is 1½d. per share; not exceeding 30s., 3d. per share; not exceeding 40s., 4½d. per share, and so on.

There are minimum charges of 10s. on less than £100 and £1 on over £100.

When a company is issuing new shares, a broker is sometimes asked to "place shares." He then circulates prospectuses to his clients; and should he induce any to apply for shares and should allotments be made to them, he is paid a commission by the company.

160. A **STOCK-JOBBER** is a member who is always ready to buy shares from or sell shares to the public, the brokers acting as the go-between. Each jobber confines himself in transacting business in a group of securities known as a "market." No member can be a broker and jobber at the same time, but he may change from one to the other if he desires. The difference between the price at which the jobber will buy and that at which he will sell is known as "**the turn of the market**" or the "**jobber's turn.**" As prices vary from day to day, this difference does not necessarily represent the jobber's profit, but it acts as an approximate measure of it.

161. A simple **STOCK EXCHANGE TRANSACTION** is described as follows—

1. A instructs his broker, X, to sell £1,000 G.E.R. Stock.
2. X goes to a jobber, J, and without saying whether he wishes to buy or sell, asks him to quote a price. J quotes, say, 33⅞-34⅛, which means he is willing to buy at £33 17s. 6d. per £100 stock, or to sell at £34 2s. 6d. per £100 stock. X claims the right to sell to J at 33⅞, and each records the bargain.

3. Meanwhile B instructs his broker, Y, to buy £1,000 G.E.R. Stock.
4. Y asks a quotation from J, who names his prices as before. Y buys the stock from J at $34\frac{1}{8}$, and each records the bargain.
5. On "**Ticket Day**" (*i.e.*, the second day of Settlement) Y gives J a ticket bearing his own, B's name and J's name, and stating the amount and description of the stock bought. J passes the ticket on to X, and thus A and B are indirectly brought together.
6. X now prepares the transfer deed signed and sealed by A, and this, together with A's stock certificate, is sent to Y, who obtains B's seal and signature to the deed. The third day of the Settlement is set aside for the preparation of documents and is known as "**Intermediate Day.**"
7. On "**Settling Day**" (*i.e.*, the fourth and last day of Settlement), Y, who has by now received the purchase money from B, pays J, and J pays X. The latter forwards to A a cheque for the amount for which he sold the stock, less brokerage.
8. Y sends the transfer deed and the stock certificate to the G.E.R. Co., which registers B as holding the stock, and sends a certificate to Y, who then forwards it to B.

EXAMPLE (1)—

Neglecting transfer duties and registration fee, and assuming brokerage to be $\frac{1}{2}$ per cent., calculate the amounts A should receive and B pay? Also calculate the commissions of X and Y respectively, and the profit made by J.

Amount to be paid by B $= £10 \times 34\frac{5}{8} = £346$ 5s. 0d.

" received " A $= £10 \times 33\frac{3}{8} = £333$ 15s. 0d.

Commission received by X $= £10 \times \frac{1}{2} = £5$

" " Y $= £10 \times \frac{1}{2} = £5$

Amount gained by J $= £10 \times \frac{1}{4} = £2$ 10s. 0d.

NOTE 1.—Had Y purchased the stock from a jobber other than J, the ticket might have passed through the hands of several brokers and jobbers before A's and B's names are ultimately brought together.

162. CONTANGO AND BACKWARDATION.

The settlement of bargains does not take place immediately, but at the end of periods, which are usually fortnightly.

The first day of the settlement is "Contango Day." On this day those persons who wish to postpone settlement can ask for the transaction to be carried over until the next settlement. Suppose, referring to the transaction described, B is not able to pay at the proper time; then Y must find someone from whom he can borrow the money with which to pay J. The lender would receive the stock as security, and, of course, would be paid interest at a fair rate. Y would make a slightly higher charge to B, and this charge is known as **Contango**. Suppose A has not delivered the stock by Contango day; then X must find a person or persons having

£1,000 G.E.R. Stock who are willing to lend it until the next settlement, and money equivalent to the market price would be paid as security. On return of the stock, the lender or lenders would refund the money, but a commission would be charged. This commission paid to lenders of shares and stock is known as **Backwardation.** This amount, plus a little extra for the additional services of X, would be deducted from the net amount which otherwise A would receive from the sale of his stock.

163. OFFICIAL QUOTATIONS.

Nearly all kinds of securities can be bought and sold on the Stock Exchange; but those securities having good marketability are quoted in an **Official List of Prices**, which is issued daily. Newspapers publish daily the prices in the case of Government stocks and also those of shares issued by the most important companies.

The following is an extract from a list of nominal quotations—

BRITISH FUNDS

Consols 2½%	82⅛	82⅜
,, 4%	111⅛	111⅜
War Loan 3½%	103⅜	103⅝
Fdg. Loan 4%	113⅝	113⅞
Victory 4%	111⅛	111⅜
,, Sm Bds.	111⅛	111⅜
Conversion 3½%	102⅝	102⅞
,, 4½%	109⅞	110¼
,, 5%	118¾	119¼
Metropolitan Water "B"	95½	96½
,, ,, "A"	97	98
Port of London 3% "A"	93½	94½
,, ,, 4¾%	110	111
,, ,, 3½%	103	104

DOMINION AND COLONIAL

Australia 5% 1945–75	109	109½
,, 4% 1943–48	101½	102
Canada 4% 1940–60	102	102½
,, 3½% 1930–50	99½	100
Cape 4% 1916–36	102	102½
,, 3½% 1929–49	102	102½
,, 3% 1933–43	101½	102
Natal 3½% 1934–44	100¾	101¼
New S. Wales 3½% 1950	99½	100
,, 5¼% 1947–57	112½	113
New Zealand 5% 1956–71	119¾	120¼
,, 3% 1945	99½	100
Nigeria 5% 1947–57	115	115½
Queensland 5% 1940–60	104¾	105¼
South Africa 5% 1945–75	115½	116
South Australia 5% 1945–75	109	109½

BRITISH RAILWAYS

Great Western	46½	47½
,, 4% Deb.	109½	111½
,, 5% Gtd.	122	124
,, 5% Pref	111	113
London Midland & Scottish	17	18
,, ,, 4% Deb.	101½	103½
,, ,, 4% Gtd	97	99
,, ,, 4% Pref	77½	79½
L. & N. Eastern, Def	5	5½
,, ,, Preferred Ord.	9	10
,, ,, 3% Deb	77	79
,, ,, 4% 1st Gtd.	95½	97½
,, ,, 4% 1st Pref	53	55
,, ,, 4% 2nd Pref.	19	21
Metropolitan Assented	83	85
Southern, Deferred Ord	17½	18½
,, Preferred Ord.	75	77
,, 4% Deb	109	111
,, 5% Gtd	123	125
,, 5% Pref	111	113
London Transport 5% A	131½	133½

INDUSTRIALS

Courtaulds Ord.	54/3	54/9
,, 5% Pref		26/4½
Dunlop Rubber	41/3	41/9
Home and Colonial (4/–)	5/9	6/1½
Imperial Airways	46/10½	47/3
Lever 7% Pref.	30/1½	30/6
,, 5% Deb.	110½	
Lyons	134/6	
,, 7% Pref.	34/9	35/3

164. INVESTMENTS.

Persons buy stocks and shares either as an investment or as a speculation.

A good investment is one in which there is little risk of loss, and which produces a regular and fair income in proportion to the amount of money

invested. A person wishing to make a good investment by buying shares of a company, should, first, make sure the company is on a sound financial basis; secondly, examine what dividends have been paid in the past and what are likely to be paid in the future; and, thirdly, the market price of the shares. Dividends are payable as a percentage on the paid-up values of stocks and shares, so that in order to compare the relative merits of two or more investments, the income on equal amounts of cash must be estimated.

The **Yield** is the actual percentage return on cash invested; and, in practice, it is considered as the sum of money produced as annual income by the investment of £100 cash.

EXAMPLE (ii)—

With $2\frac{1}{2}\%$ Consols at $54\frac{1}{2}$, calculate the yield, (1) neglecting brokerage, (2) taking brokerage into consideration

(1) £$54\frac{1}{2}$ cash brings in an income of £$2\frac{1}{2}$

$\therefore$ £100 „ „ £$\dfrac{2\frac{1}{2} \times 100}{54\frac{1}{2}}$

i.e., Yield = £4 11s. 9d. to nearest 1d.

(2) £$54\frac{3}{4}$ cash brings in an income of £$2\frac{1}{2}$

$\therefore$ £100 „ „ £$\dfrac{2\frac{1}{2} \times 100}{54\frac{3}{4}}$

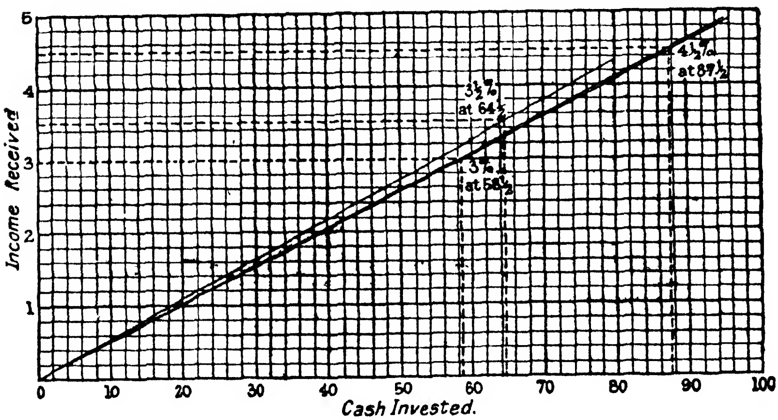
i.e., Yield = £4 11s. 4d. to nearest 1d.

NOTE 2.—As the income received by any one investment is proportional to the sum invested, the graph connecting income and cash invested will be a straight line through the origin. (See Paragraph 90.) By means of squared paper, therefore, investments can very readily be compared.

EXAMPLE (iii)—

Which is the best investment of the following—

3% stock at $58\frac{1}{2}$; $3\frac{1}{2}\%$ stock at $64\frac{1}{2}$; $4\frac{1}{2}$ pref. shares (£10) at $8\frac{3}{4}$?



Ansr. { The best investment is $3\frac{1}{2}\%$ stock at $64\frac{1}{2}$. Pref. shares (£10) at $8\frac{3}{4}$ is slightly better than 3% stock at $58\frac{1}{2}$.

When investments are to be compared without the use of squared paper the following methods are quickest: (i) When two investments only are to be compared, find the income on the product of the prices of £100 stock, *e.g.*, comparing 3% stock at $58\frac{1}{2}$ with $3\frac{1}{2}$% stock at $64\frac{1}{2}$, it is seen that the income on £$58\frac{1}{2} \times 64\frac{1}{2}$ is £$64\frac{1}{2} \times 3$, *i.e.*, £$193\frac{1}{2}$ in the first and £$58\frac{1}{2} \times 3\frac{1}{2}$; *i.e.*, £$204\frac{3}{4}$ in the second investment, so that the latter is the better of the two. (ii) When more than two investments are to be compared, find the sum of money to be invested in order to produce an income of £1, *e.g.*, referring to the three investments of example (iii), the sums of money to be invested to produce incomes of £1 are—

$$£\frac{58\frac{1}{2}}{3} \text{ (i.e., £}19\tfrac{1}{2}\text{) } £\frac{64\frac{1}{2}}{3\frac{1}{2}} \text{ (i.e., £}18\tfrac{1}{7}\text{), and } £\frac{87\frac{1}{2}}{4\frac{1}{2}} \text{ (i.e., £}19\tfrac{1}{9}\text{) respectively.}$$

It is obvious that the smaller the sum of money to be invested to produce an income of £1, the better will be the investment; and, consequently, the best is $3\frac{1}{2}$% stock at $64\frac{1}{2}$ and the least desirable is 3% stock at £$58\frac{1}{2}$.

165. SPECULATION.

A considerable amount of speculation takes place on the Stock Exchange, for there are many persons always ready to buy stocks and shares if they think there is a possibility of the price rising, so that in a short time they can sell out at a profit. Huge sums have been gained in this way; but, on the other hand, ruinous losses have been incurred.

A "Bear" is a person who persistently sells stock which he has not yet himself bought, with the idea of forcing prices down, so that just before settlement he can buy the stock at cheaper prices than those at which he has contracted to sell.

A "Bull" is one who persistently contracts to buy stock, which he cannot necessarily pay for, with the idea of forcing prices up so that just before settlement he can sell the stock he has bought at a higher price than that he has undertaken to pay. The combined efforts of "Bears" and "Bulls" have the effect of steadying prices.

Some examples illustrating the subject-matter of the present chapter are given as follows—

EXAMPLE (iv)—

What is the cost to purchase (i) £640 L.C.C. 3 per cent. Stock at 63, brokerage $\frac{1}{4}$ per cent.? (ii) 450 Dunlop Rubber Shares at $10/1\frac{1}{2}$, brokerage 3d. per share?

$$\text{Cost of £640 L.C.C. Stock, plus brokerage} = £\frac{640 \times 63\frac{1}{4}}{100}$$
$$= £404\ 16s.$$

$$\text{Cost of 450 Dunlop Rubber Shares, plus brokerage} = £\frac{450 \times 10\frac{3}{8}}{20}$$
$$= £233\ 8s.\ 9d.$$

NOTE 3.—In the case of shares, a whole number must be bought, for they are not divided; but in the case of stock, fractions of £100 stock can be bought.

NOTE 4.—The actual cost is slightly greater than the above, for there is a small registration fee to pay; while the value of the stamp on the transfer of most stocks or shares is for amounts exceeding £300, 10s. for every £50 purchase money.

EXAMPLE (v)—

What percentage dividend must the Dunlop Rubber Co. pay on the shares of the previous question in order that the yield should be at least as high as the L.C.C. 3 per cent. Stock?

Let $r \equiv$ the percentage dividend,

$$\text{then } \frac{r \times 20}{100 \times 10\frac{3}{8}} = \frac{3}{63\frac{1}{4}}$$

$$\therefore r = \frac{15 \times 83 \times 4}{8 \times 253} = 2{\cdot}46 \text{ approx.}$$

Ansr.—Dividend of $2\frac{1}{2}\%$

NOTE 5.—£r can be regarded as the income on £100 nominal value of shares, which can be bought by an expenditure (including brokerage) of £$\frac{100 \times 10\frac{3}{8}}{20}$. As the yield is to be the same in each case, the ratio $\frac{\text{Income received}}{\text{Cash invested}}$ must have the same value.

EXAMPLE (vi)—

How much must be invested in 6 per cent. Preference Shares of £10 each at $9\frac{1}{8}$, in order to produce an income of just over £175 per annum? Find the total amount to be paid if brokerage is 1s. per share, and stamp 10s. per £50 purchase money.

£6 is the income obtained by purchasing 10 shares

$\therefore$ £175 „ „ „ $\frac{10 \times 175}{6}$ shares

i.e., $291\frac{2}{3}$ shares

Cost of 292 shares at £$9\frac{1}{8}$ each = £2,664 10s.

$\therefore$ £2,664 10s. invested will produce an income of £175 4s. per annum.

Brokerage = £14 12s.

Value of Stamp = £54 × $\frac{1}{2}$ = £27

$\therefore$ Total cost = £2,706 2s.

EXAMPLE (vii)—

Certain $4\frac{1}{2}$ per cent. £5 preference shares were at a premium of 11s. 6d.: later on, the price fell and the shares could be obtained at a discount of 3s. 9d. Neglecting brokerage and transfer duty, find by how much the percentage yield increased.

Yield, when the price is £5 11s. 6d. = £$4\frac{1}{2} \times \frac{5}{5\frac{23}{40}}$

„ „ „ £4 16s. 3d. = £$4\frac{1}{2} \times \frac{5}{4\frac{13}{16}}$

$$\therefore \text{ Increase in the yield} = £4\tfrac{1}{4} \times 5 \times \left(\tfrac{16}{77} - \tfrac{40}{223}\right)$$
$$= £34 \times 5 \times \left(\tfrac{2}{77} - \tfrac{5}{223}\right)$$
$$= £\frac{34 \times 5 \times 61}{77 \times 223}$$
$$= \underline{\underline{12\text{s. } 1\text{d.}}} \text{ to nearest } \tfrac{1}{4}\text{d.}$$

NOTE 6.—When the dividend of a stock is fixed, the yield is inversely proportional to the price of the stock. Thus, Yield = Dividend $\times \dfrac{\text{Nominal value}}{\text{Market price}}$

EXAMPLE (viii)—

What is the price of $4\frac{1}{4}$ per cent. stock, if £3,136 10s. invested produces an income of £153? (Brokerage, $\frac{1}{4}$ per cent.)

Let $£x \equiv$ price of £100 stock plus brokerage,

$$\text{then } \frac{x}{4\frac{1}{4}} = \frac{3136\frac{1}{2}}{153}$$

$$\therefore x = \frac{6273 \times 17}{2 \times 4 \times 153} = 87\tfrac{1}{8}$$

$\therefore$ Price of Stock = $\underline{86\frac{7}{8}}$.

EXAMPLE (ix)—

A man sold £6,450 India $3\frac{1}{2}$ per cent. Stock at $64\frac{1}{2}$ and with the proceeds bought as many Lipton's 5 per cent. Preference £1 Shares at 16s. 3d. as he could. What was the gain in income derived by this change of investment? (Brokerage, $\frac{1}{4}$ per cent. for India Stock, 3d. per £1 share.)

$$\text{Income due to investment in India Stock} = £\frac{6450}{100} \times 3\tfrac{1}{2}$$
$$= \underline{\underline{£225 \text{ 15s. 0d.}}}$$

$$\text{Amount received by selling India Stock} = £\frac{6450}{100} \times 64\tfrac{1}{4}$$

$$\therefore \text{ Number of Lipton's Shares bought} = \frac{6450 \times 64\frac{1}{4}}{100 \times \frac{33}{40}}$$
$$= 5{,}023, \text{ neglecting } \tfrac{2}{11} \text{ of a share.}$$

$$\text{Income derived from purchase of Lipton's shares} = £\frac{5023}{100} \times 5$$
$$= \underline{\underline{£251 \text{ 3s. 0d.}}}$$

$$\therefore \text{ Gain in income} = \underline{\underline{£25 \text{ 8s. 0d.}}}$$

EXAMPLE (x)—

A man sells out £6,333 6s. 8d. $2\frac{1}{2}$ per cent. Consols at $54\frac{5}{8}$, and invests the proceeds partly in $4\frac{1}{2}$ per cent. railway stock at $80\frac{1}{4}$ and partly in $3\frac{1}{2}$ Colonial stock at 71. His income is thereby increased by £24 3s. 4d. How much of each stock did he buy? (*Brokerage*—Consols, $\frac{1}{4}$ per cent.; railway stock, $\frac{1}{8}$ per cent.; Colonial stock, $\frac{1}{4}$ per cent.)

Income from Consols $= £63\frac{1}{3} \times 2\frac{1}{2} = £158$ 6s. 8d.

$\therefore$ Income from railway and Colonial stock $= £182$ 10s. 0d.

Proceeds from sale of Consols $= £63\frac{1}{3} \times 54\frac{3}{8} = £3{,}443\frac{3}{4}$.

Let $£x \equiv$ Cash, plus brokerage paid for railway stock

then $£(3443\frac{3}{4} - x) \equiv$,, ,, ,, Colonial ,,

then $£\dfrac{100\,x}{80\frac{3}{4}} \equiv$ Amount of railway stock bought

and $£\dfrac{100(3443\frac{3}{4} - x)}{71\frac{1}{4}} \equiv$,, Colonial ,, ,,

$$\therefore \frac{4\frac{1}{2}x}{80\frac{3}{4}} + \frac{3\frac{1}{2} \times (3443\frac{3}{4} - x)}{71\frac{1}{4}} = 182\frac{1}{2}$$

$$i.e., \frac{36x}{323} + \frac{7 \times (13775 - 4x)}{285} = 365$$

$$i.e., 10260x + 323 \times 96425 - 9044x = 323 \times 104025$$

$$i.e., 1216x = 323 \times 7600$$

$$i.e., x = \frac{323 \times 7600}{1216} = 2018\tfrac{3}{4}$$

$\therefore$ Amount of railway stock bought $= £\dfrac{2018\frac{3}{4} \times 100}{80\frac{3}{4}} = £2{,}500.$

and ,, Colonial ,, ,, $= £\dfrac{1425 \times 100}{71\frac{1}{4}} = £2{,}000.$

Example (xi)—

A man invested $\frac{1}{2}$ of his capital in 5 per cent. stock at 98, $\frac{1}{3}$ of his capital in $4\frac{1}{2}$ per cent. stock at $78\frac{3}{4}$, and the remainder in 4 per cent. stock at 72 What is the average yield on the combined investments? If the entire income be £45 4s. 2d., what is the amount of his capital? (Neglect brokerage.)

Let his capital be considered as 100 units,

$$\text{then average yield} = \frac{100}{2} \times \frac{5}{98} + \frac{100}{3} \times \frac{4\frac{1}{2}}{78\frac{3}{4}} + \frac{100}{6} \times \frac{4}{72}$$

$$= 2{\cdot}55102 + 1{\cdot}90476 + {\cdot}92593$$

$$= 5{\cdot}3817 \text{ correct to 4 dec. places.}$$

Amount of capital invested $= £\dfrac{45{\cdot}20833}{{\cdot}053817} = £840.$

Example (xii)—

At what price must 5 per cent. stock be bought in order that, after deducting income tax at 5s. in the £, the net income would be at the rate of 4 per cent. per annum on the money invested?

Net income produced by £100 stock $= £5 \times \frac{3}{4} = £3\frac{3}{4}$.

$\therefore$ Price at which £100 stock must be bought $= £100 \times \dfrac{3\frac{3}{4}}{4}$

$= £93\frac{3}{4}$.

Example (xiii)—

A man bought £4,500 Colonial $3\frac{1}{2}$ per cent. stock when the price was 71. He did not pay at the time of settlement. What total sum would he have to pay by the time of the following settlement, the charges being as follows: Brokerage, $\frac{1}{4}$ per cent.; stamp, 5s. per £100 stock; contango on purchase money for 14 days at 5 per cent. per annum? If, before the time of the next settlement, he sold out at 73, what was his gain?

Cost of stock, including brokerage and stamp duty $= £45 \times 71\frac{1}{2}$

$= £3,217\ 10s.$

Contango $= £\dfrac{45 \times 71 \times 14 \times 5}{36500}$

$= £6\ 2s.\ 7d.$

$\therefore$ Total sum to be paid $= £3223\ \ 1s.\ 7d.$

Proceeds of sale of stock, less brokerage $= £45 \times 72\frac{3}{4}$

$= £3,273\ 15s.\ 0d.$

$\therefore$ Gain $= £50\ 2s.\ 5d.$

Example (xiv)—

Between the times of one settlement and the next, a man speculated as follows in 5s. shares issued by a certain Oil Company: He bought 3,000 shares at par, 4,000 at a premium of 4d., 2,500 at a premium of 9d., 4,000 at a premium of 1s. 3d., and 5,000 at a premium of 1s. 6d. He then sold 10,000 at 6s. $7\frac{1}{2}$d. each and the remainder at 6s. $2\frac{1}{2}$d. each. What did he gain, the brokerage being $1\frac{1}{2}$d. per share?

Total above par value paid for shares $= 4,000 \times 4 + 2,500 \times 9$
$+ 4,000 \times 15 + 5,000 \times 18$ pence.
$= 188,500$ pence

Total above par value received for shares $= 10,000 \times 19\frac{1}{2} + 8,500$
$+ 14\frac{1}{2}$ pence
$= 318,250$ pence.

Total brokerage, buying and selling $= 2 \times 18,500 \times 1\frac{1}{2}$ pence
$= 55,500$ pence

$\therefore$ Gain $= 318,250 - 244,000$ pence
$= 74,250$ pence
$= £309\ 7s.\ 6d.$

TEST EXERCISES IV, 3

(1) Reckoning brokerage at $\frac{1}{4}$ per cent. and transfer duties at 5s. per £100 stock, calculate the sums of money necessary to buy the following amounts of stock.

£2,200 4 per cent. stock at 88; £4,560 5 per cent. stock at 92; £7,162 10s. 5 per cent. stock at 80.

(*N.B.*—Transfer duty is 5s. for the fractional parts of £100 stock.)

(2) Calculate the net amounts received by the sale of the stock of the previous question, if sold at 86, 90, 78 respectively.

(3) What sums of money will be required in order to make the following investments, reckoning brokerage at $\frac{1}{2}$ per cent. and transfer duties at 10s. per £50 purchase money?—

£7,000 4 per cent. stock at 40; £2,745 5 per cent. stock at 86; £720 4 per cent. stock at 70.

(*N.B.*—Transfer duty is 10s. for the fractional parts of £50 purchase money.)

(4) Calculate the net amounts received by the sale of the stock of the previous question, if sold at 37, 83, 68 respectively.

(5) Neglecting brokerage and transfer duties, what amounts of stock would be obtained by making the following investments?—

(i) £560 invested in $4\frac{1}{2}$ per cent. Loan at $98\frac{1}{8}$.

(ii) £1000 invested in 3 per cent. stock at $65\frac{1}{2}$.

(6) What incomes will be obtained by the investments of the previous question?

(7) Calculate the yield per £100 in the case of each of the investments of Question (5).

(8) What numbers of shares could be obtained with the following sums of money? (Neglect transfer duties.)—

(i) £735 invested in 5 per cent. Pref. shares at 18s. $7\frac{1}{2}$d. (brokerage 3d. per share).

(ii) £1000 invested in 6 per cent. Pref. shares at 21s. $4\frac{1}{2}$d. (brokerage 3d. per share.

(9) What incomes will be obtained from the investments of the previous question?

(10) Calculate the yields on purchase money, including brokerage, on the investments of Question (8).

(11) Certain £10 shares (£5 paid) can be bought at £12 each. How many could be bought with £7,500? (Neglect brokerage.) If a dividend of 12 per cent. be given, what income would be obtained? Also calculate the yield.

(12) A sold 4,000 shares at 17s $4\frac{1}{2}$d. and B purchased them at 17s. $7\frac{1}{2}$d. If brokerage be 3d. per share and transfer duty at 10s. per £50 purchase money, find the net amount received by A and the total amount paid by B.

(13) How many shares at 11/9 (brokerage $1\frac{1}{2}$d. per share) could be bought from the proceeds of the sale of £520 stock at $78\frac{3}{4}$ (brokerage $\frac{1}{4}$ per cent.)?

(14) What income would be dervied from investing £785 in $4\frac{1}{4}$ per cent. stock at $98\frac{1}{2}$?

(15) What must be the price of $3\frac{1}{2}$ per cent. stock in order that an income of £120 can be secured by investing £2,730?

(16) What must have been the price of 4 per cent. stock such that after income tax at 3s. 6d. in the £ was deducted the net income per £100 cash was £4? (Brokerage, $\frac{1}{4}$ per cent.).

(17) A man bought 715 £10 shares, fully paid, when the price was $9\frac{5}{8}$. He received a half-yearly dividend at 5 per cent. per annum and, later on, sold the shares when the price was $11\frac{5}{16}$. What was the total gain derived by the investment, reckoning brokerage at 1s. per share and transfer duty 10s. per £50 purchase money?

(18) A man received £1,583 3s. 9d. from the sale of £1,735 stock. If brokerage at $\frac{1}{4}$ per cent. had been deducted, at what price was the stock sold?

(19) Calculate what percentage the brokerage is of the purchase-money in each of the following cases: Consols $54\frac{1}{2}$, brokerage $\frac{1}{8}$ per cent.; Metropolitan $21\frac{3}{4}$, brokerage $\frac{1}{2}$ per cent.; Houlder Line Pref. £5 shares $4\frac{1}{8}$, brokerage 6d. per share; Eastman's £1 shares 10s. 3d., brokerage 3d. per share.

(20) A man bought 450 2s. shares at a premium of 4d. and sold them at a discount of 9d. per share. What was the total extent of his loss, being given that the brokerage was $1\frac{1}{2}$d. per share and the transfer duty at 5s. per £25 purchase money?

(21) Find graphically which is the best investment of the following—

(i) Transvaal 3% at 69½.
(ii) Irish Land 2¾% at 54½.
(iii) Gold Coast 4% at 79.
(iv) Tasmania 3½% at 72.
(v) Salt Union 7% at 1¼ per £1 share.

(22) A man invested £2,000 in the 3¾ per cent. stock at 88. How much stock at the same price could he buy with his half-yearly instalment of interest ?

(23) What sum should a man invest in 2½ per cent. Consols at 54½, in order that, after paying income tax at 5s. in the £, he may have a clear income of £500 per annum ? (Neglect brokerage and duty.)

(24) By finding the amounts of cash respectively to be invested in order to obtain an income of £1, arrange the following in the order of profitableness: (i) 5 per cent., £1 shares at 21s. 9d.; (ii) 7 per cent., £5 shares at $7\frac{1}{16}$; (iii) 4½ per cent., £10 shares, £4½ paid at 4⅝; (iv) 18 per cent., £12 shares at 41½.

(25) A company bought £5,500 Consols when the price was 74⅝. By how much had the value diminished when the price was $55\frac{3}{16}$? (Neglect brokerage.)

(26) What dividend must be paid on Price's Patent Candle £16 shares at 41½, in order that the yield should be the same as that on the British Cement 6 per cent. Preference £10 shares at 9⅛ ? (Neglect brokerage.)

(27) How many shares of the London, City & Midland Bank (£12, £2½ paid) must be bought, in order that should the Bank pay a dividend of 15 per cent., the income derived would be £250 per annum ?

(28) How many 15 per cent. Preference shares of £1 each at $2\frac{3}{16}$ could be bought with the proceeds of the sale of £8,400 4 per cent. Inscribed Stock at 80 ? What gain or loss of income would be derived by the change ? (Brokerage, 3d. per share and ¼ per cent.; Stamp, 10s. per £50 purchase money.)

(29) At a certain time the price of 3½ per cent. stock was at a premium of 14½. Later on, the price fell to a discount of 12½. By how much was the yield at the latter price greater than that at the former price?

(30) A sum of £45,000 is divided equally between two brothers. The first spent £22,500 in 4 per cent. stock free of income tax at 101½; and the second, £22,500 in 4½ per cent. stock at 93. Reckoning brokerage at ¼ per cent. in each case, neglecting fees and transfer duties, and assuming income tax to be at 5s. in the £, calculate the income, free of income tax, obtained by each.

(31) A man sold £4,000 India 3½ per cent. stock at 64¼ and, with part of the proceeds, invested in 5 per cent. bank stock at 82, and with the remaining part in 4 per cent. Corporation stock at 71¾. The income was increased by £21 4s. 2d. Calculate how much of each stock (to nearest £1) he bought, reckoning brokerage at ¼ per cent. in each case.

(32) A man had £100 to invest. He invested one-half in 3 per cent. stock at 62½, one-eighth in 4 per cent. stock at 88, and the remainder in 5 per cent. stock at 101. Neglecting brokerage, etc., calculate what his total income would be. Also calculate to the nearest penny what his total income would have been had he invested £5,435 17s. 6d.

(*N.B.*—Express income on £100 in £ correct to 5 decimal places and multiply by 54·35875, contracted method.)

(33) A man invested £800 in 4½ per cent. stock at 91 and £1,800 in 3½ per cent. stock at 68. He also invested £1,200 in 5 per cent. stock at such a price that, neglecting brokerage, etc., the yield on his money was 5⅛. Calculate, to the nearest sixteenth, the price at which he purchased the 5 per cent. stock.

(34) Two-fifths of a man's capital was invested in 4½ per cent. stock at 92½ and the remainder in 2½ per cent. stock at 53½. The total income obtained was £215 10s. 6d. Reckoning brokerage at ½ per cent. in each case, find the amount of the man's capital.

(35) A man bought 450 £1 shares at 33s. 6d., brokerage 3d. per share; and 5,200 2s. shares at a premium of 4d., brokerage 1½d. per share. On the £1 shares he received a dividend of 12½ per cent., but received no dividend on the 2s. shares. Calculate the total amount of money paid, the total income, and the average yield.

(36) Twelve days before contango day a man bought 12,000 £1 shares at a premium of 7s. 9d. Eight days later he sold them at a premium of 21s. 3d. Assuming brokerage to be 3d. per share and the total of all other expenses incurred by his broker on his behalf to be £347 5s., calculate the amount he should receive from his broker after the settlement.

(37) A fortnight before the next settlement a "bear" started persistent selling of £1 shares of a certain mining company. He sold 10,000 shares at 21s. 3d., 15,000 at 20s. 9d., 20,000 at 19s. 7½d., 15,000 at 19s. 4½d. He then bought 35,000 at 19s. 3d. each and the remaining shares at 19s. 10½d. each. Reckoning brokerage at 3d. per share, what was his gain?

(38) Supposing the "bear" of the previous question had been forced to buy as follows: 10,000 at 19s. 6d., 20.000 at 21s., 30,000 at 21s. 6d., find the extent of his loss.

(39) Calculate the total cost of the following: £7,500 stock at 74⅝, brokerage ¼ per cent.; contango on purchase money for 12 days at 4⅓ per cent. per annum; stamp duty at 10s. per £50 purchase money; fees, 12s. 6d.

(40) £5,000 5 per cent. stock at 102 is transferred to 3 per cent. stock, and a gain in income of £25 is obtained. What is the price of the 3 per cent. stock? (Neglect brokerage, etc.)

(41) What sum of money to nearest £1 would produce £22 10s. more income if invested in 4½ per cent. stock at 94 than in 3½ per cent. stock at 74½? (Brokerage, ¼ per cent.)

(42) Dividends paid during the year on 5s. shares in a certain company amounted to 7½d. per share. What rate per cent. is this? If the shares could be bought at 12s. 9d. each, what would be the yield? (Neglect brokerage.)

(43) A man invests £10,521 partly in 6 per cent. stock at 104¼ and partly in 4½ per cent. stock at 83¾, so that the total income is £594. How has he divided his capital and how much of each stock does he buy?

CHAPTER XXI.

ANNUITIES.

***166. MONEY ACCUMULATING AT COMPOUND INTEREST** is increased by interest at certain dates; and the interval between one date and the next is, in practice, either a year, a half-year, or a quarter-year. For convenience, in the present chapter, these intervals are termed "interest periods."

The interest on P units of money for n days occurring within an interest period at a nominal rate of i per unit per annum is $\frac{Pni}{365}$ and therefore the amount is $P\left(1 + \frac{ni}{365}\right)$.

Also P units of money at the same nominal rate of interest for exactly N periods, there being p equal interest periods in a year, amount to $P\left(1 + \frac{i}{p}\right)^N$

Combining these results, it is seen that for a length of time consisting of n days immediately before an interest period plus N interest periods, each $\frac{1}{p}$ of a year plus m days immediately after the N^{th} interest period, P units of money amount to

$$P\left(1 + \frac{ni}{365}\right)\left(1 + \frac{i}{p}\right)^N\left(1 + \frac{mi}{365}\right) \text{ units of money}$$

so that the interest is

$$P\left[\left(1 + \frac{ni}{365}\right)\left(1 + \frac{i}{p}\right)^N\left(1 + \frac{mi}{365}\right) - 1\right] \text{ units of money}$$

The **effective rate** of interest is the equivalent rate, if interest were payable yearly instead of p times in a year.

Calculations involving the compound interest formula can quickly be made by the aid of logarithms, and this chapter should not be read by the student unless he is able to use the logarithmic tables. In obtaining answers to the examples and exercises, seven figure logarithms have been used. If the logarithm tables given in the Appendix be used, in most cases answers correct to 3 significant will be obtained.

EXAMPLE (i)—

A man wished to deposit a sum of money on 5th May, 1930, in a bank paying interest at $2\frac{1}{2}$ per cent. per annum, interest payable half-yearly on 31st October and 30th April, such that the amount by the time his son would reach the age of 18 years, on 4th June, 1941, would be £1,000. How much money should he deposit?

Number of days before 31st Oct., 1918	=	179
,, complete interest periods	=	21
,, days after 30th April, 1929	=	35

Let £$V \equiv$ the money that should be deposited,

$$\text{then } V\left(1 + \frac{179 \times \cdot 025}{365}\right)\left(1 + \frac{\cdot 025}{2}\right)^{21}\left(1 + \frac{35 \times \cdot 025}{365}\right) = 1000$$

$$\text{i.e., } V \times \frac{369 \cdot 475}{365} \times 1 \cdot 0125^{21} \times \frac{365 \cdot 875}{365} = 1000$$

$$\therefore \quad V = \frac{1000 \times 365^2}{369 \cdot 475 \times 1 \cdot 0125^{21} \times 365 \cdot 875}$$

$$= 759 \cdot 2293 \text{ (using seven-figure logarithms).}$$

Ansr.—£759 4s. 7d.

EXAMPLE (ii)—

What is the nominal rate per cent. per annum, interest payable quarterly equivalent to an effective rate of 4 per cent. per annum ?

Let $r \equiv$ nominal rate per cent.,

then, equating the amounts of 1 unit after one year,

$$\left(1 + \frac{r}{400}\right)^4 = 1 \cdot 04$$

$$\therefore 1 + \frac{r}{400} = \sqrt[4]{\cdot 104}$$

$$\therefore r = 400(\sqrt[4]{1 \cdot 04} - 1)$$

$$= 400 \times \cdot 009853$$

$$= 3 \cdot 9412.$$

EXAMPLE (iii)—

Certain debentures paying interest at 5 per cent. per annum, half-yearly on 1st April and 1st October are repayable on 1st October, 1936, at 102 per cent. Assuming debentures were bought on 1st October, 1931, at par, calculate the effective rate of interest.

Let $r \equiv$ effective rate per cent. per annum,

then, equating the amount of £100 on 1st October, 1936,

$$100\left(1 + \frac{r}{100}\right)^5 = 100 \times 1 \cdot 025^{10} + 2$$

$$\therefore \left(1 + \frac{r}{100}\right)^5 = 1 \cdot 025^{10} + \cdot 02 = 1 \cdot 30008$$

$$\therefore 1 + \frac{r}{100} = \sqrt[5]{1 \cdot 30008} = 1 \cdot 0539$$

$$\therefore r = 5 \cdot 39..$$

NOTE.—The method adopted assumes that instalments of interest are invested at 5% per annum.

EXAMPLE (iv)—

In what time would £100 amount to £435, if compound interest at 3½ per cent. per annum, interest payable quarterly, be allowed ?

Let $N \equiv$ number of quarter-years,

$$\text{then } 100(1{\cdot}00875)^N = 435$$
$$\therefore\ 1{\cdot}00875^N = 4{\cdot}35$$
$$\therefore\ N \log 1{\cdot}00875 = \log 4{\cdot}35$$
$$\text{i.e., } N \times {\cdot}0037837 = {\cdot}6384893$$
$$\therefore\ N = \frac{{\cdot}6384893}{{\cdot}0037837} = 168{\cdot}7474$$

Ansr.—42 years 68 days.

*167. An **ANNUITY** is a periodic payment, payable either yearly or at more frequent intervals, and lasting during a period called the **status**, which might be a fixed term of years or which might depend on the continuance of a given life or lives. When the status is a fixed term of years, the annuity is called an **annuity-certain**; while a **life-annuity** is one which is payable during the lifetime of a person or of the survivor of a number of persons.

Whether an annuity be payable yearly or at more frequent intervals, it is measured by the total sum payable in one year. Thus, if a person be entitled to receive £50 every three months, he is in possession of an annuity of £200 payable by quarterly instalments.

A **Perpetuity** is an annuity that is to last indefinitely.

Unless otherwise stated, the first payment of an annuity is understood to be made at the end of the first interval. Thus if a person come into possession of the above annuity now, he would be entitled to receive the first instalment of £50 three months hence. Should, however, the first payment be made at the beginning instead of at the end of the first interval, the annuity is called an **annuity-due**.

A **Deferred Annuity** is an annuity which does not begin until after the lapse of a certain number of years. Thus the first payment of an annuity (interval, one year) deferred m years is made at the end of $m + 1$ years, although it is said to be entered on at the end of m years.

An annuity left unpaid for a certain number of years is said to be **forborne** for that number of years. The total of the amounts

of each instalment left unpaid and accumulating at compound interest is called the **amount** of the annuity for so many years.

The **present value** of an annuity is the sum of money which at the present time is equivalent in value to the annuity. For example, a freehold estate yields a perpetuity known as rent, so that the value of the estate can be regarded as the present value of a perpetuity equal to the rent. Similarly, the value of certain leasehold property is the present value of an annuity-certain equal to the rent, the status being the number of years to the time the lease terminates. Estates are frequently bought at so many **years' purchase**: this means that if x is the number of years' purchase, then the present value of the estate is x times the rent which the estate yields.

*168. AMOUNT OF AN ANNUITY.

Let $a \equiv$ the annuity, payable yearly.
,, $i \equiv$ interest per unit per annum, payable yearly.
,, $A \equiv$ the amount of the annuity for n years.
,, $V \equiv$ the present value of the annuity for n years.

i is the interest on 1 for 1 year

$\therefore a$,, ,, $\frac{a}{i}$,, 1 ,,

Now, $\frac{a}{i}$ after n years amounts to $\frac{a}{i}\left(1+i\right)^n$, and this includes the amount of the annuity a for n years, together with the original principal $\frac{a}{i}$

$$\therefore\ A = \frac{a}{i}\left[(1+i)^n - 1\right]$$

If the annuity and also interest be payable half-yearly,

$$\text{then } A = \frac{\frac{a}{2}}{\frac{i}{2}}\left[\left(1+\frac{i}{2}\right)^{2n} - 1\right] \quad \text{i.e., } A = \frac{a}{i}\left[\left(1+\frac{i}{2}\right)^{2n} - 1\right]$$

Similarly, if each be payable p times a year,

$$A = \frac{a}{i}\left[\left(1+\frac{i}{p}\right)^{pn} - 1\right]$$

Consideration of the general case when the number of times during a year interest is payable is unequal to the number of instalments of the annuity payable each year is beyond the scope of the present book, as an advanced knowledge of Mathematics is required. It is interesting to note, however, that in the theoretical treatment of the subject of Finance the compound interest law is exclusively employed; thus the interest for $\frac{1}{q}$ of a year on 1 unit at a nominal rate of i per

unit per annum payable p times a year would be $\left(1+\frac{i}{p}\right)^{\frac{p}{q}}-1$ not $\frac{i}{q}$, even if the period of $\frac{1}{q}$ of a year came within an interest period. The index $\frac{p}{q}$ arises because $\frac{1}{q}$ of a year is $\frac{p}{q}$ of an interest period, and thus $\frac{p}{q}$ replaces the whole number N in the compound interest formula. In other words, the compound interest law is regarded as holding equally well for fractions as for whole numbers of interest periods.

*169. PRESENT VALUE.

In n years, V amounts to $V(1+i)^n$. Now, as V is equivalent in value to the annuity, the amount after n years must be the same as the amount of the annuity.

$$\therefore \quad V(1+i)^n = \frac{a}{i}\left[(1+i)^n - 1\right]$$

$$\therefore \quad V = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

If the annuity and also interest be payable p times a year,

$$\text{then} \quad V = \frac{a}{i}\left[1 - \frac{1}{\left(1+\frac{i}{p}\right)^{pn}}\right]$$

As $\frac{a}{i}$ yields an interest of a each year, $\frac{a}{i}$ invested produces an annuity of a indefinitely.

$\therefore$ The present value of a perpetuity of $a = \frac{a}{i}$.

This result is also obtained by making n an infinitely large number and using the formula $V = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$. When n is infinitely large $\frac{1}{(1+i)^n}$ becomes zero, and V becomes $\frac{a}{i}$.

Let $V_m \equiv$ present value of the annuity to commence at the end of m years and to continue n years,

$$\text{then} \quad V_m(1+i)^m = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$\therefore \quad V_m = \frac{a}{i(1+i)^m}\left[1 - \frac{1}{(1+i)^n}\right]$$

If the annuity and also interest be payable p times a year,

$$\text{then} \quad V_m = \frac{a}{i\left(1+\frac{i}{p}\right)^{pm}}\left[1 - \frac{1}{\left(1+\frac{i}{p}\right)^{pn}}\right]$$

Also by making n infinitely large, it is seen that the present value of a perpetuity of a deferred m years is

$$\frac{a}{i(1+i)^m} \quad \text{or} \quad \frac{a}{i\left(1+\frac{i}{p}\right)^{pm}}$$

according as to whether the annuity and also interest are payable yearly or payable p times a year.

An annuity-due which consists of n yearly payments of a can be considered as an immediate payment of a, together with an annuity for $n-1$ years, so that the amount at the time the last payment is made is

$$a(1+i)^{n-1} + \frac{a}{i}\left[(1+i)^{n-1} - 1\right]$$

and the present value is

$$a + \frac{a}{i}\left[1 - \frac{1}{(1+i)^{n-1}}\right]$$

If the annuity-due be paid p times a year and interest also be payable p times a year, then the amount is

$$\frac{a}{p}\left(1+\frac{i}{p}\right)^{pn-1} + \frac{a}{i}\left[\left(1+\frac{i}{p}\right)^{pn-1} - 1\right]$$

and the present value is

$$\frac{a}{p} + \frac{a}{i}\left[1 - \frac{1}{\left(1+\frac{i}{p}\right)^{pn-1}}\right]$$

To transform the formulae from the case when the annuity and interest are each payable yearly to that when both are payable p times a year, it is only necessary to change a to $\frac{a}{p}$, i to $\frac{i}{p}$, and n to pn; and in the case of deferred annuities, m to pm. It should be noted also that it has been assumed that the times between consecutive payments of the annuity coincide with the interest periods. For example, suppose the annuity to be paid on 31st May of each year and interest added to principal on 31st December of each year, then the application of the above formulae would give results slightly differing from the actual practical results.

*170. LEASES AND SINKING FUNDS.

Suppose that a man having purchased a lease yielding a rent of a annually, and lasting for n years, wishes, after x years have elapsed, to renew the lease for an additional y years, he must pay a sum of money which is called the **fine** for renewing y years of the lease. This fine is the present value of an annuity a for y years deferred $n - x$ years, and so the formula giving the present value of a deferred annuity can be applied in order to calculate the extent of the fine.

When capital has to be replaced, say, n years hence, it frequently happens that a **sinking fund** is formed consisting of n equal yearly payments, which are such that, accumulating, they together

amount to the sum that has to be replaced at the end of the n years. Thus, the sinking fund can be regarded as an annuity amounting to the original capital under the stated conditions, and so the formula giving the amount of the annuity can be applied to the solution of this problem.

An extract from a table giving the present value of lease, freehold estate, or annuity is as follows—

Years.	3%	4%	5%	6%	Years.	3%	4%	5%	6%
5 .	4·58	4·45	4·33	4·21	55 .	26·77	22·11	18·63	15·99
10 .	8·53	8·11	7·72	7·36	60 .	27·67	22·62	18·93	16·16
15 .	11·94	11·12	10·38	9·71	65 .	28·45	23·04	19·16	16·29
20 .	14·88	13·59	12·46	11·47	70 .	29·12	23·39	19·34	16·38
25 .	17·41	15·62	14·09	12 78	75 .	29·70	23·68	19·48	16·45
30 .	19·60	17·29	15·37	13·76	80 .	30·20	23·91	19·59	16·51
35 .	21·49	18·66	16·37	14·50	85 .	30·63	24·11	19·68	16·55
40 .	23·11	19·79	17·16	15·05	90 .	31·00	24·27	19·75	16·58
45 .	24·52	20·72	17·77	15·46	95 .	31·32	24·40	19·80	16·60
50 .	25·73	21·48	18·26	15·76	100 .	31·60	24·50	19·85	16·62
					Perpetual	33·33	25·00	20·00	16·66

N.B.—The numbers in the column give the present value of an annuity of 1 unit of money or, what is the same numerical value, the number of years' purchase of any annuity. The present value of a perpetuity of a units is $\frac{a}{i}$ units, so that the number of years' purchase is $\frac{1}{i}$, which equals $\frac{100}{\text{rate per cent. per annum.}}$ The figures in the above table refer to annuities paid once a year, and interest is assumed to be payable yearly.

*171. LIFE-ANNUITIES.

The values of life-annuities depend upon certain mortality tables prepared by actuaries. The approximate value, however, at different rates of interest can be calculated by using the data given in a table called the *expectation of life*, an extract from which is as follows—

Age.	Mean Expectation of Life.		Age.	Mean Expectation of Life.		Age.	Mean Expectation of Life.	
	Male.	Female.		Male.	Female.		Male.	Female.
0	44·13	47·77	30	33·07	35·39	60	12·93	14·10
5	53·50	55·79	35	29·24	31·52	65	10·34	11 27
10	49·63	51·97	40	25·64	27·82	70	8·05	8·78
15	45·21	47·61	45	22·20	24·20	75	6·15	6·70
20	41·02	43·44	50	18·90	20·64	80	4·62	5·05
25	37·01	39·37	55	15·79	17·24	85	3·45	3·80

N.B.—When this table is used in any of the questions that follow, the numbers of years should be taken as the nearest whole numbers above the mean expectation of life. The figures referring to males are not applicable to men having liability for military service during times of war.

The following worked examples illustrate the principles dealt with in this chapter—

EXAMPLE (v)—

What is the amount and the present value of an annuity-certain of £150 for 12 years, reckoning interest at $3\frac{1}{2}$ per cent. per annum?

$$i = \cdot 035,\ (1 + i) = 1\cdot 035.$$

$$\text{Amount of the annuity} = £\frac{150}{\cdot 035} \times (1\cdot 035^{12} - 1)$$

$$= £\frac{150 \times \cdot 511066}{\cdot 035}$$

$$= \underline{£2{,}190\ 5s.\ 8d.}$$

$$\text{Present value of the annuity} = £\frac{150}{\cdot 035} \times \left(1 - \frac{1}{1\cdot 035^{12}}\right)$$

$$= £\frac{150 \times \cdot 3382157}{\cdot 035}$$

$$= \underline{£1{,}449\ 9s.\ 11d.}$$

EXAMPLE (vi)—

A man built property having a lease of 99 years. The annual rent was such that interest at 8 per cent. per annum, payable quarterly, was obtained on the initial value of the property. Assuming the rent to be paid by quarterly instalments, what is the number of years' purchase? Hence, calculate what rent was charged if the initial value of the property was £750?

Let $x \equiv$ number of years' purchase,

then £x = present value of an annuity of £1 for 99 years

$$\therefore\ x = \frac{1}{\cdot 08} \times \left(1 - \frac{1}{1\cdot 02^{396}}\right)$$

$$= \frac{\cdot 999607}{\cdot 08} = \underline{12\cdot 49509}$$

$$\therefore\ \text{Annual rent} = £\frac{750}{12\cdot 49509}$$

$$= \underline{£60\ 0s.\ 6d.}$$

EXAMPLE (vii)—

A tenant owns an estate, the annual rent of which is £135, and the lease lapses at the end of 8 years. What fine should he pay for renewing 10 years of the lease, reckoning that the rent is paid by half-yearly instalments and that interest is at $4\frac{1}{2}$ per cent. per annum, payable half-yearly?

$$\text{Fine for renewing 10 years of the lease} = £\frac{135}{\cdot 045 \times 1\cdot 0225^{16}} \times \left(1 - \frac{1}{1\cdot 0225^{20}}\right)$$

$$= £\frac{135 \times \cdot 3591828}{\cdot 045 \times 1\cdot 0225^{16}}$$

$$= \underline{£754\ 15s.\ 9d.}$$

EXAMPLE (viii)—

The cost price of a machine which will last 15 years is £3,750. There is also a similar machine of better quality that will last 20 years. Reckoning interest at 3 per cent. per annum, at what price should the more durable machine be purchased, in order that in the long run it may be as cheap as the former one?

Considering the cost of each machine to consist of equal annual amounts spread over the times the machines last, then for one machine to be as cheap as the other the annual cost of one must equal that of the other.

Let $£x \equiv$ annual cost of each machine,

" $£V \equiv$ cost of the more durable machine,

$$\text{then } 3750 = \frac{x}{\cdot 03} \times \left(1 - \frac{1}{1\cdot 03^{15}}\right) \ldots\ldots (1)$$

$$\text{and } V = \frac{x}{\cdot 03} \times \left(1 - \frac{1}{1\cdot 03^{20}}\right) \ldots\ldots (2)$$

From (1) and (2), by division,

$$\text{then } \frac{V}{3750} = \frac{1 - \frac{1}{(1\cdot 03)^{20}}}{1 - \frac{1}{(1\cdot 03)^{15}}}$$

$$\therefore V = 3750 \times \frac{\cdot 4463236}{\cdot 3581376}$$

$$= 4673\cdot 381$$

Ansr—£4,673 7s. 7d.

EXAMPLE (ix)—

A man is 40 years of age. Reckoning interest at 2½ per cent. per annum, what sum should be set aside each year so that on attaining the age of 65 he could purchase a life-annuity of £200 per annum?

Let $£x \equiv$ sum set aside each year,

$$\text{then } £\frac{x}{\cdot 025} \times \left(1\cdot 025^{25} - 1\right) \equiv \text{amount saved after 25 years.}$$

Reckoning the man's expectation of life as being 11 years, this amount is the present value of an annuity of £200 for 11 years.

$$\therefore \frac{x}{\cdot 025} \times \left(1\cdot 025^{25} - 1\right) = \frac{200}{\cdot 025} \times \left(1 - \frac{1}{1\cdot 025^{11}}\right)$$

$$\therefore x = 200 \times \frac{1 - \frac{1}{1\cdot 025^{11}}}{1\cdot 025^{25} - 1}$$

$$= \frac{200 \times \cdot 237856}{\cdot 853942}$$

$$= 55\cdot 70776$$

Ansr.—£55 14s. 2d.

EXAMPLE (x)—

Using the tables, find what a lady 45 years of age should pay for a life-annuity of £120, reckoning interest at 4 per cent. per annum.

Expectation of life of a lady 45 years of age is nearly 25 years.

$$\therefore \text{ Price of the annuity} = £15{\cdot}62 \times 120$$
$$= £1{,}874 \ 8s.$$

EXAMPLE (xi)—

A certain life assurance company has an endowment assurance table giving the premiums to secure £100 payable at death or at the expiration of certain numbers of years. The premium (payable at the beginning of each year) for a man aged 35 to obtain the money, if living, after 25 years, is £4 5s. 1d. Assuming the man lives to receive the money and reckoning interest at 3 per cent per annum, calculate the profit of the insurance company at the time the money is paid. Also find the loss to the company at the time of payment should the man die just before the end of 10 years.

Amount of annuity due of £4 5s.1d. at the beginning of the 25th year $= £\dfrac{4{\cdot}254167}{{\cdot}03} \times (1{\cdot}03^{25} - 1)$

$\therefore$ Amount at end of 25th year $= £\dfrac{4{\cdot}254167 \times 1{\cdot}093774 \times 1{\cdot}03}{{\cdot}03}$

$= £159 \ 15s. \ 2d.$

$\therefore$ Gain by the Insurance Company at the end of 25 years $= £ \ 59 \ 15s. \ 2d.$

Amount of annuity due of £4 5s. 1d. at the beginning of the 10th year $= £\dfrac{4{\cdot}254167}{{\cdot}03} \times (1{\cdot}03^{10} - 1)$

$\therefore$ Amount at end of 10th year $= £\dfrac{4{\cdot}254167 \times {\cdot}343916 \times 1{\cdot}03}{.03}$

$= £50 \ \ 4s. \ 8d.$

$\therefore$ Loss by the Insurance Company at the end of 10 years $= £49 \ 15s. \ 4d.$

*EXAMPLE (xii)—

What is the present value of an annuity consisting of payments of a, $a + d$, $a + 2d$ . . . $a + (n-1)d$ at the end of 1, 2, 3 . . . n years respectively?

Let $i \equiv$ interest on 1 per annum, interest payable yearly.

,, $V \equiv$ present value of the annuity.

$$\text{Then } V = \frac{a}{1+i} + \frac{a+d}{(1+i)^2} + \frac{a+2d}{(1+i)^3} + \ldots\ldots + \frac{a+(n-1)d}{(1+i)^n}$$

$$\therefore \frac{V}{1+i} = \qquad \frac{a}{(1+i)^2} + \frac{a+d}{(1+i)^3} + \ldots\ldots + \frac{a+(n-2)d}{(1+i)^n} + \frac{a+(n-1)d}{(1+i)^{n+1}}$$

$$\therefore V\left[1 - \frac{1}{1+i}\right] = \frac{a}{1+i} + \frac{d}{(1+i)}\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \ldots\ldots + \frac{1}{(1+i)^{n-1}}\right] - \frac{a+(n-1)d}{(1+i)^{n+1}}$$

The quantity in the bracket on the R.H.S. is the present value of an annuity of 1 for $(n-1)$ years $\therefore$ the quantity $= \frac{1}{i}\left[1 - \frac{1}{(1+i)^{n-1}}\right]$

$$\therefore V \times \frac{i}{1+i} = \frac{a}{1+i} + \frac{d}{i(1+i)}\left[1 - \frac{1}{(1+i)^{n-1}}\right] - \frac{a+(n-1)d}{(1+i)^{n+1}}$$

$$\therefore V = \frac{a}{i} + \frac{d}{i^2}\left[1 - \frac{1}{(1+i)^{n-1}}\right] - \frac{a+(n-1)d}{i(1+i)^n}$$

$$= \frac{1}{i}\left[a + \frac{d}{i} - \frac{d + i\{a + nd\}}{i(1+i)^n}\right]$$

*EXAMPLE (xiii)—

A certain appointment carries with it a salary commencing at £120 per annum, rising by annual increments of £15 to a maximum of £450, and the holder must retire after 40 years' service. Assuming the holder will complete the 40 years' service; that the salary is payable monthly; and reckoning interest at 3 per cent. per annum, interest payable yearly, what single sum is equivalent, at the time of his taking up the appointment, to the total salary for the 40 years?

Considering any complete year, let $£x \equiv$ each monthly instalment.

$$\text{Total interest for the year} = \frac{x \times \cdot 03}{12} \times [11 + 10 + \ldots + 1]$$

$$= \frac{x \times \cdot 03 \times 66}{12} = \cdot 165x$$

$\therefore$ Twelve monthly payments of $£x$ are equivalent to one payment of $12{\cdot}165x$ at the end of the year.

$\therefore$ The salary can be regarded as commencing at $£\frac{12\cdot 165}{12} \times 120$

i.e., $£1{\cdot}01375 \times 120$, and rising by $£1{\cdot}01375 \times 15$ annually to a maximum of $£1{\cdot}01375 \times 450$, the payments being made at the end of each year.

After 23 years, the first complete year's payment at £450 will be obtained.

Let $£V \equiv$ present value of salary for the first 23 years' service,

$$\text{then} \quad V = \frac{1\cdot 01375}{\cdot 03}\left[120 + \frac{15}{\cdot 03} - \frac{15 + \cdot 03 \times 465}{\cdot 03 \times 1\cdot 03^{23}}\right]$$

$$= \frac{101\cdot 375}{3}\left[120 + 500 - \frac{28\cdot 95}{\cdot 03 \times 1\cdot 03^{23}}\right]$$

Let $£V_1 \equiv$ present value of salary for the remaining 17 years,

$$\text{then} \quad V_1 = \frac{1\cdot 01375 \times 450}{\cdot 03 \times (1\cdot 03)^{23}}\left[1 - \frac{1}{1\cdot 03^{17}}\right]$$

$$\therefore V + V_1 = \frac{101\cdot 375}{3}\left[620 - \frac{965}{1\cdot 03^{23}} + \frac{450}{1\cdot 03^{23}} - \frac{450}{1\cdot 03^{40}}\right]$$

$$= \frac{101\cdot 375}{3}\left[620 - \frac{515}{1\cdot 03^{23}} - \frac{450}{1\cdot 03^{40}}\right]$$

$$= \frac{101\cdot 375}{3}[620 - 260\cdot 9465 - 137\cdot 9509]$$

$$= \tfrac{1}{3} \times 101\cdot 375 \times 221\cdot 1026$$

$$= 7471\cdot 423$$

Ansr.—£7,471 8s. 6d.

EXAMPLE (xiv)—

A company purchased machinery for £40,000. One year later £2,000 was set aside; another year later, another sum of £2,000 was set aside; and so on. After what time would the sum of £40,000 be replaced, reckoning interest at $3\frac{1}{2}$ per cent. per annum ?

Let $n \equiv$ number of years,

$$\text{then } 40000 = \frac{2000}{\cdot 035}\,[1{\cdot}035^n - 1]$$

$$\therefore\ 1{\cdot}035^n - 1 = \frac{40 \times 35}{2000}$$

$$\therefore\ 1{\cdot}035^n = 1{\cdot}7$$

$$\therefore\ n \log 1{\cdot}035 = \log 1{\cdot}7$$

$$\therefore\ n = \frac{\cdot 2304489}{\cdot 0149403}$$

$$= 15{\cdot}42465$$

$$\text{Amount of annuity of £2,000 for 16 years} = £\frac{2000}{\cdot 035}\,[1{\cdot}035^{16} - 1]$$

$$= £\frac{2000 \times \cdot 733983}{\cdot 035}$$

$$= £41{,}941\ 17\text{s. } 9\text{d.}$$

Thus £40,000 will be obtained at the end of 16 years by setting aside £2,000 at the end of each of the first 15 *years* and the sum of £58 2s. 3d. at the end of the 16th year.

NOTE 1.—Supposing the total sum to be replaced had been £39,000, the value of n would have been just over 15, but before the end of the 16th year the amount would have reached £39,000 without a 16th payment having been made at all. Now, the amount of an annuity of £2,000 for 15 years at $3\frac{1}{2}$ per cent. per annum is £38,591·2, so that if $x \equiv$ the number of days after the end of 15 years when the amount reaches £39,000, then $408{\cdot}8 = \frac{38591{\cdot}2 \times 3{\cdot}5x}{36500}$, from which $x = \frac{408{\cdot}8 \times 36500}{38591{\cdot}2 \times 3{\cdot}5}$ $= 110{\cdot}47$. Thus a trifle over **£39,000** is obtained after **15 years 111 days** by setting aside **£2,000** at the end of each of the first **15** years.

NOTE 2.—It has been assumed that interest is payable on the date on which the machinery was purchased. If this were not assumed, say, the machinery was purchased on 10th May and interest payable on 31st December, it would be necessary to find the amount of each instalment at the end of the year in which it was set aside.

NOTE 3.—The amount would be £40,000 approx. after 15·42465 years, if the final payment at the end of this period were ·42465 of £2,000.

TEST EXERCISES IV, 4.

(1) £2,000 was deposited on 16th April, 1930, in a bank paying interest on 30th June and 31st December at the rate of $2\frac{1}{2}$ per cent. per annum. Calculate the amount on 10th August, 1937, and also on 17th May, 1939.

(2) What is the effective rate per cent. per annum if the nominal rate is 5 per cent. per annum, interest payable half-yearly ?

(3) What is the nominal rate per cent. per annum, interest payable quarterly, when the effective rate is 8 per cent. per annum ?

(4) Calculate the effective rate of interest in the case of debentures paying interest at 5 per cent. per annum half-yearly on 1st April and 1st October, and repayable on 1st October, 1938, at 103 per cent., assuming they are purchased at par on 31st March, 1932.

(5) What is the effective rate of interest in the case of 5 per cent. debentures paying interest half-yearly on 1st April and 1st October, and repayable 1st October, 1941, at 105 per cent., if purchased on 30th September, 1932, at 101?

(6) Interest at 10 per cent. per annum is payable half-yearly on 1st March and 1st September respectively. What is the effective rate for the year ending 31st December?

(7) A man deposited £500 on 30th March, 1932, in a bank paying interest at 3 per cent. per annum on 30th June and 31st December. On what date should the amount as near as possible be £900?

(8) A five-year note is issued on 1st May, 1931, at 98 per cent., bearing interest at $4\frac{1}{2}$ per cent. per annum. What is the yield per cent. if the note is held until its redemption at par? Interest is payable yearly on 1st May.

(*N.B.*—Amount of £98 after 5 years $= 98 \times \left(1 + \frac{4{\cdot}5}{98}\right)^5 + 2$)

(9) Find the amounts of the following annuities—

(*a*) £150 payable in half-yearly instalments for 15 years at 4 per cent per annum, interest payable half-yearly.

(*b*) £20 per month for 10 years at 3 per cent. per annum, interest payable half-yearly.

(*c*) Annuity-due of £150, payable in half-yearly instalments for 15 years at 4 per cent. per annum, interest payable half-yearly.

(*d*) Annuity-due of £20 per month for 10 years at 3 per cent. per annum, interest payable half-yearly.

(*N.B.*—In (*b*) and (*d*), first find the amount of six payments at the end of 6 months.)

(10) Find the present value of the following annuities—

(*a*) £80 for 20 years at $3\frac{1}{2}$ per cent. per annum.

(*b*) Annuity-due of £80 for 20 years at $3\frac{1}{2}$ per cent. per annum.

(*c*) Perpetuity of £15 per month at 4 per cent. per annum.

(*d*) £100 to commence after 10 years and to last 15 years, interest at 3 per cent. per annum.

(*e*) £100 by quarterly instalments to commence after 8 years and to last 10 years, interest at 4 per cent. per annum, payable half-yearly.

(*N.B.*—£25 at the end of each quarter is equivalent to £$50\frac{1}{4}$ at the end of each half-year.)

(11) Find the number of years' purchase of—

(*a*) An annuity-certain for 15 years at 3 per cent. per annum.

(*b*) An annuity-due ,, ,, ,,

(*c*) A perpetuity ,, ,,

(12) The number of years' purchase of a house is $13\frac{1}{2}$. Reckoning interest at 6 per cent. per annum, find to the nearest year the duration of the lease.

(13) When the lease has 60 years to run, the value of a house is £500. Reckoning interest at 5 per cent. per annum, calculate the value of the house, assumed to be in good repair, 30 years later. (*See* Ex. viii.)

(14) What is the rent of the house of Question (13)?

(15) Certain property yields a rent of £225 per annum, and the lease terminates at the end of another 15 years. What fine must be paid to renew 10 years of the lease? Assume that rent and interest, which is at 5 per cent. per annum, are payable at the end of each year.

(16) Referring to Question (15), calculate the fine in the case when the rent is paid by quarterly instalments; and (i) interest is payable yearly, (ii) interest is payable half-yearly.

(17) A company issues £150,000 debenture stock, which is repayable in 10 years' time at 104 per cent. Reckoning interest at $3\frac{1}{2}$ per cent. per

annum, what sum should be set aside at the end of each year to enable the company to make the repayment?

(18) A man at the beginning of a certain year resolved to deposit £50 at the end of each year in a bank. For the first 8 years the rate of interest was 3 per cent. per annum; and for the next 12 years, $2\frac{1}{2}$ per cent. per annum. What was the amount at the time the 20th deposit was made?

(19) Referring to Question (18), what would have been the amount had the 7th, 8th, and 9th deposits not been made?

(20) A life-annuity of £1 can be bought for £17 17s. 10d. by a man of 40, and for £19 18s. by a woman of 40. Reckoning interest at $2\frac{1}{2}$ per cent. per annum, calculate approximately the expectation of life of a man of 40 and a woman of 40.

(21) A person of 24 years of age of either sex could, by paying the Government £54 7s., insure the payment of £100 at the age of 55 or sooner in the event of death. If the person should reach the age of 55, what would the Government gain at the time the £100 payment is made? Also calculate the loss to the Government if the person should die at the age of 37. Reckon interest at $2\frac{1}{2}$ per cent. per annum.

(22) The insurance of the previous question can also be effected by a person aged 24 years by payment of an annual premium of £2 17s. 6d., the first payment to be paid immediately. Calculate the gain and loss of the Government if (i) the person reaches 55 years of age, and (ii) the person dies at the age of 37, reckoning interest at $2\frac{1}{2}$ per cent. per annum.

(23) A man whose age next birthday is 30 years can secure, by a premium of £3 7s. 9d. paid annually to a certain insurance company, the first paid immediately, £100 at death or at the expiration of 30 years. In addition, the company adds a bonus of £1 10s. for each year on which the premiums had been paid. Calculate (i) the profit to the company if the man does not die before the expiration of the time, and (ii) the loss if he dies just before the expiration of the 10th year. Reckon interest at 3 per cent. per annum.

(24) A man retires at the age of 60 years, and his employer gives him a pension of £200 a year paid in quarterly instalments for the rest of his life. Reckoning his expectation of life to be 13 years, and that interest is at 4 per cent. per annum payable half-yearly, what single sum is equivalent to this pension?

(25) A man directed in his will that a life-annuity of £140 should be purchased for his daughter aged 45. Reckoning interest at $3\frac{1}{2}$ per cent. paid half-yearly, the annuity paid in half-yearly instalments and the expectation of life of the daughter to be $24\frac{1}{2}$ years, calculate the price the executors should pay for the annuity.

(26) Draw a graph connecting the present worth of an annuity of 1 with the number of years from 0 to 40. Hence approximately find (i) the present worth of an annuity of 100 dollars for 22 years, and (ii) the present worth of an annuity of 1,000 francs for 28 years. Take interest to be at 3 per cent. per annum and use the figures given in the table.

(27) Draw a graph connecting the ages of females from 30 to 55 years, with their expectations of life. Using this graph and the table giving the present worth of an annuity of 1, find approximately the value of a life-annuity of £100 for a lady 42 years of age, reckoning interest at 4 per cent. per annum.

(28) A man is 45 years of age. Reckoning interest at $3\frac{1}{2}$ per cent. per annum, find how much he should set aside at the end of each year to enable him to purchase a life-annuity of £150 when he is 65 years of age.

(29) Find the present worth of 45 years' salary which, commencing at £80, rises by annual increments of £10 to a maximum of £250 per annum. Consider salary to be paid monthly and interest to be at 4 per cent. per

annum, payable half-yearly. [*N.B.*—£x per annum paid monthly is equivalent to £$\frac{12\cdot221x}{12}$ paid at the end of the year, and effective rate of interest is 4·04 per cent. per annum.]

(30) A company purchased plant for £25,000 at the beginning of a year, and at the end of the year and each subsequent years placed £1,500 in a sinking fund to enable new machinery to be bought later on in place of the old. Reckoning interest at 3 per cent. per annum, find the time which must elapse before £25,000 is available for this purpose.

(31) Referring to the above question, what would the time be if £1,000 could be obtained by the sale of the old machinery.

(32) What perpetuity has the same value as an annuity of £75 for 15 years, reckoning interest at 3½ per cent. per annum ?

(33) The present value of each of a number of annuities is £5,000. Draw a graph connecting the annual payments with the number of years the annuities have to run by calculating the former when the latter is 4, 6, 8, 10, 12, and 14 years respectively. Use the graph to find the annuity when the status is 9 years. Reckon interest at 5 per cent. per annum.

SECTION V.

CHAPTER XXII.

INSURANCE.

172. Insurance plays a very important part in commercial life, for without it merchants would be faced continually with the risk of serious losses brought about by fires, thefts, floods, shipwrecks, etc. However carefully goods may be guarded, unavoidable accidents may cause destruction or damage. By means of insurance, losses so incurred are spread more or less evenly over the whole community, the share borne by any individual being represented by the premiums he has paid.

Problems in connexion with the insurance of property against loss by fire or burglary present little difficulty, but life insurance involves very intricate actuarial calculations which are beyond the scope of this book.

This section deals chiefly with simple problems that arise in connexion with marine insurance. There is a mass of detail connected with this subject that cannot be considered in this book, but it is hoped that the broad principles are made clear. This being so, the student should find no difficulty over this section, for the arithmetical work is of a very simple character.

173. INSURANCE.

The various risks to which goods are exposed in the course of their transport by land or by sea evoked the need at an early date of some form of transport insurance. This first arose in the form of marine insurance, which can be traced back into the thirteenth and fourteenth centuries. It is now considered a matter of course that every merchant should insure his goods in any case against marine risks, although these have been greatly reduced in consequence of improved means of communications. But although accidents in peace time are proportionally small in comparison with those of early days, it would be considered a very risky undertaking on the part of the exporter or importer who did not insure his goods during the sea voyage under the impression that he would save the amount of the premiums.

With an insurance company a large number of contracts is concluded, so that the risk is spread evenly over a period of time. Insurance companies also reduce their risk by re-insuring with other companies, thus bringing about a further equalization of loss in the event of accidents.

In this way it becomes possible to obtain an idea of the magnitude of the average risk, and thus to arrive at a basis for the determination of the insurance premium. The solution of the problem of insurance lies in the uniting of a large number of persons, all of whom contribute proportionally towards the amount required for covering the loss to which they are collectively exposed. The risks of damage and loss which enter into consideration during transport by sea are incurred through storms, ice, shipwrecks collision, damage by sea-water, fire, lightning or explosion, piracy, etc.; besides destruction or damage by enemy ships in time of war.

The centre for marine insurance business is in London, where the majority of transactions are effected either with members of Lloyd's Corporation—in which each member carries on business on his own account—or with one of the large marine insurance companies.

174. MARINE INSURANCE AT LLOYD'S.

The institution known as Lloyd's had its origin in a coffee-house kept by a certain **Edward Lloyd** about 1690. It was the resort of persons interested in shipping matters, and the proprietor had established a means of obtaining shipping news. The name of this enterprising proprietor is now perpetuated in the world-wide celebrity of the institution, which is still unique in its facilities for obtaining news concerning the world's shipping.

Generally, insurances at Lloyd's are effected through the medium of **brokers**, who receive their remuneration from the **underwriters** in the form of a reduction of 10 per cent. on the premiums they arrange to pay. The course of business is, for the broker to write on a slip the particulars of the insurance he desires to effect with any special arrangement he may desire to make. The slip having been made out, the broker presents it to one or more of the underwriters with whom he usually does business. The underwriters note on the slip the share of the insurance they will take; and

since the widest possible distribution of risks is desirable, it is seldom that any one of the underwriters will take a larger sum or "*line*" (as it is called) than £300.

175. MARINE INSURANCE AT INSURANCE COMPANIES.

In effecting insurances with these companies, much larger lines are, of course, taken. Where several offices are concerned, it is usual to disclose to each the names of the companies where other parts of the same insurance are placed—and if any part is done at Lloyd's, the amount that is placed there—so that each may know its share of the risk.

176. KINDS OF POLICIES.

Policies are divided into various classes—

Voyage Policies and **Time Policies**, in which property is insured for transit from one point to another only, or for a certain period of time.

Valued Policies and **Open Policies**, in the former of which the value of the insured object is definitely stated at the time of drawing up the contract. In open policies there is no such statement; but the value of the goods lost or damaged must be proved if required: as a rule, the basis is formed by the invoice value, in addition to shipping charges and a certain percentage on account of profit.

Named Policies, in which the name of the vessel on which the risk is taken is definitely stated; and **Floating Policies**, where the wording is wide enough to cover the insured property by whatever ship it may come.

177. Almost all claims on underwriters are made in consequence of loss due to perils of the sea, which include—

(1) *Particular Average.*

(2) *General Average.* (3) *Total Loss.*

Particular Average Claims are those which arise from unavoidable causes, such as storms at sea, fire, or accident of any description. It is usually agreed between the underwriters and the insured that no claim for loss by these causes shall be made unless it amounts to 3 per cent. of the value declared in the policy

On some goods it is necessary that the claim should amount to 5 per cent. whilst on others no claim for particular average loss can be made at all

Among the former class are sugar, tobacco, hemp, flax, hides, and skins; and among the latter are corn, fish, salt, fruit, flour, and seed. The reasons for these exceptions are that these two classes of goods are exceptionally hazardous by reason of their peculiar liability to spontaneous injury.

The measure of any Particular Average loss is the difference between the amount realized for the goods concerned in their damaged condition and the sum they would have realized had they arrived sound and uninjured.

General Average Claims arise in another form, and may be distinguished from those made under Particular Average as follows:

(1) Such losses are distributed over all the persons having any share or interest in the ship and her cargo.

(2) They always arise out of some act done voluntarily by the master of the ship involved, and for the purpose of rescuing the ship and her cargo from some loss which would have caused her to become a total loss.

For example, suppose a ship is being blown towards the shore and the crew are unable to prevent her from being cast on to it. The master may call in the assistance of a steam tug, which is able to keep the ship from becoming a wreck by towing her to a place of safety. The remuneration for the services of the steam-tug is not the time and labour expended as in ordinary towage, but is proportional to the value of the property saved from destruction, and thus may amount to a considerable sum. The arrangement of the amount to be paid, and the payment of it, fall upon the shipowner; but he is entitled to recover a part of the sum from those others whose interests are concerned These interests are generally—

(1) The ship.
(2) The sum she is earning for freight on the voyage.
(3) The cargo she is carrying

Suppose the expense for towing amounted to £1,000 and that the ship is worth £9,000, the freight £1,000, and her cargo £10,000; that is, a total of £20,000. Then—

the ship would have to pay $\frac{9}{20}$ of £1,000; i e, £450,
„ freight „ „ „ $\frac{1}{20}$ „ „ £50,
„ cargo „ „ „ $\frac{10}{20}$ „ „ £500.

In a similar manner, if the master of a ship deem it necessary for the safety of the property under his control to sacrifice any part of it so as to secure the safety of the ship and the remaining cargo, he is entitled to do so; but the owners of the lost property are entitled to receive payments on the basis of General Average from those whose property has been made secure by the master's action.

Total Loss Claims are generally easier to settle than those already mentioned. They may be divided into two classes—

For example, a ship may founder at sea and never again be heard of. After a time, when she is posted as "missing," the underwriters pay the sum for which they have insured. Or the vessel may sink after collision, when likewise the underwriters must pay. Again, she may run on shore and there be more or less broken to pieces, when the owners of ship and cargo alike may call upon the underwriters to accept an *abandonment*. The

latter will do this if the demand is justified and accept for total loss, taking as their own so much of the insured property as is saved from the wreck.

178. PROPERTY PARTIALLY INSURED.

It sometimes happens, whether in connection with marine insurance or with respect to insurance against fire, burglary, etc., that property is insured for a certain amount, which is less than the full value. In the event of the property being totally lost, an amount equal to that for which it was insured can be claimed. If, however, a portion only of the property be lost or damaged, a claim for the full amount of the loss, even if less than the amount of the policy, could not be made.

Supposing property whose value was £1,000 was insured for £600 and that $\frac{1}{4}$ of the same was totally destroyed. The underwriter or the company would consider that $\frac{3}{5}$ of the whole property was fully insured and that $\frac{2}{5}$ was not insured at all, and that $\frac{1}{4}$ of the insured part and $\frac{1}{4}$ of the part not insured was destroyed. Consequently the greatest amount for which liability would be accepted would be $\frac{1}{4}$ of the amount for which insurance was made; that is, the owner of the property would recover an amount equal to $\frac{1}{4}$ of $\frac{3}{5}$ i.e., $\frac{3}{20}$ of the total value of the property and thus lose $\frac{1}{4}$ of $\frac{2}{5}$ (i.e., $\frac{1}{10}$ of the total value).

179. In general, if property be insured for an amount greater than its value, an amount greater than the actual value could not be claimed in the event of loss. There are exceptions to this, however; for it is legitimate to insure for such an amount that, in the event of loss, a sum of money representing the value of the property, together with the premium paid for the insurance, can be claimed. Again, in connection with the importation of goods, it is quite usual to insure for an amount obtained by adding together the importer's invoice price, the freight, commission to agents, insurance, and an addition of 10 per cent. profit to cover estimated profits.

EXAMPLE (i)—

Property whose value was £2,250 was insured for £1,650. Find what amount the owner could claim, if property of value £840 was totally destroyed, and (1) the remainder was undamaged, (2) the remainder was damaged, causing its value to depreciate by 60 per cent.

(1) Amount to be claimed $= £\frac{840}{2250} \times 1650$

$= £616$

(2) „ „ $= £616 + £\frac{1410}{2250} \times \frac{60}{100} \times 1650$

$= £1{,}236\ 8s.\ 0d.$

EXAMPLE (ii)—

Goods valued at £7,860 are to be insured at 22s. 6d. per cent. For what amount to the nearest £5 should they be insured in order that in the event of total loss the value, together with the premium paid, could be recovered ?

Let 100 units $\equiv$ amount for which goods are insured,
then $1\frac{1}{8}$ unit $\equiv$ premium paid,
„ $98\frac{7}{8}$ units $\equiv$ actual value of goods insured.

$\therefore$ Amount for which the goods should be insured $= £\frac{100}{98\frac{7}{8}} \times 7860$

Ansr.—£7,950.

NOTE 1.—An alternate method is as follows—

Let $£x \equiv$ amount for which insured,

$$\text{then } £x \times \frac{1\frac{1}{8}}{100} = \text{premium paid}$$

$$\therefore 7860 + \frac{9x}{800} = x$$

$$\text{i.e., } 7860 \times 800 + 9x = 800x$$

$$\text{i.e., } x = \frac{7860 \times 800}{791}$$

EXAMPLE (iii)—

A ship, damaged in a collision, was assisted by lighters. Some of the cargo was transferred to the lighters, but some was jettisoned in order to enable the ship to reach port. Particulars regarding ship, cargo, etc., and the losses incurred are given in the following tables—

Contributory Values for General Average.	£
Ship (less cost of repair, which is a Particular Average item) . . .	10,450
Freight	1,020
A & Co.'s goods . . .	7,200
B & Co.'s „ . . .	2,750
C & Co.'s „ . . .	12,500
D & Co.'s „ . . .	7,500
E & Co.'s „ . . .	3,580
Total of contributory values	45,000

General Average Statement.	£
Shipowner's Losses— Charges to lighters, harbour dues, warehousing of cargo, wages of crew, preparing claims, etc., etc.	4,800
A & Co.'s goods lost .	5,140
B & Co.'s „ „ .	1,100
C & Co.'s „ „ .	2,220
„ „ damaged sound value	2,600
D & Co.'s goods damaged sound value	1,540
Total of losses . . .	17,400

Calculate the net amounts received or paid out by each.

			£	s.	d.		
Shipowner	pays $£\frac{174}{450} \times$ 11,470,	i.e.,	4,435	1	4	and receives	£4,800
A	„ $£\frac{174}{450} \times$ 7,200,	„	2,784	–	–	„	£5,140
B	„ $£\frac{174}{450} \times$ 2,750,	„	1,063	6	8	„	£1,100
C	„ $£\frac{174}{450} \times$ 12,500,	„	4,833	6	8	„	£4,820
D	„ $£\frac{174}{450} \times$ 7,500,	„	2,900	–	–	„	£1,540
E	„ $£\frac{174}{450} \times$ 3,580,	„	1,384	5	4	„	*nil*
	Total . . .		£17,400	–	–	Total .	£17,400

		£	s.	d.
∴ Net amount received by shipowner	=	364	18	8
" " A	=	2,356	-	-
" " B	=	36	13	4
Total	=	£2,757	12	-
" paid by C	=	13	6	8
" " D	=	1,360	-	-
" " E	=	1,384	5	4
Total	=	£2,757	12	-

NOTE 2.—The column stating what each owner pays and what each receives should be carefully studied. It is assumed that the ship, freight, and cargo is fully insured; and by the principle of general average, the losses allocated to the various owners are £4,435 1s. 4d., £2,784, £1,063 6s. 8d., £4,833 6s. 8d., £2,900, and £1,384 5s. 4d. respectively. These amounts then could be claimed by the owners from the respective underwriters who undertook the insurance. Now, the shipowner can only claim £4,435 1s. 4d. from his underwriter, whereas his loss amounts to £4,800: therefore he is entitled to receive £364 18s. 8d. from those owners who could claim an amount from their underwriters greater than the amount of their individual loss. Thus E actually has sustained no loss as regards his cargo but, legally, he can be called upon by the other owners for £1,384 5s. 4d., which represents the share of the loss to be borne by him, and he, in turn, can claim this amount from the underwriters who undertook the insurance of his goods. Actually a person or firm is employed to attend to the entire business. This person or firm, known as an average adjustor, first collects £4,435 1s. 4d., £2,784, £1,063 6s. 8d., £4,833 6s. 8d., £2,900, and £1,384 5s. 4d. from the underwriters who have undertaken the insurance of the goods of the shipowner, A, B, C, D, and E respectively; and thus obtains a total sum of £17,400, with which sum he is enabled to send £4,800, £5,140, £1,100, £4,820, and £1,540 to the shipowner, A, B, C, and D respectively.

The cost of undertaking the work of the settlement of claims falls at first upon the shipowner; but as it is included in the shipowner's losses, it is actually borne by each owner proportionally to the value of his goods.

C and D have received £4,820 and £1,540 respectively; but this is greater than the loss actually incurred, for the damaged goods may be sold, say, at a price half that at which the undamaged goods could be sold. The proceeds of the sale of C & D's damaged goods, however, would be paid to the respective underwriters of C and D.

EXAMPLE (iv)—

Of 18 cases of goods of same kind and quality, 4 were damaged by sea-water. The gross weight of 4 cases of goods in sound condition was 18 cwt. 1 qr. 24 lb., tare and draft 3 qr. 8 lb. The gross weight of the 4 cases of goods in the damaged state was 18 cwt. 3 qr. 26 lb. The price of the undamaged goods was 1s. 1½d. per lb. and that of the damaged goods was estimated at 9¾d. per lb. If the goods were insured for £550, calculate the sum that should be paid in respect of the insurance.

Net weight of 4 cases of goods when sound = 18 cwt. 1 qr. 24 lb. − 3 qr. 8 lb.

= 1,976 lb.

" " " damaged = 1,976 lb. + 58 lb.

= 2,034 lb.

Value of 4 cases of sound goods = 1,976 × 1⅛ shillings

= 2,223 "

Value of 4 cases of damaged goods $= 2{,}034 \times 9\frac{1}{2}$ pence

$= 1{,}610\frac{1}{4}$ shillings

$\therefore$ Loss $= 612\frac{3}{4}$ „

Total value of 18 cases of sound goods $= 2{,}223 \times \frac{18}{4}$ shillings

$\therefore$ Ratio of loss to total value $= \dfrac{612\frac{3}{4}}{2{,}223 \times \frac{9}{2}}$

$\therefore$ Amount of claim under Particular Average $= £\dfrac{2{,}451 \times 2}{4 \times 2{,}223 \times 9} \times 550$

$= £33$ 13s. 9d.

TEST EXERCISES V.

(1) Calculate the premium to be paid to insure goods for £8,950 at 11s. 9d. per cent.

(2) The sum of £143 16s. 11d. was paid to insure goods for £87,400. Calculate the rate per cent. of the premium.

(3) The net weight of goods was 43 cwt. 1 qr. 15 lb. and the invoice price was 1s. 4½d. per lb. Freight was at 17s. 3d. per measurement ton of 40 cub. ft. on 644 cub. ft. plus 10 per cent. primage. Allowing 12½ per cent. for additional expenses and profit, for what amount to the nearest £5 should the importer insure the goods? Calculate the amount of the premium at 7s. 4½d. per cent.

(4) A merchant estimates he will be able to sell his goods for £4,735. He wishes to insure so that in the event of total loss he can claim £4,735 together with the sum paid as premium. If the premiums be at 14s. 6d. per cent., calculate to the nearest pound the amount for which he should insure. Also calculate the total amount of the premium.

(5) Goods are insured for £2,400 at 18s. 6d. per cent. If the sum of £2,400 be arrived at by increasing the actual value of the goods by 10 per cent. and then adding the cost to insure, calculate to the nearest pound the actual value of the goods.

(6) Goods valued at £3,650 are insured for £2,500 against fire. In the event of the destruction of goods to the value of £2,500, what sum could be obtained from the insurance company?

(7) A quantity of a certain commodity is insured for 65 per cent. of its value: 32 per cent. is utterly destroyed; 40 per cent. is damaged and thereby depreciates in value by 60 per cent.; while the remainder is undamaged. Neglecting the amounts paid as premium, what percentage of the initial value of the goods is the net loss incurred by the owner?

(8) Certain goods were partially damaged. The undamaged goods were sold for £495 and the damaged goods for £325. It was estimated that the goods, if undamaged, could have been sold for a total of £1,240. If the goods were insured for £1,000, calculate the amount the owner was entitled to receive from the underwriters with whom the insurance was effected.

(9) A ship ran aground, and certain cargo was thrown overboard in order to free the ship. The total loss incurred, including damage to ship, loss of freight, and cargo, and various expenses, amounted to £4,647 10s. 6d. Jones & Co. had goods to the value of £1,100, which were saved free from damage. If the total value of ship, freight, and cargo amounted to £11,750, what would the underwriters who undertook the insurance of the goods of Jones & Co. be called upon to pay, assuming the goods were insured to their full value?

(10) Referring to the previous question, calculate what Jones & Co. and

his underwriters would respectively have to pay if Jones & Co.'s goods had been insured for £924.

(11) Referring to Question (9), calculate the amount the underwriters of Jones & Co. would have to pay if two-thirds of the latter's cargo had been lost. Also find how much the firm of Jones & Co. would have been entitled to receive, assuming the goods were fully insured.

(12) A ship, damaged in a storm, was assisted into port by tugs; but some of the cargo was jettisoned and some was damaged. From the following tables calculate what the respective underwriters would be called upon to pay, assuming ship, freight, and cargo were fully insured. Also write down the amounts the respective owners should receive.

Contributory Values for General Average.	£	General Average Statement.	£
Ship (less cost of repair)	14,500	Shipowner's losses (including expenses and charges)	3,240
Freight	1,260	A & Co.'s goods lost	4,160
A & Co.'s goods	8,400	„ damaged (sound value)	1,840
B & Co.'s „	3,580	B & Co.'s „ lost	1,425
C & Co.'s „	4,150	C & Co.'s „ „	2,075
D & Co.'s „	2,200	D & Co.'s „ „	760
X & Co.'s „	1,910		
Total of contributory values.	36,000	Total of losses	13,500

(13) Referring to the previous question, if B & Co., C & Co., and D & Co had not had their goods insured at all, calculate the net losses incurred by them respectively.

(14) If the damaged goods of A & Co. were sold for £472 10s, what would be the net loss incurred by A & Co.'s underwriter?

(15) Owing to an accident, a ship was damaged, and the cargo partly lost and partly damaged. The total of contributory values was £11,400, and the total of losses including the sound values of the damaged goods was £3,720. Roberts & Co. had goods on board to the value of £1,455, of which one-quarter was lost and one-third was damaged. The goods were fully insured, and the damaged goods fetched 45 per cent. of their sound value. Calculate the net loss incurred by the underwriter of Roberts & Co.

(16) A merchant insured 500 qr. of corn valued at 57s. 6d. per quarter, and the underwriter undertook to pay Particular Average only if damage amounting to not less than 5 per cent. of the insured value was incurred. 36 qr. were damaged and were sold at 23s. 9d. per quarter. If the merchant had effected the insurance on four quantities of 125 qr. each separately, and if each quantity was insured for £375, calculate the amounts that could be claimed from the underwriter if (i) the damage was spread equally over the four quantities; (ii) no damage was done to two quantities, 10 qr. of the third quantity was damaged, and the remainder occurred in the fourth quantity; (iii) all the damage occurred in one of the quantities.

(17) A consignment of fruit consisted of 30 cases, of which 8 cases of fruit were damaged. The gross weight of these 8 cases of fruit when in sound condition was 8 cwt. 1 qr. 20 lb., and tare and draft 2 qr. 24 lb. In the damaged condition the gross weight was 9 cwt 0 qr. 18 lb, and the fruit was sold at 1s. 2d. per lb.; whereas the sound fruit could be sold for 1s. 11d. per lb. If the consignment was insured for £360, calculate the sum the underwriters should pay under Particular Average.

(18) A merchant imported 50 bales of wool, which he insured for £1,080. Altogether 10 bales of wool were damaged; 6 bales were sold at 11½d. per lb., and the remaining 4 bales at 1s. 0½d. per lb. The gross weight of the 6 bales

in sound condition was 18 cwt. 3 qr. 16 lb., tare and draft 2 qr. 10 lb.; while the gross weight in the damaged condition was 19 cwt. 2 qr. 20 lb. The gross weight of the 4 bales was 12 cwt. 3 qr. 10 lb., tare and draft 1 qr. 17 lb.; while the gross weight after the wool was damaged was 13 cwt. 0 qr. 12 lb. The wool in the undamaged state could be sold for 1s. 3d. per lb. Calculate the total amount to be paid by the underwriters in respect of the insurance.

(19) 40 bales of cotton were imported and insured for £545. 14 bales of cotton were damaged, particulars being as follows—

	Gross Weight (Undamaged).			Tare and Draft.			Gross Weight (Damaged).			Price at which sold.
	CWT.	QR.	LB.	CWT.	QR.	LB.	CWT.	QR.	LB.	
7 bales . .	25	2	10	1	0	14	27	0	8	6d. per lb.
4 ,, . .	14	2	11		2	16	15	2	7	6½d. ,,
3 ,, . .	10	3	26		1	26	11	2	3	7d. ,,

If the price of cotton in the sound condition was 8d. per lb., calculate the percentage deterioration, correct to 3 decimal places, in each of the three quantities of damaged goods.

(20) Using the data and the answers of the previous question, calculate the total sum to be paid in respect of the insurance.

MISCELLANEOUS QUESTIONS.

The following questions are selected from the examination papers of various examining bodies, such as the Royal Society of Arts, the London Chamber of Commerce, The Institute of Bankers, etc.

(Arranged approximately according to difficulty.)

(1) Make out an invoice for the following—

$2\frac{1}{2}$ lb. tea at 3s. 4d. lb.

10 oz. coffee at 1s. 10d. lb.

3 lb sugar at $5\frac{3}{4}$d. lb.

4 lb. 7 oz. bacon at 2s. 8d. lb.

3-lb jar marmalade at 10d. lb. (jar 1d. extra).

3 lb. 3 oz cheese at 1s. 5d. lb.

1 tin, biscuits of $6\frac{1}{2}$ lb at 1s. 3d. lb. (tin, 1s. 2d. extra).

$1\frac{1}{2}$ doz. eggs at $2\frac{1}{2}$d. each.

Allow for 5 jars at $1\frac{1}{2}$d. each already returned.

(2) The following table gives the value of production per year, the number of wage-earners, and the horse-power employed in the United Kingdom and the United States for two trades—

		Production per Year.	Horse-power used	No. of Wage-earners.
Boots and Shoes	United Kingdom .	£ 20,095,000	20,171	117,565
	United States .	£102,359,000	96,362	198,297
Clothing .	United Kingdom .	£ 62,169,000	17,837	392,084
	United States .	£190,566,000	65,019	393,439

Calculate for each country and each trade the value of production per wage-earner, and the horse-power used per thousand wage-earners, giving your answers to the nearest whole number.

(3) In one year we imported 105,222,638 cwt. of wheat, which cost £44,160,884. Find, to the nearest penny, the average cost of 1 cwt.

(4) A square foot of sheet lead $\frac{1}{8}$ in. thick weighs 7 lb. 6 oz. Given that a gallon of water weighs 10 lb. and that the volume of a gallon of water is $277\frac{1}{4}$ cub. in., find how many times lead is heavier than water.

(5) The prices for grain are: Wheat, 45s. 6d. per sack of 504 lb.; oats, 29s. 6d. per sack of 336 lb.; Barley, 38s. 4d. per sack of 448 lb. By how much per cent. are the prices of oats and barley below the price of wheat?

(6) A bought goods for £21 17s. 6d. and sold them to B at a profit of 12 per cent. B sold the goods to C for £26 6s. 9d. What profit per cent. did B make?

(7) A banker's assets are £1,226 5s. 4d. and his debts are £7,357 12s. How much can he pay in the £?

(8) Find the sum which will amount to £185 12s. in 4 years, simple interest being reckoned at 5 per cent. per annum.

(9) Make out a bill for the following articles bought from Henry Smith, corn dealer, Tiverton, by Thomas Jones—

23 trusses of hay at 4s. 6d. per truss.
$\frac{1}{2}$ cwt. mixed corn at 7s. 1d. per stone.
700 lb. of bone meal at £9 10s. per ton.

Receipt the bill, and allow 1s. in the £ discount for cash.

(10) What is the freight on 42 cases of goods at 24s. 8d. per ton of 40 cub. ft. and 10 per cent. primage, if each case measures 4′ 2″ × 3′ 3″ × 2′ 9″?

(11) Two steers weighed 16 cwt. 3 qr. 26 lb. and 16 cwt. 1 qr. 10 lb. The dead weight is $\frac{7}{10}$ of the live weight. Find their carcase value at 11s. 4d. per stone of 8 lb.

(12) The cost of gravelling a rectangular piece of ground, the sides of which are in the ratio of 2 to 3, is £32 8s. at the rate of 9d. per square foot. Find the length of the sides.

(13) Find the cost of 2 tons 17 cwt. 3 qr. 14 lb. when 7 cwt. 28 lb. cost £39 13s. 10$\frac{1}{2}$d.

(14) The cost of the gas for a stove burning 10 hr. a day for 28 days, and using 15 cub. ft. of gas per hour, is 11s. 8d. Another stove, burning 18 cub. ft. of gas per hour for 25 days cost 15s. For how many hours a day was this second stove alight?

(15) What was the price realized for 3 ac. 3 rd. 29 sq. per. 11 sq. yds. of land at £60 per acre? (Practice.)

(16) An iron safe is cubical in shape and measures on the outside 2 ft. 3 in. each way. The sides are 1 in. thick. Find the weight of the safe if a cubic foot of iron weighs 487 lb.

(17) 1,430,000 crans of herrings (about 1,000 herrings to a cran) were caught, during a certain season, off the coast of Scotland. Their value was estimated at £3,500,000. At this rate, how many herrings were worth half-a-crown?

(18) A field was sold for £561, and a loss sustained thereby of 6$\frac{1}{4}$ per cent. on the cost. Find what would have been the gain per cent. if it had been sold for £612.

(19) 16 annas make a rupee. If the value of the rupee in English money is 1s. 4$\frac{5}{32}$d., find the number of rupees and annas you ought to get for £25.

(20) On a map the plan of an estate has an area of 2·61 sq. in. The scale of the map is 6 in. to the mile. Find, in acres, the area of the estate.

(21) Under a property, with an area of 3 sq. ml., lies a coal-seam supposed to be about 2 ft. 8 in. thick. Find, to the nearest million tons, the probable weight of coal in the property. (1 cub. ft. of coal weighs 80 lb.)

(22) Find, in acres, roods, perches, to the nearest perch, the area of a rectangular field, the length of which is 9 ch. 28 lk., and the breadth 5 ch. 76 lk.

(23) Reduce 7s. 7$\frac{1}{2}$d. to the decimal of £1. Make a table showing the value, in decimals of £1, of 1, 2, 3, 4, 5, 6, 7, 8, 9 articles, each costing 7s. 7$\frac{1}{2}$d. Use the table to find the value of 753 such articles.

(24) A man held a bill of exchange for £355, dated 10th January, 1932, and payable 90 days after date. If he discounted it on 8th February at 4 per cent., how much did he receive?

(25) A grocer buys raw coffee at £4 14s. 6d. per cwt.; the duty is 1½d. per lb. in addition. He roasts 1 cwt., which yields him 93 lb. of roasted coffee. What does the coffee cost him per lb. when roasted? If he sells it at 1s. 8d. per lb., what profit per cent. does he make (*a*) on his return? (*b*) on his outlay?

(26) The following table exhibits the traffic, etc., at a certain port in 1911 and 1931.

	1911.	1931.
Tons of Cargo Inwards . . .	235,510	674,318
Passengers	118,096	302,405
Length of Quays	11,067 ft.	22,150 ft.
Shed Area	486,198 sq. ft.	953,026 sq. ft.

Find to the nearest integer the percentage increase in each case.

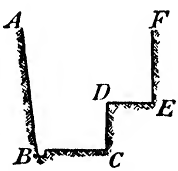
If the shed area had increased in proportion to the length of the quays, find to three significant figures what would have been the shed area in 1931

(27) The Geira is a Brazilian measure of land and equals 600 sq. Bracas. A Braca is 2·2 metres nearly, and a metre is nearly 39·4 in. Express a Geira as the decimal of an acre (two places of decimals).

(28) Verify that $\frac{714 \times 79 \times 3{\cdot}7}{36500} + \frac{715 \times 34 \times 3{\cdot}7}{365} + \frac{814 \times 82 \times 3{\cdot}7}{36500}$

$$= \frac{3{\cdot}7}{36500} \times [714 \times (79 + 34 + 82) + 34 + 8200]$$

Express $£x$ per cwt. in shillings per lb. and in pence per oz.

(29) 

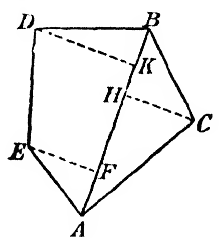
The figure shows the section of a trench. $AF = 5' \; 6''$, $BC = 3'$, $CD = 2'$, $DE = 1' \; 6''$, and $EF = 3'$. Reckoning that the volumes of earth to be excavated by the average untrained man to be: 1st hour, 30 cub. ft; 2nd hour, 25 cub. ft.; 3rd hour, 15 cub ft.; each subsequent hour, 10 cub. ft., find what length of trench ought to be dug in 6 hr. by a working party of 50 men.

(30) Calculate the import duty on—

13 cwt. 2 qr 20 lb. sugar at 5s. 8·3d per cwt.
7 cwt. 1 qr. 10 lb. „ 8s. 1·6d. „

(31) By the minimum amount of arithmetical calculation, evaluate the following—

(*a*) ·3078 × 178·255 + ·3078 × 37·275 − 215·51 × ·2998.
(*b*) 1·784 × 16·92 − 1·783 × 16·91.

(32) 

ACBDE represents a field. The dimensions are given as follows—

	Chains	
	10·05 *B*	
5·24 *D*	8·32 *K*	
	6·25 *H*	3·72 *C*
3·84 *E*	2·56 *F*	
	From *A*	

Calculate the area in acres, roods, and square poles to the nearest square pole, and find the rent at £9 10s. per acre.

(33) If a ship were chartered at 29s. 3d., and she carried 3,100 tons of cargo, of which 1,250 tons was re-let at 31s. 6d., 740 tons at 30s. 4d., and

310 tons at 29s., and if the bills of lading were filled up at these rates, at what rate would the bill of lading for the remaining tons have to be made out?

(34) If the population of a town increases every year by 1·8 per cent. of the population at the beginning of that year, in how many years will the total increase of the population be 30 per cent.?

(35) A Rubber Company has a plantation of 850 ac. and a capital of £35,000. It expects this year to produce 233,600 lb. of rubber and to sell it at an average price of $3\frac{1}{16}$d. lb. What must be the total expenses per acre if the Company expects to earn 7 per cent. on its capital?

(36) Find the dividend at the rate of 12s. 8d. in the £ on—

(*a*) £748 15s. (*b*) 174 francs 50 centimes. (*c*) 645 dollars 75 cents.
(1 franc = 100 centimes and 1 dollar = 100 cents.)

(37) In Brazil, a book of railway ticket coupons for 6,000 Km. costs 120 milreis. Taking 1 milreis as worth $4\frac{1}{32}$d. in English money, and taking 8 Km. as equal to 5 miles, find whether this is cheaper or dearer than the rate of 1d. per mile.

(38) It is required to raise £2,500 by means of a rate levied on a district assessed at £23,500. If the expenses of collection are 3 per cent. of the money collected, find, to the nearest farthing, how much in the £ the rate must be?

(39) A banker discounts a bill which has 25 days to run before it is legally due at $5\frac{1}{16}$ per cent. per annum. This discount amounts to £1 0s. 3d. For what sum was the bill drawn?

(40) An article which costs 8s. in London, sells for 4 dollars in New York. Find to the nearest penny the profit made by a man who buys 1,200 of these articles in London and sells them in New York, after paying $4\frac{1}{2}$d. per article for freight and a Customs duty of 20 per cent. on the New York selling price. (£1 = 3·88 dollars.)

(41) Find, to 3 significant figures, a multiplier for converting a price in shillings per lb. into a price in francs per kilogramme. (£1 = 89·25 francs; 1 Kg. = 2·205 lb.)

(42) In a certain year, £124,652,674 5 per cents. was converted into 4 per cents. £3,787,292 was paid off to those people unwilling to accept the reduction. £100 of the 5 per cent. was, however, converted into £105 of the 4 per cent. Find the yearly saving to the country.

(43) Given that the rate of exchange between New York and London is 3·57 dollars = £1, obtain the value of 1 dollar as the decimal of £1, retaining 7 digits after the decimal point; then write down in a column the value of 1, 2, 3 up to 9 dollars in terms of £1 to the same degree of accuracy. Make use of this table in determining the values of these three amounts, viz., 4,695 dollars 75 cents, 7,698 dollars 50 cents, and 1,056 dollars 25 cents in terms of the £1, converting the final result in each case into £ *s*. *d*.

(44) Draw up a table for the conversion of pounds, shillings, and pence into francs and centimes, showing in 3 columns the values in francs and centimes of 1, 2, 3, up to £9; 1, 2, 3, up to 9 shillings; and 1, 2, 3, up to 9 pence—and use this table to write down the equivalents of £496 17s. 8d. and £7,309 13s. 6d. in francs and centimes. (£1 = 82·26 francs.)

(45) A circular pond in a public garden is 72 ft. in diameter, and is surrounded by a gravel path 12 ft. wide. Find the cost, at 8s. 3d. per cubic yard, of fresh gravel for the path, to cover it to a depth of 3 in. ($\pi = \frac{22}{7}$.)

(46) By how much per cent. is freight at 23s. 9d. per ton of 40 cub. ft. greater or less than freight at 19s. 6d. per ton of 50 cub. ft. together with 8s. 6d. per ton of 20 cwt., if each case of goods measured 5′ 6″ × 4′ 3″ × 3′ and has a gross weight of 14 cwt. 2 qr. 18 lb.?

(47) A person arranges to receive an advance of £50 on the 1st day of each month from January to June inclusive. On 30th June he pays off

the loans with interest on them at 6 per cent. per annum. What does the settling payment amount to?

(48) Given that 1 gall. = $277\frac{1}{4}$ cub. in., and that fine gold is 19·26 times heavier than water, find to the nearest grain the weight of 1 cub. in. of fine gold.

(49) A tea dealer blends 75 lb. of tea at 1s 8d. per lb. with 45 lb. of tea at 1s. 11d. per lb. What is the lowest price per lb. at which he can sell the mixture so as to gain at least 25 per cent.?

(50) A bankrupt fails for £10,400. His assets are estimated at £8,530. A first dividend of 10s. 6d. in the £ is paid. On the estate being wound up, the assets realize only 90 per cent of their estimated value, and the expenses of the Receiver absorb 15 per cent. of that realized value. Find how much in the £ can be paid as final dividend.

(51) A Bill of Exchange for £400 dated 1st April, 1932, and payable 60 days after date, was discounted on 30th April for £398 13s. 11d. What rate of interest per annum was charged?

(52) Decimalize £5 13s. $8\frac{1}{2}$d. and, multiply the result by 6,938 correct to 3 decimal places, and finally express this product as pounds, shillings, and pence.

(53) Given that an English gallon is 277·274 cub. in., that 6 United States gallons are equal to 5 English gallons, and that a litre is 61 cub. in., express the United States gallon in litres, to 2 places of decimals.

(54) An investor in this country applied for 10,000 francs nominal capital of the 5 per cent. Rentes of the French National Defence Loan issued in December, 1915, paying for them in one sum at the rate of £3 3s. 6d. per 100 francs. A few years later, the exchange became £1 = 124·21 francs. What income did the investor obtain then and what was the yield of the investment?

(55) If the Russian measure of length, the arshin, is equal to 28 in., and the Russian measure of area, the dessiatine, is 21,600 sq arshins, express a dessiatine in acres.

(56) A manufacturing firm is accustomed to allow to trade customers a discount or rebate of 15 per cent. on the prices in its published list. This has given it a profit of 19 per cent. on the cost of manufacture. The cost of manufacture goes up 12 per cent., and the firm issues a new price list with all the prices put up 10 per cent. If the firm continues to allow its customers the same rate of discount, what percentage of profit will it now make on the cost of manufacture?

(57) The £100 shares of a company are quoted on the Stock Exchange at £$127\frac{1}{2}$. The company offers to the shareholders more of the same shares at the price £110, allowing each shareholder to apply for one new share for every two shares he already holds. Show that a shareholder, taking up his new shares, virtually receives a bonus of $5\frac{5}{8}$ per cent. on the nominal value of all the shares he now holds.

(58) A merchant has a debt of 10,000 doll. to discharge in New York. Find what it will cost him in English money (i) if he remits direct to New York at 3·60 doll. for £1; and (ii) if he remits to an agent in Paris at 88·60 fr. for £1, and his agent remits to New York at 24·60 fr. for 1 dollar.

(59) A ship with its cargo is worth £99,882, and is insured at 7 per cent. by its owners, A, B, and C, for such an amount that in case of loss the value of the ship and the premium will be recovered. A has an interest in the ship of £52,319, B of £33,294, C of £14,269. How much will each have to pay for premium (to within £1)?

(60) A bank allows a customer to overdraw to the extent of £1,000, the interest chargeable being 4 per cent. for the first month, $4\frac{1}{2}$ per cent. for the second month, and 5 per cent. for the third and fourth months or part

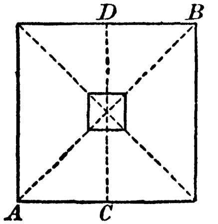
thereof. The account was overdrawn by £940 for 100 days from the 1st of May. What was the interest?

(61) Jones, Smith & Brown are partners in a firm, and the capital each has subscribed is: Jones, £15,000; Smith, £4,000; Brown, £1,000. On dissolving partnership and meeting all liabilities except capital, there remains £8,800. What should each receive or pay out if (i) no agreement as to division of losses exists? (ii) by agreement they are to bear 55 per cent., 25 per cent., and 20 per cent. of the loss?

(62) A merchant fails for £10,520 and pays a first dividend of 11s. 6d. in the £1, and afterwards a final dividend of 5s. 6d. in the £1 on what was then due. How much altogether did he pay in the £, and what must his assets have amounted to?

(63) A dealer sells two horses for £66 16s. 6d. each. He gains 10 per cent. of his outlay on one and loses 10 per cent. in the other. What is his total gain or loss?

(64)



A garden in the shape of a square contains a square flower bed placed symmetrically as shown. If the length of the sides be 95 ft. and 18 ft. respectively, calculate the shortest distance a person would have to walk to go from (i) A to B; (ii) C to D.

(65) A hectare is an area of 10,000 sq. metres. Taking 1 metre = 39·37 in. and £1 = 25·22 fr., find, to three significant figures, a multiplier for converting £s per acre into francs per hectare.

(66) A man invests the same amount of money in each of three different stocks. The rates of interest paid by these stocks are 3, $3\frac{1}{2}$, and 5 per cent.; and their prices, including brokerage, are 80, 90, and 110 respectively. Find, to the nearest farthing, the average rate of interest that he receives on the money he invests.

(67) Given that the interest on £100 for 1 day at 4 per cent. is £0·0109589, make a table showing the interest at that rate for 1, 2, 3 . . . 9 days. Use the table to find the interest on £269 for 271 days. Answer to the nearest penny.

(68) The price of a certain article on a manufacturer's price list is twice the cost of manufacture. On the list price the retailer is allowed 35 per cent. trade discount. The retailer gets a profit of $15\frac{1}{2}$ per cent. on what he charges to the public. What percentage is the cost of manufacture of the price that the public pays?

(69) A bookseller takes off 3d. in the shilling for ready money, and then makes a profit of 17 per cent. on what the books cost him. What will be the price in his list for a book which he buys from the publisher at the rate of 20s. 10d. per dozen, 13 copies of the book going to the dozen?

(70) I borrow £100 from a money-lender, and undertake to repay by four instalments of £40 each, payable at the end of 3, 6, 9, and 12 months. What rate of Simple Interest is he charging?

(71) A roadway 376 yds. long and 51 ft. broad is paved with granite blocks which have a depth, as laid, of 6 in. The interstices between the blocks have a volume which is 2 per cent. of the volume of the blocks. Find what the blocks cost if they are sold at 15s. per ton, and 1 cub. ft. of granite weighs 168 lb.

(72) A and B become partners, A's capital being £2,000 and B's £1,000. It was arranged that capital and withdrawals of capital should bear interest at 5 per cent. per annum, and that half the net profit should be added to each partner's capital account. After 1 month, A withdrew £60 and B withdrew £130. The net profit after 3 months' trading was £246 8s. 10d. How much capital did A and B respectively possess then?

(73) A stock of timber which will be sufficiently seasoned for sale in eight months will then be worth £4,650. Find its present value reckoning interest at 5 per cent. per annum.

(74) In a certain borough a poor rate of 1s. 3d. in the £ is just sufficient to provide for the guardians' requirements for the year. In the following year the rateable value of property in the borough increases 10 per cent., and the poor rate is raised 1d. in the £. Find by what percentage the amount raised by the rate increases.

(75) Three men, Jones, Brown, and Robinson, together rent a field of 19 ac. at £2 10s. per acre. During a certain year Jones puts in 11 sheep for 5 months, Brown 18 sheep for 4 months, and Robinson 21 sheep for 3 months. How much should each of them pay towards that year's rent ?

(76) The capital of a company consists of £225,000 of 4 per cent. debentures, £150,000 of 5 per cent. preference stock, and £250,000 of ordinary stock. What income would be derived by a man who held £100 of each (*a*) when the total amount divided was £13,500 ? (*b*) when the total amount divided was £25,250 ?

(77) The average measurements of a rick are: Length, 27 ft.; width, 17 ft.; height to the eaves, 16 ft.; and height to top, 20 ft. Find the weight of hay if 1 cub. ft. weighs 6 lb. 14 oz., and the value at 4s. 10d. per truss of 60 lb.

(78) Genuine milk contains 88·75 per cent. by bulk of water, 2·75 of fat, and 8·5 of non-fatty solids. A purchaser buys 8 gall. of milk at 6d. per quart; the milk on being analysed is found to contain 90·84 per cent. of water, 2·24 of fat, the residue being non-fatty solids. Find whether anything besides water has been added to it; and find the sum of which the purchaser has been defrauded.

(79) The shape of a field whose area is stated to be 1 ac. $7\frac{1}{4}$ sq. ch. is that of an equilateral triangle. If a plan is required to be drawn on a scale of $\frac{1}{4}''$ to 1 yd., what should be the length of each side of the triangle drawn on the plan ?

(80) The annual cost of warehousing, apart from loss of interest, is £154, and the average value of stock warehoused is £5,350. The average stock is turned over 2·65 times during the year. Calculate the total cost, including loss of interest at 6 per cent. per annum, of warehousing each £100 worth of stock until it is sold.

(81) A retailer bought 1 tub of butter, the list price being £8 17s. 6d. He was allowed a trade discount of $12\frac{1}{2}$ per cent. and a cash discount at $2\frac{1}{2}$ per cent. one month. What was his gain if he sold all the butter at 2s. 4d. per lb. ?

(82) A disc of metal $\frac{1}{4}''$ thick has a radius of 10·24 in. Four circular holes, each having a radius of 1·2 in. are bored out. Given that 1 cub. ft. of the metal weighs 515 lb., calculate the weight of the piece that remains.

(83) An agent receives each year from his employer a certain sum for travelling expenses, and also a certain percentage of the amount of his sales as commission. In three successive years his sales amounted to £12,800, £18,400, £17,600 respectively. His income (including the allowance for expenses) was for the first year £350, and for the second year £437 10s. What was it for the third year ?

(84) A circular marble table-top is of uniform thickness $1\frac{1}{4}$ in. and weighs 37 lb. 2 oz. Given that a cubic foot of the marble weighs 156 lb., find, to an inch, the diameter of the table. ($\pi = \frac{22}{7}$.)

(85) A mortgage of £1,575, rate of interest $4\frac{1}{2}$ per cent. per annum, was neglected for $3\frac{1}{2}$ years. What was the total amount then due, compound interest being charged ? (Do not use logarithms.)

(86) Goods were bought at £2 7s. 6d. with four months' credit and sold forthwith at £2 11s. with such credit as would give $6\frac{3}{4}$ per cent. profit on the cost. How much credit was given, reckoning interest at 4 per cent. per annum ?

(87) The Simple Interest on £100 for 100 days at $3\frac{1}{2}$ per cent. is £0·9589041. Find, from this, the interest on £100 for 200 days, for 40 days, and for 7 days Hence obtain the interest on £100 at $3\frac{1}{2}$ per cent. for 247 days, and give the answer in £ *s. d.*

(88) A manufacturer sells goods to a wholesale dealer at a profit of 10 per cent. on what they cost him; the wholesale dealer sells the *same* quantity of goods to several retailers at a profit of 8 per cent. on what they cost him. The difference between what the goods cost the manufacturer and what the wholesale dealer sold them for was £169 4s. Find their cost to the manufacturer.

(89) A man owns a house which he lets at £70 a year, but has to do the repairs, which may be reckoned at about one-sixth of the rent. If he were to convert it into two flats at an outlay of £107 10s., and let the flats at 19s. 6d. a week each, he paying the local rates at 6s. 10d. in the £ on an assessment of £64, find how much better off he would be in the year, after allowing 4 per cent. as interest on the money sunk, and allowing an average of £5 a year for each flat for repairs.

(90) Without logarithms, find, to 5 decimal places of £1, the Compound Interest on £100 for 4 years at $2\frac{1}{2}$ per cent. Hence write down the Interest for the same time and rate on £2,000, on £500, and on £40, and give the answer in each case in £ *s. d.* to the nearest penny.

(91) Formerly, certain goods were selling at a profit of 15 per cent. above cost price. Since then the cost price has decreased by 40 per cent., and the selling price has been lowered 45 per cent. What is the percentage profit of cost price?

(92) A manufacturer's list price shows a profit of 30 per cent. over the cost of production, and he sells to the retailer at 20 per cent. off list price. If the latter allows the public 5 per cent. discount off the list price, what percentage of profit does he make on his return? and what does it cost to produce an article for which the public pays 7s. 10d.?

(93) If the diameter of a penny is $1\frac{3}{16}$ in., and 3 pennies weigh an ounce, and a cubic foot of the metal of which the pennies are made weighs 548 lb., find the average thickness of a penny to the nearest thousandth of an inch. ($\pi = \frac{22}{7}$)

(94) Walter Standford & Co , London, purchased from James Heath & Son the following: 5 pcs. 33 in. Flannel (F. 169), 315 yd. at 1s. $2\frac{1}{4}$d.; 4 pcs. 36 in. Flannel (F. 94), 254 yd. at 1s. $3\frac{1}{8}$d.; 5 pcs. 30 in. Flannelette (F.T. 37), 308 yd. at $7\frac{3}{4}$d.; 3 pcs. Calico (C. 147), 298 yd. at $5\frac{1}{2}$d. Make out the invoice, dated 4th June, 19.., deducting 10 per cent. trade discount and $1\frac{1}{2}$ per cent. cash discount.

(95) A dealer offers to sell an article for £10 cash down, or for £2 cash and five payments of £2 in addition, at intervals of one month. Reckoning Simple Interest, calculate the rate charged per annum.

(96) Make out an Account Sales for 80 hf.-cs. Tea *ex* ss. *Clive*, sold by T. Hampton & Son for account of Messrs. Tomlin & Ratcliffe, Colombo.

32	hf.-cs.	net weight	1,978 lb.	@	2s. $1\frac{1}{2}$d.
18	,,	,,	1,112	,,	1s. $11\frac{3}{4}$d.
30	,,	,,	1,854	,,	1s. $10\frac{1}{2}$d.

less $1\frac{1}{2}$ per cent. discount.

Charges: Entries, Dock Charges and Rent, £9 14s. 7d.; Insurance on £500 at 4s. 3d. per cent.; Brokerage, $\frac{1}{2}$ per cent.; Commission, $2\frac{1}{2}$ per cent.

(97) 1 lb. of a certain powder occupies 34 cub. in. What is the least diameter of a cylindrical tin $4\frac{1}{2}$ in. high that would hold $\frac{1}{2}$ lb. of the powder?

(98) A man on 4th May retired a bill for \$5,400 payable at the London branch of a New York bank, the date of maturity being 15th June, by paying £1,124 16s. Given that the rate of exchange was $4{\cdot}77\frac{3}{4}$, calculate to the nearest eighth the rate per cent. at which the rebate had been calculated.

(99) The price of certain goods in France is 49 fc. 63 centimes per kilogram, and the carriage to England costs 4s. 6d. per cwt. Find to the nearest penny the price per lb. at which the goods must be sold in England so as to make a profit of 10 per cent. (1 kg. = 2·204 lb.; £1 = 88·27 fc.)

(100) A man invests some money in a 3 per cent. stock at 96, an equal amount in a $3\frac{1}{2}$ per cent. stock at 105, and a further sum in a 5 per cent. stock at 120. If he gets an all-round $3\frac{3}{4}$ per cent. on all the money invested, compare his holding in the 5 per cent. stock with his holding in the 3 per cent.

(101) A man buys £3,000 of Treasury Bills redeemable at par in 6 months and £2,000 redeemable in 3 months. He purchases them at a rebate calculated at the rate of $5\frac{3}{4}$ per cent. per annum for the time they have to run, and he has to pay $\frac{1}{16}$ per cent. brokerage. What does he pay altogether, and what is the actual rate of interest per annum thereon, calculated in £ *s. d.* to the nearest penny?

(102) The time taken in emptying a canal lock by a sluice is proportional to the square root of the head of water and inversely proportional to the area of the sluice. If, in a certain lock, the time taken is 3 min. 45 sec. with a certain sluice and a head of 18 ft., find the time taken in a similar lock with a sluice the area of which is $12\frac{1}{2}$ per cent. greater, but in which the head is 16 ft., giving the time to the nearest second.

(103) Make out a statement rendered by Wolff & Son, Huddersfield, on 2nd September for the following transactions in July with R. Scott, Blackpool—

Goods delivered: 4th, £47 2s. 9d.; 10th, £37 1s 6d.; 18th, £54 13s 8d.; 26th, £29 14s. 10d.

Goods returned: 20th, £14 3s. 4d.; 28th, £8 11s. 2d.

Cash received on account: 26th, £85 10s.

Allow $1\frac{1}{2}$ per cent. discount on the goods kept by R. Scott.

(104) Construct a graph connecting prices on £ per cwt. with prices in pence per lb., such that each of the latter is 10 per cent. greater than the corresponding former price. Use the graph to find (i) at what price per lb. to the nearest $\frac{1}{4}$d. goods costing £5 14s. per cwt. should be sold so as to gain 10 per cent. on the outlay; and (ii) at what price per cwt. to the nearest shilling, goods should be bought so that by selling at $7\frac{1}{2}$d a lb. a profit of 10 per cent. on outlay should be obtained.

(105) The alloy from which our bronze coinage is minted contains 95 per cent. by weight of pure copper. Out of 38 tons of this bronze, an *equal* number of pennies, halfpennies, and farthings are coined. What sum of money will all these coins represent? what is their number? and what is the value of the pure copper in them if pure copper is now selling at £40 per ton?

(Take 1 oz. as the weight of 3 pennies, or of 5 halfpennies, or of 10 farthings, and 1 ton = 2,240 lb., and 1 lb. = 16 oz.)

(106) A company whose capital was 30,000 $4\frac{1}{2}$ per cent. cumulative preference shares of £10 each and 500,000 ordinary shares of £1 each, was formed in 1928, when the shares were allotted and fully paid. No dividend was paid for two years, then for the next four half-yearly periods the following amounts were respectively available for dividends: £12,500, £15,000, £18,750, £32,400. What were the percentage dividends paid to each class of shareholders each half-year?

(107) A. Townsend was a partner in the following firms, which were dissolved owing to his becoming bankrupt—

Firm.	Kind of Partner.	Capital.	Townsend's Capital.	Loss.	Townsend's Share of Losses.
		£	£	£	
A. & Co	Ordinary	15,000	8,000	20,000	one-half
B & Co.	,,	20,000	15,000	24,500	two-thirds
C & Co.	Limited	10,000	3,000	12,600	one-third
D & Co.	Ordinary	12,000	4,000	8,400	one-third

The net surplus of his private estate was £624, and the other partners of D & Co. were solvent. What should each of the other firms have received ?

(108) A London merchant buys goods in Paris valued at 57,200 fr., for which he gives a 3 months' bill. At the same time he sells goods to a French firm for £[illegible]75, and receives in payment a 6 months' bill. He sells this latter bill to a firm of discounters in London. How much additional English money must he give them so that they may meet the 3 months' bill when due, discount in both countries being calculated at 4 per cent. per annum, and exchange in London and Paris being 88·00 fc.?

(109) A company whose issued capital consists of 90,000 cumulative 6 per cent. preference shares of £5 each fully paid, 500,000 cumulative 6 per cent B preference shares of £1 each fully paid, and 625,000 ordinary shares of £1 each fully paid, distributes dividends in the year amounting to £166,375 Apportion this amount amongst the three classes of shareholders and calculate the rate per cent. which the ordinary shareholders received in the year.

(110) A contractor submits an estimate for a job, which will give him a profit of 20 per cent. on his outlay for wages and material, the cost of materials being twice that for wages. If the cost of materials goes up 20 per cent. and wages go up 10 per cent., by what percentage must he increase his estimate so as to get the same amount of profit as before ?

(111) Thomas, Cook & Co., Bradford, received an indent from Messrs. Telford & Co., Bombay, by which 5 cases marked T.C. 1 to 5, containing altogether 30 pcs., 48″ coloured serges, were to be purchased and shipped to Bombay. The goods were purchased by Thomas, Cook & Co. as follows—

12/52½, 6/52 @ 4s. 10½d. per yard, and 8/52½, 4/52 @ 5s. 2d. per yard less 2½ per cent. discount.

The charges were: Packing, 7s. 9d. per case; carriage to Liverpool, £1 9s. 4d.; Dock Charges and Stamps, 16s. 9d.; Fire Insurance, 4s. 3d.; Freight @ 47s. 3d. and 10 per cent. primage per ton of 40 cub. ft., each case measuring 51″ × 24″ × 16″; Marine Insurance on £380 @ 7s. 2d. per cent. plus 5 per cent. and 6d. stamp; Insurance against War Risks on £380 @ 42s. 6d. per cent.; Commission, 2½ per cent.

Make out a " Loco " Invoice in the proper form.

(112) In the yearly balance sheet of a manufacturing company, machinery, which was originally bought for £5,290, is depreciated 10 per cent. every year at stocktaking. At what value does it stand at the end of 12 years ?

(113) The capital of a company is made up as follows—

Share Capital.	£
5 per cent. Preference Stock	600,000
Ordinary Stock	2,500,000
Debenture Capital.	£
4 per cent. First Debenture Stock	750,000
4½ ,, Second Debenture Stock	700,000

The net profits for 1931 amounted to £204,000. If all this were paid away as dividend, how much of it was available for the Ordinary Stock, and at what rate could dividend on that stock be paid ?

(114) Explain the meaning of—

(i) Promissory Note (ii) Bill of Exchange.

What is meant by discount, and how is it calculated on a bill ?

Find the cost to an English merchant of settling a debt of 29,900 fc. by buying from a bill broker 6 months' bills on Paris at 88·40.

Rate of discount, 3½ per cent.; Brokerage, 1 per mille.; Stamp duty, ·5 per mille.

(115) Referring to Question (111), make out a F.O.B. invoice and a C.I.F. invoice, taking 1 rupee as equivalent to 1s. 4½d.

(116) The strength of a beam is inversely proportional to its length, directly proportional to its width, and directly proportional to the square of its depth. A wooden beam of rectangular cross-section is 5 ft. long, 3 in. wide, 4 in. deep, and is supported at its ends. The breaking load is 650 lb. placed at its centre. Find the breaking load of the same beam
(i) when the supports are each brought 4 in. from the ends;
(ii) when the supports are at the ends and the beam is placed so as to be 4 in. wide and 3 in. deep.

(117)

Stocks.			Prices.	
Canada	3	per cent.	$69\frac{1}{2}$	$70\frac{1}{2}$
Natal	3	,,	$62\frac{3}{4}$	$63\frac{3}{4}$
N.S.W.	4	,,	$83\frac{3}{4}$	$84\frac{3}{4}$
N.S.W.	3	,,	$70\frac{1}{8}$	$71\frac{1}{8}$
Tasmania	$4\frac{1}{4}$	,,	$86\frac{1}{8}$	$87\frac{1}{8}$

Find the change in income due to selling out from the stocks in the above table and re-investing as follows, assuming brokerage $\frac{1}{8}$ per cent. in each case.
(a) £8,400 Canadian 3 per cent. into N.S.W. 4 per cent.
(b) £10,600 Tasmanian $4\frac{1}{4}$ per cent. into Natal 3 per cent.
(c) £6,500 N.S.W. 3 per cent. into Tasmanian $4\frac{1}{4}$ per cent.

(118) A merchant imported 35 bales of wool and insured the same for £750. 20 bales of wool were delivered undamaged and were sold at 1s. $2\frac{1}{2}$d. per lb. From the particulars of the damaged wool given below, calculate the total amount to be paid by the underwriters to the merchant.

	Gross Weight (Undamaged).			Tare and Draft.			Gross Weight (Damaged).			Price.
	CWT.	QR.	LB.	CWT.	QR.	LB.	CWT.	QR.	LB.	
8 bales	25	2	18		1	16	26	1	6	11d. per lb.
7 ,,	22	1	22		1	10	22	3	26	$10\frac{1}{2}$,,

(119) When the numbers of articles manufactured per week were 2,000, 3,000, and 4,000, the costs per article were approximately 18s. 6d., 17s. 6d. and 16s. 8d. respectively. When the selling prices were £1 5s., £1 2s., and 17s., the weekly sales were 1,600, 3,200, and 4,500 respectively. By graphical means, find approximately how many articles should be manufactured per week, the cost price and the selling price, so that a maximum profit will be obtained?

(120) Find the value in English money of a ten rupee piece which contains 75·3344 gr. of fine gold when fine gold is priced at 118s. per oz. tr.

(121) The import duty on wine is inreased from 50 per cent. to 60 per cent. of the cost price. In consequence the retail price is increased from 10d. to 11d. per glass. The retailer originally made 20 per cent. profit on his outlay. What profit per cent. does he now make?

(122) The length of an iron rail is 43·254 . . . ft. on a cold day; what will it be on a day when each foot of the iron has expanded to a length of 1·00324 . . . ft.? Give your answer as accurately as you think the data allow.

(123) £18 18s. 4d. was paid up on each £20 share of a company. A bonus was given so as to discharge the call. The dividend before the bonus was given was 18 per cent., and that after was $17\frac{1}{2}$ per cent. Calculate the amounts received by a person who owned 25 shares, assuming that income tax at 5s. in the £ had been deducted.

(124) A French manufacturer sells an article for 86 fr., which yields him a profit of $7\frac{1}{2}$ per cent. on his outlay. If he could double his sales, the cost of manufacture would be reduced by 5 per cent. At what price now could he afford to sell the article so as to make a profit of $12\frac{1}{2}$ per cent. on his outlay?

(125) Machinery costing £2,065 in London was shipped by W. Todd & Co. London, to Messrs. Leblanc & Son, Lyons, per ss. *Seagull* from London to Marseilles.

The charges were: Packing, £7 5s.; Cartage and Dock Charges, £4 11s. 6d.; Freight on gross weight of 3 tons 7 cwt. 2 qr. 14 lb. at 31s. 6d. per ton weight plus 5 per cent.; Marine Insurance at 3s. 8d. per cent. on £2,400 and 5 per cent.; Insurance against War Risks on £2,400 at 34s. 6d. per cent.; Dock Charges, etc., at Marseilles; 519·35 fcs.; carriage to Lyons and delivery 248·20 fcs.

Assuming W. Todd & Co. wished to gain $7\frac{1}{2}$ per cent. profit on their purchase price of £2,065 and to receive a commission of $2\frac{1}{2}$ per cent. on the total charges, make out a "Franco" invoice in the proper form, taking £1 = 89·76 francs.

(126) If the cheque exchange in London on Paris is 88·065 fc. per £1, and the rate of discount for a 3 months' bill in London is $5\frac{1}{16}$ per cent. per annum, what debt in Paris can be discharged by a person in London who holds a 3 months' bill on London for £3,500?

(127) Find the cost to a London merchant of settling a debt of 47,125 fr by buying from a bill broker 3 months' bills on Paris at 89·05.

Rate of Discount	3 per cent.
Brokerage	1 per mille
Stamp Duty	0·5 per mille

(128) A broker borrowed £2,000 from a bank for a week at $3\frac{1}{4}$ per cent He bought a bill for 22,000 fl. (Dutch) at 10·51 exchange. In the same week he sold the bill at $10{\cdot}49\frac{1}{4}$ exchange, and 6 days after borrowing the money he repaid the loan with interest. What did he gain?

(129) Without using logarithms, calculate the following correct to five significant figures, in each case obtaining the result by the minimum amount of arithmetical work.

(*a*) $$\frac{179{\cdot}43 \times 27{\cdot}24 \times {\cdot}7892}{{\cdot}09076 \times 791{\cdot}4}$$

(*b*) $$\sqrt{317{\cdot}90463 \times 13{\cdot}149287}$$

(130) Calculate the date on which the balance of the following sums of money should be made so as to discharge all debts.

Dr.			*Cr.*	
Aug. 4.	Goods, £150; credit 3 mos.		Aug. 25.	By Cash, £90
Sept. 20.	„ £35 10s.; „ 2 „		Oct. 7.	„ „ £50
Nov. 10.	„ £75; „ 2 „			

Also calculate the balance to be paid on 31st December, reckoning interest at 5 per cent. per annum.

(131) A steam engine employed in pumping water from a coal mine is found to consume an amount of coal proportional to the product of the amount in gallons of the water pumped up, and the depth in feet from which the water is raised. If 180 lb. of coal is burnt in pumping 8,500 gall. of water from a depth of 405 ft., how much coal will be burnt when 9,000 gall. are raised from a depth of 595 ft.?

(132) The capital of a company consists of 420,000 shares of £1 each, and 500 management shares of £10 each. There are also £200,000 of debentures paying 5 per cent. The management shares are entitled to half the surplus profits after paying the interest on the debentures and 7 per cent. on the ordinary shares. One year the dividend on the ordinary shares was $12\frac{1}{2}$ per cent. In the following year 210,000 more ordinary shares were issued and taken up, and the dividend that could be paid on the ordinary shares was $10\frac{1}{2}$ per cent. for that year. Find whether the dividend on each management share was greater or less than in the year before, and by what amount. Find also by what amount the profits increased.

(133) Make out an Account Current to 31st December for the following

transactions between Brent & Co., London, and Dodd & Co., Singapore, debiting and crediting interest (including "red" interest) at 5 per cent. per annum.

			£	s.	d.
Dr.					
July	15.	To Invoice per ss. *Swallow*	405	17	6
Aug.	30.	,, ,, ,, *Brenda*. . . .	750	10	-
Oct.	12.	,, 3 m/s Draft on us due 20th Dec. .	500	–	–
Dec.	15.	,, Invoice per ss. *Perseus*	350	–	–
Cr.			£	s.	d.
Aug.	2.	By A/S Fruit *ex* ss. *Bombast*	715	–	–
Oct.	5.	,, A/S ,, ,, *Nelson*	546	–	–
Dec.	7.	,, remittance 3 m/s due 30th Jan. . .	250	–	–

(134) If the rate of exchange in London on New York be 3 59$\frac{1}{4}$ for 60 days' bills and 3·54$\frac{15}{16}$ for cheques, what is the equivalent rate of discount ?

(135) A retailer gets 25 per cent. trade discount off the list prices of a wholesaler and, in addition, is allowed a cash discount of 5 per cent. He wished to price his articles such that, after allowing his customers a discount of 5 per cent., he will make a profit of 10 per cent. on his outlay. By how much per cent. are the prices he marks his goods less than the prices in the wholesaler's list. Construct a graph connecting these prices, and thus find the price he should mark an article priced at 35 guineas in the wholesaler's list

(136) A man invests £7,325, partly in a 3$\frac{1}{2}$ per cent. stock at 95, and partly in a 4$\frac{3}{4}$ per cent. stock at 108. He gets an all-round 4 per cent. on the money invested. How much does he invest in each stock ?

(137) Given that 1 metre = 39·37 in., find what rate of exchange would make a price of 1s. per yard equivalent to a price of 45 fr. for 10 metres. (Answer in francs per £ to the nearest centime.)

(138) A merchant buys goods for £127 16s. 3d. payable six months hence; he immediately sells the goods for £152 5s., which he is to receive four months hence. If the rate of discount is taken at 4$\frac{1}{2}$ per cent. per annum, what cash profit does this sale yield ?

(139) A wholesaler purchased goods from a manufacturer as follows:
4th April, £248, credit, 2 months; 18th April, £315; credit, 3 months; 2nd May, £420, credit, 2 months.
On what day should £983 be paid in discharge of the debts ? Also reckoning discount at 5 per cent. per annum, find what should be paid on 4th May to completely discharge the debts.

(140) A dealer sold 4 motor-cars for £257 10s., £315, £450, and £487 10s respectively, thereby gaining 8 per cent. on the total selling price. He lost 5 per cent. on the selling price by the sale of the first, but gained 4 per cent and 10 per cent. respectively by the sale of the second and fourth. What percentage gain was made by the sale of the third motor-car ? and what was the percentage gain if based on cost price ?

(141) On 31st December a deposit account at a bank was credited with £61 7s. 4d., being 5 per cent. per annum for the half-year. On 30th June the deposit stood at £1,826, and was subsequently increased to £2,826. There is an uncertainty as to the date at which the £1,000 was added. What was the date ?

(142) If 1,056 doll. earn 15 doll. 84 cents in four months, at the same rate of interest how many francs and centimes will earn 321 fr. 69 centimes in seven months ? and in what time will £276 earn £2 1s. 5d. ? Also what is the rate per cent. per annum in all these cases ?

(143) The diameter of a sphere is 4·34 in. correct to two decimal places. Calculate, to as many significant figures as the data permit, the volume of the sphere.

(144) P and Q are the only partners in a firm. P.'s capital is £5,000 and Q's £4,000: they are each entitled to £25 per month, this amount being

reckoned as part of the working expenses of the firm; they are also entitled to interest at 5 per cent. per annum payable on 31st December, and 5 per cent. is allowed or charged on balances in their respective drawing accounts. On 1st January, P and Q had credit balances of £544 and £280 respectively: on 5th May P withdrew £500 and on 14th July Q withdrew £600. They are entitled to share net profits equally, which during the year under consideration amounted to £2,260. Calculate the balances in their Drawings Account on 1st January of the next year, and find the total income of each during the complete year.

(145) How much per cent. must the list price of an article be greater than the cost of manufacture, such that by selling at a trade discount of 12½ per cent. and allowing a cash discount of 2½ per cent. the manufacturer would make a profit of 6 per cent. on the cost of manufacturing the articles?

(146) The capital of a small manufacturing company consists of 1,050 shares of £10 each. It has also issued 6 per cent. debentures of £100 each, but two of them have to be paid off at the end of every year, out of the profits of that year. At the beginning of last year there were still 32 of these debentures, the business done during the year was £5,680, materials used in the manufacture cost 51 per cent., and other expenses were 28 per cent. of the business done. Find the greatest dividend per share that could be paid at the end of the year without drawing on reserve.

(147) A bankrupt's debts amount to £12,560, and the dividend which can be paid to his creditors is reduced from 7s. 6d. to 6s. 11d. in the £1, by the admission of a claim to a preferable ranking to be paid in full. Find the amount of such claim.

(148) Make out an Account Current to 30th June for the following trans actions between Messrs. S. Ross & Co., Bristol, and N. Angard, Montreal. Reckon interest at 6 per cent. per annum.

Dr.			*Cr.*		
Jan. 1.	To Balance . .	$475·74	Feb. 4.	By A/S due 10 Apr.	$415·00
Mar. 6.	,, £312 10s. 90 d/s *ex* 3·84⅛ due 10th May		Apr. 15.	,, ,, 7 June	$950·00
June 10.	,, Freight and Insurance,	$315·00			

(149) At each stroke of an air-pump, one-twentieth of the air in the receiver is removed. Find what percentage of the air is left after 20 strokes, and how many strokes must be made to remove nine-tenths of the air.

(150) The duty on an article was reduced from 6d. to 4d., but the consumption of that commodity increased by 20 per cent. What is the decrease per cent. in the revenue derived from this tax. What increase per cent. in the consumption would have left the revenue unchanged?

(151) A stone pillar has the shape of a frustum of a cone. Its height is 60 ft., the diameter of the base is 5′ 6″, and that of the upper end 4′. Reckoning that 1 cub. ft. of the stone weighs 285 lb., calculate the weight of the pillar.

(152) A gold mining company crushed in its mills 220,610 tons of quartz and treated 289,733 tons of tailings and slimes in its cyanide works. From the mills were obtained bar gold equal to 91542·11 oz. of fine gold, and from the cyanide works bar gold equal to 26844·74 oz. of fine gold, the total fine gold realizing £501,468 18s. 7d. Find the yield in weight of fine gold per ton of ore milled, also the yield per ton from the cyanide plant and the money value of 1 oz. of fine gold. (24 gr. = 1 dwt., 20 dwt. = 1 oz.)

(153) A and B go into business in partnership, A contributing a capital of £6,000 and B £9,000. It is arranged that, after 5 per cent. interest on capital has been paid out of the year's profits, A as managing partner is to receive, should the profits so admit, an extra 1 per cent. on his capital and a yearly salary of £200. Any remaining profits are then divided in proportion to the capitals put into the business. One year A receives altogether £806. What does B get?

(154) A cargo is valued at £13,716, and it is insured at £4 4s. per £100; the policy 3s. 6d. per cent, and the commission £$\frac{3}{8}$ per cent. For what sum must the cargo be insured so that the value of the cargo and the expense of insuring may be recovered in case of loss?

(155) At the birth of his son, a parent invests £100 to provide for him a three years' scholarship at a University, starting on his eighteenth birthday. What will be the yearly value of the scholarship if Compound Interest is reckoned at 4 per cent?

(156) A speculator paid 1 per cent. of the nominal value for a "put" option for a month on £12,500 stock at 123$\frac{7}{8}$. After 3 weeks he bought at 121$\frac{1}{4}$, and at the end of the month he exercised his "put." What profit did he make?

(157) The weight of a piece of gold is 315·37 . . . gr. Given that 15·43232 . . . gr. = 1 gm., find to as many significant figures as the data permit, the weight of the gold in grammes. Also find the value of the piece of gold if it be of 9 carat and the value of 1 oz. tr. of fine gold is 120s.

(158) The annual rental of a house is £65, and local rates amounting to 6s. 8d. in the £1 are levied on its rateable value. The rateable value is obtained by deducting 15 per cent. from the annual rental. Find the percentage which the rates are of the annual rent; and if the tenant of such a house wishes to reduce his expenditure by £15 a year in rent and taxes combined, what would be the rental to the nearest 10s. of the house that he could afford to rent?

(159) In estimating the value of plant and machinery for the annual balance sheet of an engineering firm, 4$\frac{3}{4}$ per cent. is written off for depreciation on the valuation at the beginning of the year. What amount should be quoted as the value of plant and machinery purchased 19 years previously for £10,000?

(160) A company has an issued capital of 356,000 ordinary shares of £1 each, 500,000 5 per cent. cumulative preference shares of £1 each, 400,000 6 per cent. preferred ordinary shares of £1 each, all fully paid. In addition to this, it has £300,000 1st Mortgage Debenture Stock 4 per cent., and £82,000 2nd Mortgage Debenture Stock 4$\frac{1}{2}$ per cent. The entire sum distributed in the year in dividends on shares and interests on debentures amounts to £215,990. How much of this did the ordinary shareholders receive and what rate per cent. did it represent on their capital? What rate per cent. does £215,990 represent on the combined total of issued capital and Debenture Stock?

(161) An investor puts $\frac{1}{4}$ of his capital in an undertaking yielding 10 per cent. per annum, $\frac{1}{2}$ of his capital in debentures yielding 4 per cent. per annum, and the remainder in preference shares yielding 6 per cent. per annum. His entire income from these investments is £760; what is the average rate of interest that he is receiving on his capital, and what is the amount of this capital?

(162) The nominal capital of a company consists of 40,000 4$\frac{1}{2}$ per cent. preference shares of £5 each, 4,000 ordinary shares of £100 each, and 200 management shares of £1 each. The whole of the preference and management shares have been allotted and paid up. All the ordinary shares have been allotted, but only £65 has been paid up on each share. Moreover, £80,000 of 4 per cent. debentures have been created and taken up. After payment of the fixed interest on debentures and preference shares, nine-tenths of the remaining available profits are distributed as dividend on the ordinary shares, and one-tenth goes to the management shares. If each management share receives a dividend of £13, find the total profit earned and the rate of interest paid on the ordinary shares.

(163) A "call" option was secured for £103 2s. 6d. on £8,250 stock at 159$\frac{1}{4}$. What was the price of the stock when the speculator used his option if his profit on the transaction was £175 6s. 3d.?

(164) The subscribed capital of a bank was 229,341 shares of £100 each, only £15 10s. being paid up on each share. A *half-yearly* dividend at the rate of 10 per cent. per annum on the paid-up capital, less the income tax deducted, amounted to £149,071 13s. What was the average rate of income tax per £1 deducted?

(165) A certain 4 per cent. stock has been issued at the price 97, payable as follows—

£5 on application on 5th December, 19. . ; £22 on allotment on 12th Dec.; £35 on 15th January; £35 on 15th February. Investors will receive a full half-year's dividend on 1st April next.

Regarding the investment as made on 1st April, and allowing for the dividend then received, and also allowing for the interest at 4 per cent. on the various payments that have been made, how much should an investor reckon that he has paid for each £100 stock?

(166) A person borrowed £22,000 for two months at 5 per cent. per annum. At the end of the time the interest was added on, and the debt renewed for another two months. This was continually repeated, till at the end of two years the debt and interest were paid. Find what the debt and interest amounted to.

(167) A tradesman spends £1,200 in buying cloth in England at 3s. 9d. per yard, takes it to France at a cost of 2d. per yard, and pays duty on it at 1·28 fc. per metre. Half of the cloth is damaged, and he sells this at a loss of 20 per cent. on its gross cost to him. The remainder he sells at 28·80 fc per metre. Find his gain. (88·00 fc. = £1, 1 metre = 39⅜ in.)

(168) An industrial company has an issued capital of 360,000 5 per cent. cumulative preference shares of £1 each, fully paid; and 360,000 ordinary shares of £1 each, also fully paid. The dividends on the preference shares are paid quarterly and on the ordinary shares an interim dividend of 6d. per share was paid at the end of the half-year, and a final dividend of 9d. per share at the end of the year. After paying these dividends and placing to General Reserve £15,000, to Capital Reserve £2,000, and to Income Tax Reserve £750, a balance of £11,380 7s. 11d. is carried forward to the next year's account. What must the net profits for the year's trading have been if a balance of £5,357 0s. 2d. was brought forward from the previous year? Give also the rate per cent. per annum received by the ordinary shareholder.

(169) What is the value, to the nearest pound, of a reversion to £3,250 on the death of a lady who is 70 years of age?

[Compound interest at 3 per cent. Expectation of life, at 70, for a female is 9 years.]

(170) Calculate the value of the immediate annuity that may be purchased by the payment of £1,000 at the age of 55, the purchaser being male. (Allow interest at 2½ per cent. per annum; expectation of life at 55 years for a male is 18 years.)

(171) Find the number of years and fraction of a year that a sum of money will treble itself at Compound Interest at 4 per cent. per annum.

(172) An investor buys 100 ordinary £1 shares, fully paid up, in an industrial company, for each of which he paid 54s. He holds these shares for a year, receiving two half-yearly dividends of 1s. and 3s. respectively per share. In addition to these dividends, he receives a bonus of 1 ordinary share for every 2 shares which he held, this additional capital being met by a withdrawal of an equivalent sum from a large reserve fund. If he now sells out his entire holding at 40s. a share, what gain has he made in the year, and what rate per cent. does it amount to?

(173) A shareholder holds 100 shares in a gold mining company, which pays three dividends during the year of 4d., 5d., and 8d. respectively per share. When the income tax is deducted from these dividends, the shareholder receives respectively £1 10s. 2d., £1 17s. 3d., and £2 17s. 7d. in net dividends. Find the three rates of income tax that have been deducted.

If the shares are 2s. 6d. shares, what is the rate per cent. per annum that the company is paying before any income tax is deducted?

(174) Calculate the interest on £1,500 from 10th May, 1917, to 1st February, 1918, interest at 5 per cent. per annum being paid on 30th June and 31st December.

(175) A Home Corporation Stock paying 3½ per cent. per annum could be bought recently at 84½ per cent. At this price, how much per cent. per annum would an investment in this stock yield? If this stock is redeemable at par in 14 years' time, and an investor now held on until it was so repaid, what percentage per annum roughly will it have yielded him during that time?

(176) If a person borrows £1,100 at 5 per cent. per annum Compound Interest, and agrees to repay principal and interest in eleven equal annual instalments, how much must each instalment be, the first being paid at the end of the first year?

(177) The value of a lease having 45 years to run was £1,750. Reckoning interest at 6 per cent. per annum, calculate the value of the lease 15 years later.

(178) Using the tables in Paragraphs 170 and 171, draw a graph giving the present value of an annuity of 1, interest at 5 per cent. Use the graph to find (i) the present value of an annuity of 1 for 28 years; (ii) the approximate value of a life-annuity of 1 in the case of a female child just 1 year old.

(179) A person having £95,000 stock of the 4½ per cent. War Loan, 1925–1945, dividends payable half-yearly on 1st June and 1st December, converted his holding as on 16th February, 1917, into 5 per cent. War Loan, 1929–1947, receiving in lieu £100 of the 5 per cent. War Loan for each £95 of the 4½ per cent. War Loan converted. How much of the 5 per cent. War Loan stock did he receive? and what was his first dividend on 1st June, 1917, this dividend consisting of interest at 4½ per cent. per annum, accrued from 1st December, 1916, to 16th February, 1917, upon the old holding, together with the interest at 5 per cent. per annum from the 16th February, 1917, to 1st June, 1917, upon the new holding? What will the subsequent half-yearly dividends be? (*N.B.*—No income tax reckoned.)

(180) A man invests £12 for his son every year, beginning on the day of the boy's birth. If the money has increased at 3 per cent. Compound Interest, how much will the son receive, to the nearest £, on his twenty-first birthday?

(181) What sum of money, paid now to an insurance company, should provide for an annuity of £100 per annum for 10 years, the first payment beginning 21 years hence. Reckon Compound Interest at 3½ per cent. per annum.

(182) How must a sum of £6,505 be divided, so that one portion put out at Compound Interest at 4 per cent. per annum for two years may amount to the same sum as the other portion put out at Compound Interest for four years at the same rate will amount to?

(183) An investor has allotted to him on 8th July, 1928, 1,000 cumulative 7 per cent. preference shares of £1 each issued by a public company, the terms of subscription being 2s. per share on application; 2s. per share on allotment; 5s. per share on 1st January, 1929; 5s. per share on 1st July, 1929; and the remaining 6s. per share on 1st January, 1930. He is to receive a fixed cumulative preferential dividend of 5 per cent. per annum on the capital paid up thereon, calculated from the dates fixed for the payment of instalments; and interest at the rate of 4 per cent. per annum is to be allowed on instalments paid in advance, from the date of payment until the due dates respectively, 7 per cent. dividend not beginning until 1st October, 1930. If he pays up in full on allotment, and these preferential dividends are paid half-yearly on 1st June and 1st December respectively

in each year, how much in *all* does he thus receive up to and including 1st December, 1930?

(184) An insurance company charges a certain premium to a male client who begins insuring at 33 years of age, and the calculation employed in fixing the premium is based on money earning interest at $2\frac{1}{2}$ per cent. per annum. What is the price of the premium (disregarding expenses of management, etc.) on a policy of £750?

[Expectation of life at 33 for a male is 30 years.]

(185) A building society advertises a house for sale for £350 cash down, or for £10 down and 10s. 6d. per week. How many such weekly payments ought there to be? Reckon 52 weeks to the year and Compound Interest at 3 per cent. per annum.

(186) Find the accumulated value at the end of 15 years of an unpaid annuity of £100 payable in quarterly instalments. Allow compound interest at 5 per cent. per annum, payable yearly.

(187) How many years would it take to pay off, by yearly instalments of £2,000, a debt of £50,000, if we reckon compound interest at $3\frac{1}{2}$ per cent.?

(188) A government contracts a loan of £25,000,000 at 4 per cent. per annum. What should be the annual sinking fund in order that the debt may be extinguished in 20 years?

(189) If money is worth 4 per cent. per annum, show that the present value of a deferred annuity of £100 to commence 12 years hence (*i.e.*, the first payment to be made at the end of 13 years), and to continue for 20 years, is about £848 17s.

(190) A loan of £200 to a merchant is to be repaid by half-yearly instalments of £40, the first to begin six months after the advance is made. If interest is calculated at 5 per cent. per annum on the respective outstanding balances of the loan, what is his indebtedness to the nearest penny after having paid the fifth instalment?

(191) A man buys a house from a building society, paying £10 down and £20 at the end of each year until the house is paid for. If the last payment is at the end of 23 years, what ought he to have paid for the house if he had bought it cash down? Reckon 3 per cent. compound interest.

(192) A vessel owing to damage was assisted by lighters, and some of the cargo was thrown overboard in order to save the ship. Assuming that the ship, freight, and cargo were fully insured, calculate from the following particulars what the respective underwriters would be called upon to pay.

Contributory Values for General Average.	£	General Average Statement.	£
Vessel (less cost of repair)	7,450	Shipowner's losses (including freight at risk and charges)	2,440
Freight	920	X's goods lost	750
X's goods	2,140	X's „ damaged (sound value)	620
Y's „	270	Y's „ damaged (sound value)	560
Z's „	720		
Other owners' goods	3,500		
Total of contributory values	15,000	Total of losses	4,370

Also calculate the amounts due to the respective underwriters of X and Y in respect of the sale of the damaged goods if they were sold respectively at 45 per cent. and 55 per cent. of the sound value.

(193) A bill broker bought a bill for 150,000 fcs. at 82·35 exchange and sold it almost immediately afterwards at 81·99 exchange. What did he gain? Also calculate at what exchange he should sell the bill so as to gain $\frac{1}{2}$ per cent. on his outlay.

(194) (*a*) Find the effective rate which corresponds to a nominal rate of per cent., convertible quarterly.

(*b*) Find the nominal rate, convertible quarterly, which corresponds to an effective rate of 5 per cent.

(195) The weight of a statue 7′ 3″ high is 1 ton 3 cwt. 12 lb. If a reproduction be made of the same material such that the height is 5′ 6″, what would the weight be? Also if the cost to gild the former was £17 14s., what should be the cost to gild the latter? By the aid of logarithms, calculate the height of a reproduction made with the same material such that the weight should be 10 lb.

(196) Two companies A and B amalgamated. A's capital consisted of 7,500 shares of £10 each, on which £8 5s. was paid, and B's of 100,000 shares of £1 each on which 16s. 6d. was paid. A's assets were £65,450 and £2,450 was owing to sundry creditors, while B's assets were £42,560 and £3,860 was owing to sundry creditors. No fresh capital was issued or called up, but a re-arrangement was effected whereby shares were distributed between the two groups of shareholders in proportion to the net assets. If the new company declared a dividend of 6 per cent., calculate what should be the amount due (i) to a shareholder in A who previously held 50 shares; and (ii) to a shareholder in B who previously held 500 shares; also calculate the percentage yield on paid-up capital in each case.

(197) The rate of exchange between London and Paris was 27·18, between Paris and Switzerland ·78 Swiss franc per French franc, between Switzerland and Rome 1·77 lire per franc. Calculate the arbitrated rate of exchange between London and Rome.

(197) What sum should be set aside at the end of each year by a man 35 years of age so that on attaining the age of 60 he could purchase a life-annuity of £150 per annum. Reckon interest at 4 per cent. per annum as regards the savings and 3 per cent. per annum in calculating the present value of the annuity.

(198) Machinery costing £20,000 will last 18 years and produce each year a net revenue of £9,000. A better quality machinery costing £28,000 will last 24 years and produce each year a net revenue of £10,000. Find which is the more profitable machinery to purchase by calculating the ratio of the present value of revenue to the cost of machinery in each case, reckoning interest at 5 per cent. per annum.

(199) Two sums of money are put out at interest: the first, £655, at 2½ per cent. per annum; and the second, £650, at 4 per cent. per annum. In what time will they amount to the same sum of money?

(200) It is estimated that machinery costing £35,000 will last for 15 years. If an appliance be used which, although not affecting the annual amount of work done, will enable the machinery to last an additional 5 years, calculate the value of the appliance to the owner of the machinery, reckoning interest at 5 per cent. per annum.

(201) Find the amount of 6 payments of £20, one at the end of each month, after 6 months, reckoning simple interest at 3 per cent. per annum. Hence find the present value of the money to be received by a man earning a salary of £240 per annum and paid by monthly instalments for the next 10 years, reckoning interest at 3 per cent. per annum paid half-yearly.

(202) A man of 50 years of age, having no dependents, wishes to purchase by a lump sum the right to £1,000 at the end of 15 years should he still be living, but should he die before the end of 15 years no repayment is to be made. Reckoning interest at 3½ per cent. per annum, and given that out of 530,888 men of 50 years of age, 332,344 live to the age of 65, calculate the approximate sum the man should pay to the insurance company.

(203) A company purchased a factory for £45,000. The sum of £2,000 was set aside at the end of each year to provide a sinking fund. Reckoning

interest at 4 per cent. per annum, find the time that must elapse before the sum of £45,000 is replaced.

(204) A sinking fund was formed by setting aside £1,000 at the end of the first year, and at the end of each following year money was put by such that the sum set aside at the end of any year was 15 per cent. greater than that of the previous year. Find the total amount at the end of 15 years, reckoning interest at 5 per cent. per annum.

(205) A man left £1,000 to be divided between his two sons. The elder was to receive a certain sum of money in 5 years' time and the younger was to receive an equal sum in 8 years' time. Reckoning interest at 4 per cent. per annum, calculate the sum of money each should have received.

[*N.B.*—Let £x ≡ sum each should have received,

$$\text{then } \frac{x}{(1{\cdot}04)^5} + \frac{x}{(1{\cdot}04)^8} = 1{,}000]$$

APPENDIX

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	2	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9849	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5

ANTILOGARITHMS.

·	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TABLE OF SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND RECIPROCALS.

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
1	1	1	1	1	1	26	676	17576	5·099	2·962	·03846
2	4	8	1·414	1·260	·50000	27	729	19683	5·196	3·000	·03704
3	9	27	1·732	1·442	·33333	28	784	21952	5·291	3·037	·03571
4	16	64	2·000	1·587	·25000	29	841	24389	5·385	3·072	·03448
5	25	125	2·236	1·710	·20000	30	900	27000	5·477	3·107	·03333
6	36	216	2·449	1·817	·16667	31	961	29791	5·568	3·141	·03226
7	49	343	2·646	1·913	·14286	32	1024	32768	5·657	3·175	·03125
8	64	512	2·828	2·000	·12500	33	1089	35937	5·745	3·208	·03030
9	81	729	3·000	2·080	·11111	34	1156	39304	5·831	3·240	·02941
10	100	1000	3·162	2·154	·10000	35	1225	42875	5·916	3·271	·02857
11	121	1331	3·317	2·224	·09091	36	1296	46656	6·000	3·302	·02778
12	144	1728	3·464	2·289	·08333	37	1369	50653	6·083	3·332	·02703
13	169	2197	3·606	2·351	·07692	38	1444	54872	6·164	3·362	·02632
14	196	2744	3·742	2·410	·07143	39	1521	59319	6·245	3·391	·02564
15	225	3375	3·873	2·466	·06667	40	1600	64000	6·325	3·420	·02500
16	256	4096	4·000	2·520	·06250	41	1681	68921	6·403	3·448	·02439
17	289	4913	4·123	2·571	·05882	42	1764	74088	6·481	3·476	·02381
18	324	5832	4·243	2·621	·05556	43	1849	79507	6·557	3·503	·02326
19	361	6859	4·359	2·668	·05263	44	1936	85184	6·633	3·530	·02273
20	400	8000	4·472	2·714	·05000	45	2025	91125	6·708	3·557	·02222
21	441	9261	4·583	2·759	·04762	46	2116	97336	6·782	3·583	·02174
22	484	10648	4·690	2·802	·04545	47	2209	103823	6·856	3·609	·02128
23	529	12167	4·796	2·844	·04348	48	2304	110592	6·928	3·634	·02083
24	576	13824	4·899	2·884	·04167	49	2401	117649	7·000	3·659	·02041
25	625	15625	5·000	2·924	·04000	50	2500	125000	7·071	3·684	·02000

TABLE OF SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND RECIPROCALS.

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
51	2601	132651	7·141	3·708	·01961	**76**	5776	438976	8·718	4·236	·01316
52	2704	140608	7·211	3·733	·01923	**77**	5929	456533	8·775	4·254	·01299
53	2809	148877	7·280	3·756	·01887	**78**	6084	474552	8·832	4·273	·01282
54	2916	157464	7·348	3·780	·01852	**79**	6241	493039	8·888	4·291	·01266
55	3025	166375	7·416	3·803	·01818	**80**	6400	512000	8·944	4·309	·01250
56	3136	175616	7·483	3·826	·01786	**81**	6561	531441	9·000	4·327	·01235
57	3249	185193	7·550	3·849	·01754	**82**	6724	551368	9·055	4·344	·01220
58	3364	195112	7·616	3·871	·01724	**83**	6889	571787	9·110	4·362	·01205
59	3481	205379	7·681	3·893	·01695	**84**	7056	592704	9·165	4·380	·01191
60	3600	216000	7·746	3·915	·01667	**85**	7225	614125	9·220	4·397	·01177
61	3721	226981	7·810	3·936	·01639	**86**	7396	636056	9·274	4·414	·01163
62	3844	238328	7·874	3·958	·01613	**87**	7569	658503	9·327	4·431	·01149
63	3969	250047	7·937	3·979	·01587	**88**	7744	681472	9·381	4·448	·01136
64	4096	262144	8·000	4·000	·01563	**89**	7921	704969	9·434	4·465	·01124
65	4225	274625	8·062	4·021	·01538	**90**	8100	729000	9·487	4·481	·01111
66	4356	287496	8·124	4·041	·01515	**91**	8281	753571	9·539	4·498	·01099
67	4489	300763	8·185	4·062	·01493	**92**	8464	778688	9·592	4·514	·01087
68	4624	314432	8·246	4·082	·01471	**93**	8649	804357	9·644	4·531	·01075
69	4761	328509	8·307	4·102	·01449	**94**	8836	830584	9·695	4·547	·01064
70	4900	343000	8·367	4·121	·01429	**95**	9025	857375	9·747	4·563	·01053
71	5041	357911	8·426	4·141	·01408	**96**	9216	884736	9·798	4·579	·01042
72	5184	373248	8·485	4·160	·01389	**97**	9409	912673	9·849	4·595	·01031
73	5329	389017	8·544	4·179	·01370	**98**	9604	941192	9·899	4·610	·01020
74	5476	405224	8·602	4·198	·01351	**99**	9801	970299	9·950	4·626	·01010
75	5625	421875	8·660	4·217	·01333	**100**	10000	1000000	10·000	4·642	·01000

TABLE

SHOWING THE NUMBER OF DAYS IN ANY GIVEN PERIOD OF THE YEAR

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	32	60	91	121	152	182	213	244	274	305	335
2	33	61	92	122	153	183	214	245	275	306	336
3	34	62	93	123	154	184	215	246	276	307	337
4	35	63	94	124	155	185	216	247	277	308	338
5	36	64	95	125	156	186	217	248	278	309	339
6	37	65	96	126	157	187	218	249	279	310	340
7	38	66	97	127	158	188	219	250	280	311	341
8	39	67	98	128	159	189	220	251	281	312	342
9	40	68	99	129	160	190	221	252	282	313	343
10	41	69	100	130	161	191	222	253	283	314	344
11	42	70	101	131	162	192	223	254	284	315	345
12	43	71	102	132	163	193	224	255	285	316	346
13	44	72	103	133	164	194	225	256	286	317	347
14	45	73	104	134	165	195	226	257	287	318	348
15	46	74	105	135	166	196	227	258	288	319	349
16	47	75	106	136	167	197	228	259	289	320	350
17	48	76	107	137	168	198	229	260	290	321	351
18	49	77	108	138	169	199	230	261	291	322	352
19	50	78	109	139	170	200	231	262	292	323	353
20	51	79	110	140	171	201	232	263	293	324	354
21	52	80	111	141	172	202	233	264	294	325	355
22	53	81	112	142	173	203	234	265	295	326	356
23	54	82	113	143	174	204	235	266	296	327	357
24	55	83	114	144	175	205	236	267	297	328	358
25	56	84	115	145	176	206	237	268	298	329	359
26	57	85	116	146	177	207	238	269	299	330	360
27	58	86	117	147	178	208	239	270	300	331	361
28	59	87	118	148	179	209	240	271	301	332	362
29	—	88	119	149	180	210	241	272	302	333	363
30	—	89	120	150	181	211	242	273	303	334	364
31	—	90	—	151	—	212	243	—	304	—	365

To illustrate the use of the table, let us suppose that it is required to ascertain the number of days from 28th March to 30th June. We look for 30 in the first column, and on the same line, under *June*, we find the number 181. We next look for 28 in the first column, and on the same line, under *March*, we find the number 87. Subtracting from the other number, we get the answer, namely 94 days. In the case of leap year, one day more must be added for February.

COMPOUND INTEREST TABLE

(showing the sum to which (a) £1, (b) £1 yearly, will amount at Compound Interest).

No. of Years.	Rates per Cent.					
	2·5	3	3·5	4	4·5	5
1 (a)	1·02500	1·03000	1·03500	1·04000	1·04500	1·05000
(b)	1·	1·	1·	1·	1·	1·
2 (a)	1·05063	1·06090	1·07123	1·08160	1·09203	1·10250
(b)	2·025	2·03	2·035	2·04	2·045	2·05
3 (a)	1·07689	1·09273	1·10872	1·12486	1·14117	1·15763
(b)	3·07563	3·0909	3·10623	3·1216	3·13703	3·1525
4 (a)	1·10381	1·12551	1·14752	1·16986	1·19252	1·21551
(b)	4·15252	4·18363	4·21494	4·24646	4·27819	4·31013
5 (a)	1·13141	1·15927	1·18769	1·21665	1·24618	1·27628
(b)	5·25633	5·30914	5·36247	5·41632	5·47071	5·52563
6 (a)	1·15969	1·19405	1·22926	1·26532	1·30226	1·34010
(b)	6·38774	6·46841	6·55015	6·63298	6·71689	6·80191
7 (a)	1·18869	1·22987	1·27228	1·31593	1·36086	1·40710
(b)	7·54743	7·66246	7·77941	7·89829	8·01915	8·14201
8 (a)	1·21840	1·26677	1·31681	1·36857	1·42210	1·47746
(b)	8·73612	8·89234	9·05169	9·21423	9·38001	9·54911
9 (a)	1·24886	1·30477	1·36290	1·42331	1·48610	1·55133
(b)	9·95452	10·1591	10·3685	10·5828	10·8021	11·0266
10 (a)	1·28008	1·34392	1·41060	1·48024	1·55297	1·62889
(b)	11·2034	11·4639	11·7314	12·0061	12·2882	12·5779
15 (a)	1·44830	1·55797	1·67535	1·80094	1·93528	2·07893
(b)	17·9319	18·5989	19·2957	20·0236	20·7841	21·5786
20 (a)	1·63862	1·80611	1·98979	2·19112	2·41171	2·65330
(b)	25·5447	26·8704	28·2797	29·7781	31·3714	33·0660
25 (a)	1·85394	2·09378	2·36324	2·66584	3·00543	3·38635
(b)	34·1578	36·4593	38·9499	41·6459	44·5652	47·7271
30 (a)	2·09757	2·42726	2 80679	3·24340	3·74532	4·32194
(b)	43·9027	47·5754	51·6227	56·0849	61·0071	66·4388
35 (a)	2·37321	2·81386	3·33359	3·94609	4·66735	5·51602
(b)	54·9282	60·4621	66·6740	73·6522	81·4966	90·3203
40 (a)	2·68506	3·26204	3·95926	4·80102	5·81636	7·03999
(b)	67·4026	75·4013	84·5503	95·0255	107·030	120·800
45 (a)	3·03790	3·78160	4·70236	5·84118	7·24825	8·98501
(b)	81·5161	92·7199	105·782	121·029	138·850	159·700
50 (a)	3·43711	4·38391	5·58493	7·10668	9·03264	11·46740
(b)	97·4843	112·797	130·998	152·667	178·503	209·348

log. of amount of £1 in 1 year.	0·0107238654 at 2·5 %.	0·0128372247 at 3 %.	0·01494703498 at 3·5 %
	0·0170333393 at 4 %.	0·0191162904 at 4·5 %.	0·0211892991 at 5 %.

PRESENT VALUE TABLE

(showing the present value of future payments of (a) £1, (b) £1 yearly).

Time in Years.		Rates per Cent. 2·5	3	3·5	4	4·5	5
1	(a)	0·97561	0·97087	0·96618	0·96154	0·95694	0·95238
	(b)	0·97561	0·97087	0·96618	0·96154	0·95694	0·95238
2	(a)	0·95181	0·94260	0·93351	0·92456	0·91573	0·90703
	(b)	1·92742	1·91348	1·89969	1·88609	1·87267	1·85941
3	(a)	0·92860	0·91514	0·90194	0·88900	0·87630	0·86384
	(b)	2·85602	2·82862	2·80164	2·77509	2·74897	2·72325
4	(a)	0·90595	0·88849	0·87144	0·85480	0·83856	0·82270
	(b)	3·76198	3·71711	3·67308	3·62990	3·58753	3·54595
5	(a)	0·88385	0·86261	0·84197	0·82193	0·80245	0·78353
	(b)	4·64583	4·57971	4·51505	4·45182	4·38998	4·32948
6	(a)	0·86230	0·83748	0·81350	0·79031	0·76790	0·74622
	(b)	5·50813	5·41720	5·32855	5·24214	5·15788	5·07569
7	(a)	0·84127	0·81309	0·78599	0·75992	0·73483	0·71068
	(b)	6·34939	6·23029	6·11454	6·00205	5·89271	5·78637
8	(a)	0·82075	0·78941	0·75941	0·73069	0·70319	0·67684
	(b)	7·17014	7·01969	6·87396	6·73274	6·59589	6·46321
9	(a)	0·80073	0·76642	0·73373	0·70259	0·67290	0·64461
	(b)	7·97086	7·78611	7·60769	7·43533	7·26879	7·10782
10	(a)	0·78120	0·74409	0·70892	0·67556	0·64393	0·61391
	(b)	8·75206	8·53020	8·31661	8·11090	7·91273	7·72173
15	(a)	0·69047	0·64186	0·59689	0·55526	0·51672	0·48102
	(b)	12·3814	11·9379	11·5174	11·1184	10·7395	10·3797
20	(a)	0·61027	0·55368	0·50257	0·45639	0·41464	0·37689
	(b)	15·5892	14·8775	14·2124	13·5903	13·0079	12·4622
25	(a)	0·53939	0·47761	0·42315	0·37512	0·33273	0·29530
	(b)	18·4244	17·4131	16·4815	15·6221	14·8282	14·0939
30	(a)	0·47674	0·41199	0·35628	0·30832	0·26700	0·23138
	(b)	20·9303	19·6004	18·3920	17·2920	16·2889	15·3725
35	(a)	0·42137	0·35538	0·29998	0·25342	0·21425	0·18129
	(b)	23·1452	21·4872	20·0007	18·6646	17·4610	16·3742
40	(a)	0·37243	0·30656	0·25257	0·20829	0·17193	0·14205
	(b)	25·1028	23·1148	21·3551	19·7928	18·4016	17·1591
45	(a)	0·32917	0·26444	0·21266	0·17120	0·13796	0·11130
	(b)	26·8330	24·5187	22·4955	20·7200	19·1563	17·7741
50	(a)	0·29094	0·22811	0·17905	0·14071	0·11071	0·08720
	(b)	28·3623	25·7298	23·4556	21·4822	19·7620	18·2559

log. of present value of £1 at the end of 1 year	$\bar{1}$·9892761346 at 2·5 %	$\bar{1}$·9871627753 at 3 %.	$\bar{1}$·985096503 at 3·5 %.
	$\bar{1}$·9829666607 at 4 %.	$\bar{1}$·98088370096 at 4·5 %.	$\bar{1}$·9788107009 at 5 %.

TABLE

Showing the Interest on £1 at 5 per cent. for any Period of Days in a Year

Days.	Interest.	Days.	Interest.	Days.	Interest.	Days.	Interest.	Days.	Interest.	Days.	Interest.
1	·000137	31	·004246	61	·008356	91	·012465	121	·016575	151	·020685
2	·000274	32	·004383	62	·008493	92	·012602	122	·016712	152	·020822
3	·000411	33	·004520	63	·008630	93	·012739	123	·016849	153	·020959
4	·000548	34	·004657	64	·008767	94	·012876	124	·016986	154	·021095
5	·000685	35	·004794	65	·008904	95	·013013	125	·017123	155	·021232
6	·000822	36	·004931	66	·009041	96	·013150	126	·017260	156	·021369
7	·000959	37	·005068	67	·009178	97	·013287	127	·017397	157	·021506
8	·001095	38	·005205	68	·009315	98	·013424	128	·017534	158	·021643
9	·001232	39	·005342	69	·009452	99	·013561	129	·017671	159	·021780
10	·001369	40	·005479	70	·009589	100	·013698	130	·017808	160	·021917
11	·001506	41	·005616	71	·009726	101	·013835	131	·017945	161	·022054
12	·001643	42	·005753	72	·009863	102	·013972	132	·018082	162	·022191
13	·001780	43	·005890	73	·010000	103	·014109	133	·018219	163	·022328
14	·001917	44	·006027	74	·010137	104	·014246	134	·018356	164	·022465
15	·002054	45	·006164	75	·010274	105	·014383	135	·018493	165	·022602
16	·002191	46	·006301	76	·010411	106	·014520	136	·018630	166	·022739
17	·002328	47	·006438	77	·010548	107	·014657	137	·018767	167	·022876
18	·002465	48	·006575	78	·010685	108	·014794	138	·018904	168	·023013
19	·002602	49	·006712	79	·010822	109	·014931	139	·019041	169	·023150
20	·002739	50	·006849	80	·010959	110	·015068	140	·019178	170	·023287
21	·002876	51	·006986	81	·011095	111	·015205	141	·019315	171	·023424
22	·003013	52	·007123	82	·011232	112	·015342	142	·019452	172	·023561
23	·003150	53	·007260	83	·011369	113	·015479	143	·019589	173	·023698
24	·003287	54	·007397	84	·011506	114	·015616	144	·019726	174	·023835
25	·003424	55	·007534	85	·011643	115	·015753	145	·019863	175	·023972
26	·003561	56	·007671	86	·011780	116	·015890	146	·020000	176	·024109
27	·003698	57	·007808	87	·011917	117	·016027	147	·020137	177	·024246
28	·003835	58	·007945	88	·012054	118	·016164	148	·020274	178	·024383
29	·003972	59	·008082	89	·012191	119	·016301	149	·020411	179	·024520
30	·004109	60	·008219	90	·012328	120	·016438	150	·020548	180	·024657

TABLE

Showing the Interest on £1 at 5 per cent. for any Period of Days in a Year (*continued*)

Days.	Interest.	Days.	Interest.	Days.	Interest.	Days.	Interest.	Days.	Interest.	Days.	Interest
181	·024794	212	·029041	243	·033287	274	·037534	305	·041780	336	·046027
182	·024931	213	·029178	244	·033424	275	·037671	306	·041917	337	·046164
183	·025068	214	·029315	245	·033561	276	·037808	307	·042054	338	·046301
184	·025205	215	·029452	246	·033698	277	·037945	308	·042191	339	·046438
185	·025342	216	·029589	247	·033835	278	·038082	309	·042328	340	·046575
186	·025479	217	·029726	248	·033972	279	·038219	310	·042465	341	·046712
187	·025616	218	·029863	249	·034109	280	·038356	311	·042602	342	·046849
188	·025753	219	·030000	250	·034246	281	·038493	312	·042739	343	·046986
189	·025890	220	·030137	251	·034383	282	·038630	313	·042876	344	·047123
190	·026027	221	·030274	252	·034520	283	·038767	314	·043013	345	·047260
191	·026164	222	·030411	253	·034657	284	·038904	315	·043150	346	·047397
192	·026301	223	·030548	254	·034794	285	·039041	316	·043287	347	·047534
193	·026438	224	·030685	255	·034931	286	·039178	317	·043424	348	·047671
194	·026575	225	·030822	256	·035068	287	·039315	318	·043561	349	·047808
195	·026712	226	·030959	257	·035205	288	·039452	319	·043698	350	·047945
196	·026849	227	·031095	258	·035342	289	·039589	320	·043835	351	·048082
197	·026986	228	·031232	259	·035479	290	·039726	321	·043972	352	·048219
198	·027123	229	·031369	260	·035616	291	·039863	322	·044109	353	·048356
199	·027260	230	·031506	261	·035753	292	·040000	323	·044246	354	·048493
200	·027397	231	·031643	262	·035890	293	·040137	324	·044383	355	·048630
201	·027534	232	·031780	263	·036027	294	·040274	325	·044520	356	·048767
202	·027671	233	·031917	264	·036164	295	·040411	326	·044657	357	·048904
203	·027808	234	·032054	265	·036301	296	·040548	327	·044794	358	·049041
204	·027945	235	·032191	266	·036438	297	·040685	328	·044931	359	·049178
205	·028082	236	·032328	267	·036575	298	·040822	329	·045068	360	·049315
206	·028219	237	·032465	268	·036712	299	·040959	330	·045205	361	·049452
207	·028356	238	·032602	269	·036849	300	·041095	331	·045342	362	·049589
208	·028493	239	·032739	270	·036986	301	·041232	332	·045479	363	·049726
209	·028630	240	·032876	271	·037123	302	·041369	333	·045616	364	·049863
210	·028767	241	·033013	272	·037260	303	·041506	334	·045753	365	·050000
211	·028904	242	·033150	273	·037397	304	·041643	335	·045890	—	——

Table of Depreciated Book Values

Values of £1 depreciated up to twenty-five years—i.e. Book Values of outlay of £1 at the end of each year at the given rate per cent.

Years.	2 per cent	5 per cent	7½ per cent	10 per cent
	1·000	1·000	1·000	1·000
1	·980	·950	·925	·900
2	·960	·902	·855	·810
3	·941	·857	·791	·729
4	·922	·814	·732	·656
5	·903	·773	·677	·590
6	·885	·735	·626	·531
7	·868	·698	·579	·478
8	·850	·663	·535	·430
9	·833	·630	·495	·387
10	·817	·598	·458	·348
11	·800	·568	·424	·313
12	·784	·540	·392	·282
13	·769	·513	·362	·254
14	·753	·487	·335	·228
15	·738	·463	·310	·205
16	·723	·440	·287	·185
17	·709	·418	·265	·166
18	·695	·397	·245	·150
19	·681	·377	·227	·135
20	·667	·358	·210	·121
21	·654	·340	·194	·109
22	·641	·323	·179	·098
23	·628	·307	·166	·088
24	·615	·291	·153	·079
25	·603	·277	·142	·071

Example.— A machine costing £300 will, at the end of eight years, after deducting 7½ per cent per annum from the balances in the books, be equal to (£300 × ·535) = £160·5 in value; or (£300 - 160·5) = £139·5 will then have been written off.

Sinking Fund Table

Annual amount required to be set aside and Invested with Accumulated Interest added each year to produce £1, up to twenty-five years.

Years.	2½ per cent	3 per cent	4 per cent	5 per cent
1	1·0000	1·0000	1·0000	1·0000
2	·4938	·4926	·4902	·4878
3	·3251	·3225	·3203	·3172
4	·2408	·2390	·2355	·2320
5	·1902	·1884	·1846	·1810
6	·1565	·1546	·1508	·1470
7	·1325	·1305	·1266	·1228
8	·1145	·1125	·1085	·1047
9	·1005	·0984	·0945	·0907
10	·0893	·0872	·0833	·0795
11	·0801	·0781	·0742	·0704
12	·0725	·0705	·0666	·0628
13	·0660	·0640	·0601	·0565
14	·0605	·0585	·0547	·0510
15	·0558	·0538	·0492	·0463
16	·0516	·0496	·0458	·0423
17	·0479	·0460	·0422	·0387
18	·0477	·0427	·0390	·0355
19	·0418	·0398	·0361	·0327
20	·0391	·0372	·0336	·0302
21	·0368	·0369	·0313	·0280
22	·0346	·0327	·0292	·0260
23	·0327	·0308	·0273	·0241
24	·0309	·0290	·0256	·0225
25	·0293	·0274	·0240	·0210

Example.— Amount to be set aside each year to produce £17,000 at 3 per cent c.p.i. in seventeen years (£17,000 × ·046) = £782 p.a.

Reserve Fund Table

Amount of £1 per annum accumulated with Compound Interest at various rates up to twenty-five years.

Years.	2½ per cent.	3 per cent.	4 per cent.	5 per cent.
1	1·000	1·000	1·000	1·000
2	2·025	2·030	2·040	2·050
3	3·076	3·091	3·122	3·153
4	4·153	4·184	4·246	4·310
5	5·256	5·309	5·416	5·526
6	6·388	6·468	6·633	6·802
7	7·547	7·662	7·898	8·142
8	8·736	8·892	9·214	9·549
9	9·954	10·159	10·583	11·027
10	11·203	11·494	12·006	12·578
11	12·483	12·808	13·486	14·207
12	13·796	14·192	15·026	15·917
13	15·140	15·618	16·627	17·713
14	16·519	17·086	18·292	19·599
15	17·932	18·599	20·024	21·579
16	19·380	20·157	21·825	23·657
17	20·865	21·762	23·698	25·840
18	22·386	23·414	25·645	28·132
19	23·946	25·117	27·671	30·539
20	25·545	26·870	29·778	33·066
21	27·183	28·676	31·969	35·719
22	28·863	30·737	34·248	38·505
23	30·584	32·453	36·618	41·430
24	32·349	34·426	39·083	44·502
25	34·158	36·459	41·646	47·727

Example.—The amount of £782 invested annually at 3 per cent p.a., c.p.i. in seventeen years = £(782 × 21·762) = £17,017·884.

ANSWERS TO TEST EXERCISES

I. 1 (pages 11-13)—

(1) 93,077
(3) 1,233,981 acres
(4) 50
(5) 987,924 bricks
(6) 54,185 oranges
(7) £230
(8) 40 gall.
(9) 1,719 props
(10) 468; 192
(11) 5,585,000 acres
(12) (*a*)—

Increase in Population

1911–21	1921–31
453,637	404,165
215,729	264,242
150,159	196,511
86,602	87,768
50,618	87,392
22,569	5,914
557	746
858	6,235

(12) (*b*) 218 persons per 100 sq. ml.
(13) $437\frac{1}{2}$ hr.; 280 ml.
(14) 11 days
(15) £506; 130 ac.
(16) £99
(17) $\dfrac{W - w}{n}$ lb.
(18) $\dfrac{n(x + 3y)}{z}$ pkt.
(19) $\dfrac{500}{a + b}$ times
(20) $u(n - 1)$ yd.
(21) $\dfrac{np}{p + 1}$ articles
(22) (i) 320,563; (ii) 5,232,465

I. 2 (pages 24-5)—

(1) £5 9s. 7d.
(2) $6\frac{3}{4}$d.
(3) 16 men; £6 1s. 3d.
(4) £1 13s. 8d., 19s. 7d., 13s. 9d.
(5) £862 7s. 4d.
(7) £321 11s. 3d.
(8) 48 tons 3 cwt. 105 lb.
(9) $1148\frac{7}{16}$ oz. tr.
(10) 26 min. 37 sec. past 9.0; 12 ml. 856 yd.
(11) (*a*)—

£
652,150
877,425
605,292
579,103
670,674
1,560,468
862,860
755,447
674,880

(11) (*b*) 10s. 10d.
(12) 16,964 tons
(13) £1 10s.
(14) 1,071 cwt. 0 qr. 16 lb.
(15) $5,826,525\frac{9}{16}$ tons
(16) 2 hr. 19 min. $7\frac{1}{2}$ sec.
(18) $\dfrac{112}{n}$ shillings
(19) $\dfrac{mx + ny}{m + n}$ pounds
(20) $\dfrac{2250}{11x}$ ml. per hr.
(21) £8 9s. 8d.
(22) 5,999 casks
(23) £14 1s. 4d.
(24) 51,563 barrels
(25) (i) 5·52 p.m.; 6·18 a.m.
(ii) 11.12 a.m.; 11.38 p.m.
(iii) 3.51 a.m.; 2.59 a.m.

I. 3 (pages 29-30)—

(1) 33 articles
(2) 12 lb.
(3) 1 yd. 2 ft. 3 in.
(4) 14 men; 32 squads
(5) 60 brackets
(6) 38 ml.
(7) 6 lb.
(8) 3.4 p.m.
(9) 7 yd. 1 ft. 4 in.; 49 supports; £9 7s. 10d.
(10) 140 cases
(11) 1; $6a^2b\,(5b-6c)$
(12) $£\dfrac{(2a+1)\ (20x+y)}{20}$

I. 4 (pages 42-3)—

(1) £1 18s. $11\frac{3}{4}$d.
(2) £1 4s. $10\frac{1}{2}$d.
(3) £22 10s. 5d.
(4) £156 8s. 5d.
(5) £14 6s. 3d.
(6) $\frac{11}{40}$
(7) £41 1s. 4d.
(8) 10 lb. $6\frac{183}{224}$ oz.
(9) 5′ $0\frac{1}{16}$″; $8\frac{23}{32}$″
(10) 546 tiles
(11) $409\frac{1}{2}$ bush.; 210 bush.; $15\frac{3}{4}$ ac.
(12) £6 1s. $6\frac{3}{16}$d.
(13) $37\frac{17}{90}$ bush.; $13\frac{119}{246}$
(14) 1 lb. $13\frac{3}{5}$ oz.; 5 lb. $14\frac{2}{5}$ oz.
(15) 47, 41, 37, 37 parcels respectively; $11.31\frac{1}{2}$ a.m.
(16) Loss of 13s. $6\frac{1}{2}$d.
(17) 11s. 10d.
(18) £468, £702, £585, £390, £195
(19) £24, 10s. 10d.; £11 0s. 11d.
(20) £680 17s. $2\frac{1}{4}$d.
(21) $\frac{825}{1684}$
(22) £206 15s. 4d.
(23) £83 4s. 4d.
(24) 121 rings
(25) $\frac{128}{159}$
(26) 11. 21. 30 a.m. (to nearest sec.); 52 ml. 1,651 yd. from A (to nearest yd.)
(27) 24 hr. $15\frac{15}{23}$ min.
(28) 698, 638, 582, 532 envelopes respectively; work completed in 1 hr. 1 min. $42\frac{18}{25}$ sec.
(29) $£\dfrac{1000\,(p+1)-a}{240}$
(30) 145 yd. 1 ft. 2 in.; 172 yd. 1 ft. 6 in.; 189 yd.; 230 yd.
(31) $\dfrac{120n^2}{7+2n}$ min.
(32) $\dfrac{x\,(60y+z)}{60\,(x+y)+z}$ hr.
(33) $\dfrac{npx}{12\,(36-x)}$ shillings
(34) $\dfrac{(b-a)\,(y-x)}{by}$

I. 5 (pages 58-61)—

(1) 10s. 6d.
(2) 40 sq. ft.
(3) $29\frac{1}{16}$ c. ft.; £5 1s. $8\frac{5}{8}$d.
(4) $5\frac{485}{497}$ tons per ac.; $501\frac{69}{71}$ lb.
(5) £3 10s. 10d.
(6) £28 12s.; £46 6s. 3d.
(7) 182 bush.
(8) $397\frac{5}{8}$ sq. yd.
(9) $250\frac{125}{192}$ gall.
(10) £144 12s. 4d. (to nearest 1d.)
(11) $16\frac{51}{80}$ days
(12) 10,140 pieces
(13) $140\frac{5}{8}$ bucketsful
(14) £157 10s. 3d. (to nearest 1d.)
(15) £806 13s. 4d.
(16) 68,040 c. ft.
(17) £12 7s. $4\frac{3}{4}$d.
(18) 12 cwt. 2 qr. 2 lb. 10 oz. (to nearest oz.)
(19) £5 15s. $11\frac{1}{4}$d.; $445\frac{1}{3}$ days
(20) £16
(21) 168, 205, 168; 4 sq. pl. $9\frac{5}{8}$ sq. yd.
(22) £103 8s. 5d.
(23) £6 6s.; 11 pieces, 18s. 4d.
(24) 27 tons 1 cwt. $45\frac{1}{2}$ lb.; £209 15s. 11d. (to nearest 1d.)
(25) £186 13s. 4d.
(26) £44 11s. 7d. (to nearest 1d.)
(27) £4 18s. 5d. (to nearest 1d.)
(28) $2\frac{4}{41}$ in.
(29) 1,379,840 blocks
(30) £61,111 2s. 3d.

I. 5 (pages 58–61)—*contd.*

(31) $131\frac{107}{128}$ gal. per min.
(32) 1 hr. 41 min. 12 sec. (to nearest sec.)
(33) $2\frac{11}{44}$ in.
(34) $21\frac{1}{4}$ sq. ft.; $19\frac{1}{6}$ sq. ft.
(35) 19 allotments; $7\frac{1}{47}$ yd. ; $5\frac{3}{25}$ sq. pch.
(36) $3\frac{1}{10}$ days (approx.)
(37) 6 acres 3 sq. chn 8,080 sq. lk.
(38) 2 acres 2 sq. chs. $256\frac{1}{2}$ sq. yd.
(39) $\frac{432ab}{x^2}$ tiles
(40) $\frac{160\,(y-5)}{x}$ pieces of turf
(41) $\frac{1{,}935{,}360}{wx\,(2l - h_1 - h_2)}$
(42) $\frac{3ab}{u\,(12s + t)}$ in.
(44) 11, $343\frac{3}{4}$ gal.; 32,$410\frac{5}{7}$ ton
(45) $9\frac{99}{133}$ in.

I. 6 (pages 86-9)—

(1) 650 births; 442 deaths
(2) 16·7, 15·5, 13·7
(3) Between 17,693 and 17,793
(4) 4,444 medals
(5) 10s. 1d.
(6) £134,000,000 ; £463,060,000 £386,000,000 , £352,000,000 £382,000,000 ; £387,000,000
(7) £46 9s.
(8) 236 sq. ft. 59·52 sq. in.; ·0036 in.
(9) 9·32 oz.; 9·981108 oz.; 8·706924 oz.
(10) 1·41069
(11) 138·54167 gr , 5·83333 gr , 1·45833 gr ; 83·125 gr., 3·5 gr , ·875 gr.; 41·5625 gr., 1·75 gr , ·4375 gr.
(12) 5 oz. tr. 4 dwt. 3·10 gr.
(13) 12′ 7·43″; 150 sq. ft. 2 sq. in.
(14) 13s. 1d , 42s 11d , 27s. 7d. (to nearest 1d.); 2′ 11·82″
(15) 129 ml. 1046 yd. 2 ft.
(16) 127·62 fc.; \$3·92
(17) ·6954520 ton
(18) 4·206397 ac.
(19) (*a*) 5 fur. 22 pl 1 yd. 7 in.; (*b*) 12 c. ft. 1534·2 c. in.
(20) £523,400
(21) 104 gal. $2\frac{1}{4}$ pt.
(22) 3375 ml.; 20·39 knots
(23) 1,660 kg. 635 g.
(24) 5 oz. tr. 11 dwt. 4 gr.
(25) 41·60 g.; 2 m. 40·31 cm.
(26) 111,000 persons
(27) 1,055,000 lb.
(28) £90,200
(29) 1·36″
(30) (i) 5·860; 28·698; 8·540; 1·156; 0 035; (ii) 5·86; 28·7; 8·54; 1·16; 0 0346
(31) 9,598 gal.
(32) 2s. 11d.
(33) £10 10s 3d (to nearest 1d.)
(34) £57 4s 2d (to nearest 1d.)
(35) £7 6s. 6d (to nearest 1d.)
(36) £540 18s.
(37) £210 15s 6d. (to nearest 1d).
(38) £101 6s 11d.
(39) £95 10s $4\frac{1}{2}$d.
(40) £83 16s. 7d. (to nearest 1d.)
(41) 102
(42) 348 ml. 697 yd.
(43) 278 yd. 10 in.
(44) 3·979 cm.
(45) £780 9s. $0\frac{3}{4}$d ; £203 19s. 6d.; £291 13s $2\frac{1}{4}$d , £151 14s. 9d.
(46) £1 3s. 8d. (to nearest 1d.); £9 5s. 8d., 10s 4d.
(47) 135·163 ml. , 282·333 Km. ; 369·540 tons.; 464·334 milliers; 211 ac. 33·33 sq. pl.; 5 hect. 55·33 ares.
(48) (*a*) 12·39 Km.; (*b*) 1 cwt. 6 lb. 8·3 oz.
(49) 1s. $8\frac{3}{4}$d.
(50) 195·0 lb.
(51) 9·045 c. ft.

I. 7 (pages 108-115)—

(1) 2112 : 2107 or 1·002 : 1
(2) 1·097 : 1
(3) 198 : 133 or 1·489 : 1
(4) ·86 : 1

I. 7 (pages 108–15)—*contd.*

(5) ·765 : 1
(6) 7·2″; 25 : 64
(7) £165 12s. 6d.
(8) Increased 27 : 25
(9) $94\frac{6}{19}$ lb., $23\frac{11}{19}$ lb.; 113,178 pennies
(10) 1·087 : 1; 2,529,000 females
(11) ·564 : 1
(13) 48 : 27 : 10
(14) £690 5s. 0d.; £332 1s. 1d.
(15) £10 8s. 3d.
(16) £120
(17) £2 15s. 10d.
(18) £10 17s. 6d., 5 ac. 9 sq. ch. 218 sq. yd.
(19) 4,986 tons more; £22,631
(20) 4 dy. 12·4 hr.; 24·6 knots
(21) 2950·48 fc.; 84,732 kg.
(22) 13·1 days
(23) 44·04 Kg. per are
(24) 605 tiles
(25) 1 day 15 hours; 105 acres
(26) 600 bushels; $7\frac{7}{8}$ days
(27) £86 16s.
(28) 17 men
(29) £57 15s.
(30) 3 hr. 17·1 min.
(31) 257 tins
(32) 331·6 tons
(33) 3 hr. 20 min. per day
(34) (*a*) £56 16s. 10d.; (*b*) £68 4 . 2d.
(35) (*a*) 2 ml. 266 yd.; (*b*) ·231 in.
(36) 8·823
(37) 400·13 fc.; 1·404 Kg.
(38) £5 1s.
(39) (*a*) 44·15 fc.; (*b*) 310·18 dollars
(40) 3s. $1\frac{1}{2}$d.
(41) £239 17s. 10d.; 47 days
(42) £5,333 6s. 8d.; £178
(43) 77 cwt. 104 lb.
(44) £1 16s.
(45) £719 3s. 4d.
(46) £427 4s. 9d.; £366 4s.; £254 6s. 3d.
(47) £5 3s. 1d.; £2 11s. 11d.
(48) £1 11s. 8d.; £1 6s.; £1 4s. 3d.; £1 1s. 1d.; 9s. 6d.
(49) £413 0s. 4d.; £380 11s.; £321 0s. 8d.
(50) 192 days; 73 days; 40 days
(51) £6,601 6s.; £3,009 2s.; £1,770 0s. 3d.
(52) £637 1s. 8d.; £540 16s. 8d.; £265 16s. 8d.
(53) £2,585; £775 10s.; £282
(54) 1 : 4

(55)

$3\frac{1}{2}$%	5%	$12\frac{1}{2}$%	14·7%	119%
£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
24 1 0	35 15 8	89 9 2	105 4 1	851 13 1
158 15 7	226 17 7	567 1 5	666 17 7	5,398 10 7
6·15 fc.	8·79 fc.	21·97 fc.	25·84 fc.	209·14fc.
149·28fc	213·25fc.	533·12fc.	626·96fc.	5075·35fc.
$25·00	$35·72	$89·29	$105·01	$850·08
$172·80	$246·85	$617·13	$725·74	$5875·03

(56)

£ s. d.		
+ 63 0 3	+ 16·82%	+ 14·40%
+ 71 18 0	+ 40·91%	+ 29·03%
− 75 8 6	− 14·97%	− 17·60%
+ 7 7 6	+ 3·10%	+ 3·01%

+ 5·18%, + 4·92%

(57) + $16\frac{2}{3}$%; + 17·12%; − 13·10%
(58) 6·87% less
(59)

£ s. d.	£ s. d.	£ s. d.	£ s. d.
853 6 2	858 2 8	769 19 3	770 13 1
266 16 0	268 6 3	240 14 10	240 19 2
5070·18 fc.	5098·87 fc.	4574·96 fc.	4579·08 fc.
$3343·09	$3362	$3016·55	$3019·27

(60) 13·24%
(61) + 22·34%
(62) 17·675%
(63) 1·17%
(64) 15·83%
(65) 11·9%; 62·29%
(66) 22·48%; 20·33%; 22·03%
(67) $6\frac{2}{3}$%
(68) 47·54%; £4 19s. 8d.
(69) 2s. 6d.
(71) £207 16s. per annum.
(72) 31·96%
(73) $5\frac{3}{4}$d. per lb.
(74) ·58%
(75) $11\frac{1}{9}$%
(76) (*a*) 60·10%; (*b*) 39·90%
(77) 18·58% loss
(78) £8 7s. 8d.; £154 3s. 7d.; 445·52 fcs.; $1505·23
(79) £8,000
(80) £987 7s.
(81) 13·3%
(82) 67 days
(83) £55 6s. 2d.; £46 2s. 6d.; £127 1s. 5d.
(84) £7 9s. 10d.; £7 18s. 8d.; £8 7s. 7d.
(85) £9 5s.

I. 7 (pages 108–15)—*contd.*

(86) £216 13s. 7d.
(87) £160 12s. 7d.; £437 7s. 2d.; £49 8s. 11d.; 7656·94 fcs.; $1086·31
(88) £8,226 13s. 4d.
(89) 19s. 2d.
(90) £71 16s. 4d.
(91) £2,222 9s. 10d.
(92) (*a*) £1,785 14s. 3d.; (*b*) £1,777 19s. 10d.
(93) 5·09%

I. 8 (pages 132-7)—

(1) 1″ to $16\frac{3}{8}$′; 2′ $0\frac{3}{4}$″; 1′ $10\frac{1}{2}$″; 1′ 1″ (nearly); 23 sq. ft. 98·7 sq. in.
(2) 58′ 2″ (nearly)
(3) 16′ $1\frac{3}{8}$″; 20′ $4\frac{1}{8}$″
(4) 13·761 g.; 6·054 cm.
(5) 62′ 3″
(6) 1·291″; ·707″
(7) $\frac{1}{168,960}$; 7 sq. ml. 71 acres
(8) (1) 316·8″; (2) 1,858 sq. yd 5 sq. ft. 5·76 sq. in; (3) 398 ac. 2692 sq. yd. 3 sq. ft. 16 sq. in.
(9) 1′ 3″ nearly
(10) 11 stone 9 lb.
(11) 256 tons 9 cwt. 4 lb.
(12) £1539 11s. 8d.
(13) $37\frac{1}{27}$ times
(14) £378 13s. 4d.
(15) 4·564 gal.
(16) 3 lb. $13\frac{3}{4}$ oz.
(17) 720 revolutions
(18) $3\frac{6}{8}\frac{1}{4}$
(19) 31 blocks
(20) 162 sq. yd. 8 sq. ft. 96 sq. in.
(21) £4 8s. $2\frac{3}{4}$d.
(22) 2,511$\frac{1}{4}$ sq. yd.
(23) 436 jars
(24) 33·95 bushels
(25) £6 12s. $4\frac{3}{4}$d.; 3 sq. ft. 45·5 sq in.
(26) 1,170 gal.; 4s. 2d.
(27) 152 loads
(28) 24 sq. yd. 7 sq. ft. $91\frac{3}{7}$ sq. in.
(29) 12 tons 11 cwt. 65 lb.
(30) £12 18s.
(31) 119·2 yd.; 47·8 yd.
(32) (1) 49′ 6″; (2) £1 6s. 10d.
(33) 11 c. yd. 17·16 c. ft.; 22 sq yd. $6\frac{3}{7}$ sq. ft.
(34) 43·08′; 1·64′
(35) 3 tons 14 cwt. 1 qr.
(36) 189 lb. 2 oz. per c. ft.
(37) $V = \frac{1}{3}\pi r^2(h + 2r)$; $A = \pi r(l + 2r)$
(38) (1) 118·7 pt.; (2) 6 sq. ft. 57 sq. in.; (3) 231·84 pt.; (4) 9 sq. ft. 143 sq. in.
(39) £4 11s. 9d.
(40) $\frac{\pi r^2}{3}(2r + 3H - h)$; $142\frac{10}{21}$ c. in.
(41) £61 9s. 10d.
(42) £12 9s.
(43) £11 14s. 7d.; 41 c. yd. 12·2 c. ft.
(44) 134 yd. 2 ft. 1 in.; 44 yd. 2 ft. $8\frac{1}{2}$ in.
(45) 124 tons
(46) 3′ $8\frac{1}{2}$″
(47) 12·84 cm.
(48) 2·1″
(49) ·0524 in.
(50) 88′
(51) 6·8″, 13·6″; 25 lb. 12·4 oz.
(52) 4·14″
(53) 188 yd. 1 ft. 5 in.
(54) 95·5 c. in.
(55) 720 ft. 0·2 in.
(56) ·208 in.
(57) ·046″
(58) £150, £100, £115
(59) £58 4s. per annum
(60) £102
(61) 180 oranges
(62) £275 at 4%; £225 at 5%
(63) £449 9s. 1d.; £337 1s. 10d.
(64) £2,560
(65) £17s.; £1 6s.; £2 12s. 6d.
(66) £330, £396, £495
(67) £50 per annum; £500
(68) 7 : 1
(69) £13 per ton; $9\frac{1}{11}$%, $11\frac{1}{9}$%, $13\frac{1}{23}$%
(70) 8s.
(71) 750 eggs
(72) £600
(73) 95 pears
(74) £12 12s.
(75) 48, 33, 19 articles respectively
(76) £5 5s.; £6 16s. 6d.; £8 8s.; £10 10s.

I. 9 (pages 154-160)—

(1) 41 acres approx.
(2) 3·14
(3) 32,500 sq. ml.
(4) 2,220 sq. ft.; 5·1%
(8) $\frac{13}{17}$, $\frac{31}{43}$, $\frac{51}{73}$, $\frac{23}{40}$
(9) £575 producing £25 5s.; £335 producing £13 12s.
(10) French Bank
(11) 85·7 Km.; 62·1 ml.
(12) 50·79 Kg.
(13) ·645 hect., 3 ac. 1 rd. 13⅓ sq pl.
(14) 1,633 fc.; 953 fc.; £13 12s.; £10 10s.
(15) £135
(16) £9 19s. 4d.
(17) $62\frac{2}{5}$° F.; $19\frac{2}{5}$° F.; $-17\frac{7}{9}$° C.; $31\frac{1}{9}$° C.
(18) 9s. 2d.; 15s. 9d.; £1 8s. 1d.
(19) 15s. 11d.
(20) £17,000; £4,860 (approx.); £27,000
(21) 45½ ml. per hr. (approx.)
(22) 10·37 a.m., 65¼ ml., 11.15 a.m., 89 ml.; 11·38 a.m., 17¾ ml; 12·9 p.m., 42½ ml.
(23) Two loop lines; 660 yd. from X and 660 yd. from Y
(25) 4,650,000; 6,200,000
(26) 3s. 2d.
(27) Males: 880, 852; females: 902, 874
(28) Males: 35·7, 32·4, 29·2, 26·0, 22·9, females: 38·6, 35·2, 31·9, 28·6, 25·3
(29) £14 4s. £23 8s., £42 19s.; £15, £25 13s., £49 19s.; £15 18s., £28 3s, £58 8s.; £19 6s., £20 16s.
(30) 40·7 sq. in.; 4·9 in.
(31) 3·50
(32) 5 hr. 21 min.; 18 ml. per hr.
(33) 2⅓ lb.; 640 men
(34) £270, £150, £90
(35) 4% per annum
(36) 4 yr.
(37) 3⅓ yr.; £555 11s.
(38) 22½ yr.
(39) 20⅙ yr.; 15¾ yr.
(40) £139 2s.; 10⅔ yr.
(41) 8 yr.
(42) 2′
(43) 6′ 10″, 6′ 10″, 3′ 5″
(44) 4s. 0½d.

I. 10 (pages 179-181)—

(1) (*a*) 3·0164; (*b*) ·8008; (*c*) $\bar{7}$·7787738; (*d*) 1·08278

(2) (*a*) (*b*)

Number.	Logarithms.	Logarithms.	Number.
14	1·1461280	$\bar{1}$·8850557	·76746
2075·4	3·3171018	3·6532897	4500·8
·390487	$\bar{1}$·5916066	$\bar{2}$·7038116	·05056052
214076900	8·3305698	·0048629	1·01126
·000499362	$\bar{4}$·6984155	$\bar{1}$·202718	·159484
8·700438	·9395411	5·4700000	·295121

(3) (*a*) 33027·73; (*b*) ·1954407; (*c*) 17·91483, (*d*) ·868817
(5) ·69897, $\bar{3}$·7781513, ·6532126, $\bar{1}$·8115752, 2·0511526
(6) ·30103, ·4771213, $\bar{1}$·2833012, 1·1637575
(7) £14 0s. 4d.
(8) 1,794 spheres
(9) 84·85 c. in.
(10) 1·24
(11) Height = diameter = 2·634″
(12) 66·39 c. in.
(13) (*a*) 3·12 fc. per tonne per Km.; (*b*) 38·60 fc. per m.; (*c*) 1d. per pint, (*d*) £1 11s. 5d. per acre.
(14) 8833·07 fc.
(15) 7·69 oz.
(16) 15 tons 15 cwt. 111·3 lb.; 5′ 2″
(17) 21,625
(18) 5·07%
(19) £5 12s. 8½d.
(20) 287 days
(21) £124 5s 10d.
(22) 7·72%
(23) £578 3s 1½d.
(24) £557 2s 5d.
(25) 9 yr. 150 dy.; 9 yr. 94 dy.; 9 yr. 81 dy.
(26) 5·65%
(27) 5·74%
(28) B; £3,600, £3,548 12s. 6d.
(29) 35·1 per 1,000
(30) £814 8s. 3d.
(31) £102 12s. 11d.
(32) £54 18s. 1d.
(33) £16,250; £14,225 19s. (to nearest 1s.)
(34) £1,505 8s. (to nearest 1s.)
(35) £3,605 5s. 3d.

II. 1 (pages 188-191)—

(1) £108 19s. 9d.
(2) £21,600
(3) 6 lb.; 13·08%,
(4) £534 16s. 5d.
(5) £3 9s. 3d. gain
(6) £374 4s. 9d.
(7) $5\frac{1}{4}''$, $5\frac{3}{4}''$
(8) £11 18s. 1d.
(9) $26\frac{2}{3}\%$
(10) $4\frac{1}{2}\%$: £·000123288
$4\frac{3}{4}\%$: £·000130137
5 % : £·000136986
$5\frac{1}{4}\%$: £·000143835
$5\frac{1}{2}\%$: £·000150685
(i) £4 7s. 10d ; (ii) £4 16s. 5d.
(11) ·89812; 4s. $1\frac{1}{2}$d.; 5s. $7\frac{1}{4}$d.
(12) $62\frac{2}{9}$ lb. of first, $49\frac{7}{9}$ lb. of second
(13) 149 kroner 17 ore
(14) 40 hurdles
(15) £14 15s. 2d.
(16) $18\frac{4}{7}\%$
(17) 26s. 6d. per week; 4·6% gain
(18) 17s. $1\frac{1}{3}$d.
(19) £3 13s. 9d.
(20) Y; 20 days; 5s.
(21) £9 4s 6d.
(22) 11s. 10d.
(23) £827 10s 7d.; £913 1s. 5d.
(24) 5′ 4″, 4′ 6″, 4′ 2″
(25) $36\frac{4}{11}\%$
(26) 17·12% less
(27) 1s $0\frac{3}{4}$d. per lb.
(28) 10s 6d.
(29) 4s. $3\frac{3}{4}$d.
(30) £1,479 (to nearest £)
(31) £410 15s.
(32) £417 10s. greater
(33) £175, £187, £198 16s., £204 6s.; Loss of £56 5s ; loss of £11 5s.; gain of £33, gain of £53 12s. 6d.; £47 5s.

II. 2 (pages 204-209)—

(1) £1,420, £1,316 13s. 4d.; £743 6s. 8d.
(2) (i) £1,100, £1,103 6s. 8d., £636 13s. 4d.; (ii) £1,120, £1,116 13s. 4d., £643 6s. 8d.; (iii) £800, £903 6s. 8d., £536 13s. 4d.
(3) £552 6s. 8d., £350 12s. 3d., £273 15s. 4d.
(4) A rec. £380, B rec. £85, C pays £10, D pays £5
(5) (i) £8,583 6s. 8d., £2,583 6s. 8d., £1,083 6s. 8d.; (ii) £7,625, £2,775, £1,850, (iii) £7,145 16s. 8d., £3,062 10s., £2,041 13s. 4d.
(6) £2,238 15s., £1,288
(7) £6,388 7s. 8d., £4,965 15s. 2d., £4,565 14s. 8d.
(8) £6,031 7s. 8d., £4,608 15s. 2d., £4,208 14s. 8d.
(9) £4,708 4s. 10d.; £4,989 18s. 8d.
(10) £455
(11) £1,331 3s. 4d., £13,309 13s. 4d., £10,952 5s., £6,340 5s.
(12) £4,849 12s. $10\frac{1}{2}$d., £5,053 5s. 5d.; £1,625 4s. 5d , £968 10s. 11d., £1,269 12s $10\frac{1}{2}$d., £937 5s. 5d., £658 4s. 5d., £510 10s. 11d.
(13) £1,443 6s. 8d., £918 13s. 4d., £624
(14) £1,248 9s. 7d., £789 10s. 5d.
(15) £12,734 8s 5d., £10,481 19s. 10d., £6,054 11s. 9d.
(16) £2,108 0s 7d., £736 19s. 5d.

(17)

	Watson.			Brand.			Murray.			Trustee.			Liabilities other than Capital and Loan.		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
March		—			—			—		38	17	7	933	2	5
April		1	7	500	1	10		1	5	54	17	7	816	17	7
May	612	18	0	728	1	0	534	17	10	78	3	2		—	
June	310	5	6	350	2	3	283	5	10	39	6	5		—	
July	158	8	0	158	8	0	158	8	0	19	16	0		—	

II. 2 (pages 204–209)—*contd.*

(18) Repayment of loan to Brand; May, £376 1s. 8d.; June, £123 18s. 4d. Repayment of capital less losses: Watson, £414 19s. 9d.; Brand, £569 19s. 9d.; Murray, £309 19s. 9d.
(19) 12·82%, 18·08%, 22·56%
(20) A, £636 8s. 7d ; B, £568 4s. 3d.; A, £763 7s.; B, £568 4s. 3d.; C pays £36 18s. 5d.
(21)

	Rudd.			Wilson.			Henley.		
	£	s.	d	£	s.	d.	£	s.	d.
1st Quarter	503	6	7	200	1	11	1	15	6
2nd ,,	1,946	11	11	1,750	15	11	1,038	3	8
3rd ,,	1,252	10	2	1,152	3	1	731	10	1
4th ,,	821	15	0	761	4	0	493	1	0

(22) Reduced by £158 3s. 10d.; 15·37%
(23) £685 7s. 6d., £772 17s. 6d.: 26·144%
(24) £1,826 (to nearest £)
(25) £367 17s., £100 14s. 4d.; £1,567 8s.
(26) £365 15s. 4d., £241 6s. 8d.
(27) £15 12s. 5d.
(28) T & Co. £913 4s. 6d.; S & Co. £690 12s.; V & Co. nil
(29) Total loss: £4,168 5s.; 8s. $10\frac{3}{8}$d., 13s. $10\frac{3}{32}$d.
(30) £1,134 13s. 9d , £877 10s. 9d.; £1,397 12s. 4d., £1,154 13s. 3d.

II. 3 (pages 228-236)—

(1) £36,500, £96,750, £36,250, £24,250
(2) £192,574 12s. 6d.
(3) £900; £431 5s.; £225; £2 3s. 3d.; £1 6s. 11d.
(4) £401 10s. 6d.
(5) £21 18s. 9d.; £4 12s. 4d. per annum
(6) £40,000; £25,665
(7) £525; £805
(8) £7 0s. 11d.
(9) 401 applicants; 3, 7, 71, 143 shares resp.
(10) £223 2s. 6d.
(11) 3·254%
(12) 4·81%
(13) 4%
(14) £5,600, $7\frac{3}{11}$% per annum
(15) £618 10s.; 4·88%
(16) $22\frac{18}{31}$%
(17) 5%, 6·39%, nil; (*a*) 4·47% nil; (*b*) 5%, 1·09%, nil; (*c*) 5%, 7%, 12%
(18) 5%, 7%, 3·7%; (*b*) 5%, ·058%, nil; (*b*) 5%, 1·70%, nil; (*c*) 5%, 7%, 18·15%
(19) (i) 1·986%, nil; (ii) 6%, negligible; (iii) 6%, 1·318%
(20) (*a*) 5%, 8·79%, 11·79%; (*b*) 6·28%, 8·28%, 11·28%
(21) 24·93%; 31·43%
(22) $\frac{1}{4}$%, nil; 3%, nil; 6·4%, nil; 7·85%, 2·59%; $4\frac{1}{2}$%, $6\frac{3}{16}$%; $4\frac{1}{2}$%, $11\frac{3}{16}$%
(23) 1·21%, nil; 4·84%, nil; 7·45%, 1·74%; $4\frac{1}{2}$%, 8·99%; $4\frac{1}{2}$%, 11·34%; $4\frac{1}{2}$%, 19·15%
(24) 14%; £235 8s 4d., nil; £264 16s. 10d.; £300 15s. 7d.; £257 16s. 3d., £386 14s. 4d., £351 11s. 3d., £656 5s.; 3·28%, 0%, 3·68%, 4·18%, 3·59%, 5·38%, 4·89%, 9·13%
(25) £211 11s. 5d.; £214 8s. 1d.
(26) 4·78%; 11s. 6d.
(27) £13 16s. 1d., £16 11s 6d., £17 12s. 3d., £18 12s. 4d., £20 1s. 1d.
(28) 2,500 shares
(29) 17s. 6d.
(30) 4·27%
(31) (i) $11\frac{1}{13}$%; (ii) 12%
(32) £3,164 7s., £3,168 17s.
(33) 1·5% (nearly); (i) £471 7s. 6d ; (ii) £553 12s. 3d.
(34) $6\frac{2}{13}$%; £2,569 1s. 5d.; 710 days
(35) £3,583 3s. 7d.; £4,155 7s. 10d.
(36) 5·2%; 2·7%
(37) £2,319 18s.
(38) (*a*) £100, £15 3s. 4d.; (*b*) £45 9s. 3d.
(39) Diminished in ratio 15 : 16
(40) $17\frac{3}{16}$%, £2 2s. 3d. less
(41) 80 : 69, £20,625, 16% (nearly)
(42) £7,656 5s.
(43) £152,500, £95,312 10s.
(44) 15s. 4d. less
(45) (*a*) $27\frac{3}{11}$%, (*b*) 25%; £32,000

II. 3 (pages 228–236)—*contd.*

(46) 2 new shares for 5 old; £5, £8 3s. 4d., £21
(47) 5 new shares for every 8 old; 17s. 3d.
(48) £244,000, £186,050; £410 13s. 2d.
(49) £16,610 16s. 3d., £5,721 10s. 1d., £16,241 13s. 8d.
(50) 9 new shares to 8 old; 3 new to 5 old; 27 new to 20 old; 18 new to 25 old
(51) 7% (nearly); $1\frac{3}{4}$% (just over)
(52) 1,821 shares; 45 shares
(53) £171 8s. 7d., £30 9s. 3d.
(54) 27 : 10; £3 10s. 10d., £2 10s. 7d.; £7 17s. 6d., £5 12s. 6d.
(55) 3s. $11\frac{1}{4}$d.; £186,046 17s. 6d.
(56) $144\frac{2}{3}$%; 156·9%
(57) £15,194 11s. 11d.
(58) 1s 4d, 1s. 8d., 2s. 2d., 1s. 5d.; 9,100, 8,700, 8,100, 8,500; £1,070 : 1s. 7d., £960
(59) 38,000 articles, £13,300, £2 3s., £2 10s.; 39,000 articles, £8,287 10s, £2 5s. 3d., £2 9s. 6d.; 35,000 articles, £5,250, £2 8s. 6d., £2 11s. 6d.

III. 1 (pages 250–253)—

(1) (i) 37·25%, 20·38%, 20·48%
(ii) 27·14%, 16·93%, 17%
(2) (i) 30·39%, 14·36%, 14·46%.
(ii) 23·31%, 12·56%, 12·63%
(3) £190 12s 1d.
(4) $17\frac{1}{7}$%
(5) £22 1s 8d., 12%, 10%
(6) 25·17%
(7) $4\frac{16}{21}$%
(8) $5\frac{15}{17}$%
(9) £65 11s. 1d., £29 5s. 11d.
(10) £8 17s. 9d.
(11) £16 7s. 6d.
(12) 7s 1d.
(13)

Dealer's List Price.			Dealer's B.P. for Cash.			Dealer's S.P. for Cash.		
£	s.	d.	£	s.	d.	£	s.	d.
114	15	0	75	4	6	90	13	1
37	16	0	24	15	7	29	17	3
47	5	0	30	19	6	37	6	7
54	0	0	35	8	0	42	13	3

(14) £1,867 16s. 4d.
(15) £59 3s 4d.; 28%
(16) 110 days
(17) 21·26% less
(18) £10 2s. 3d.
(19) £3 18s.
(20) 12th July; £1,758 1s 5d.
(21) 31st Aug.
(22) 14th April; £661 17s. 3d.
(23) (i) 48·8%; (ii) 65·3%; (iii) 61·3%
(24) 5d. per lb.
(25) 7·3%
(26) $19\frac{1}{21}$%
(27) $7\frac{7}{8}$d. per oz.
(28) 20%
(29) 240 tons, $6\frac{1}{4}$% decrease
(30) £23 4s. 3d.
(31) £38 11s. 8d.
(32) £37 16s. 1d.
(33) £7 15s 9d.
(34) £534 15s 2d.
(35) £101 17s.
(36) £8 13s. 9d.

III. 2 (pages 266–269)—

(1) £127 15s. 1d.
(2) 20·44%
(3) £223 7s. 5d.
(4) £309 9s. 4d.
(5) £6 4s. 6d.
(6) £1,617 6s. 8d.; 184·4%; 11s. $3\frac{1}{4}$d.
(7) £1,942 11s. 1d.
(8) £632 13s. 11d.
(9) £479 13s. 6d.
(10) £1,567 13s. 10d.
(11) £1,405 3s. 5d.; £929 12s. 1d.
(12) £5 1s. $7\frac{1}{2}$d.
(13) £2,114 9s. 9d.
(14) £265 8s. 9d.; £36 19s. 5d.
(15) £2,096 17s. 1d.; £17 12s. 8d. less.
(16) £12 13s. 1d.
(17) £1 5s. 10d.
(18) £20 8s. 9d.; £6 8s. 7d.
(19) £94 3s. 4d.; £6 18s. 9d.; £7 12s. 6d.; £401 16s. 6d.
(20) £3 6s. 7d.; 9s. 11d.

III. 3 (pages 283-285 —

(1) 288·731 metres
(2) 10·08 litros
(3) 82·878 wigtjes
(4) (i) 109·73 metres; (ii) 306·4 ch'ih.
(5) 2 centners 3·2 punds
(6) 91 qr. 3 bush. 7·2 gal.
(7) 3 tons 6 cwt. 8 lb.
(8) (i) 5 centners 73·3 punds; (ii) 7 cwt. 1 qr. 22·45 lb.
(9) ·4938; 2·0251
(10) (i) 5·823; (ii) 7·312; (iii) 9·117
(11) 4$\frac{1}{8}$d. per ml.; (ii) 26·20 lire per chilogramma; (iii) 85·09 lire per ettolitre
(12) 5·980, 11s. 0d. per bush.
(13) 6·22 in ; £4 7s. 11d.
(14) £6·6110 per ton; £11 4s. 9d.
(15) £35 9s. 5d.
(16) 294 dollars 49 cents
(17) £9 1s. 4d.; 63 rupees 1 anna
(18) £3 17s. 5d.
(19) 1·56 pence per lb.
(20) 46,195·53 fc.; 26,055·38 fc.
(21) £14 19s.; £26 3s. 10d.
(22) 2 rupees 6$\frac{3}{4}$ annas per yd.; 2 rupees 8$\frac{3}{4}$ annas per yd.
(23) 2 rupees 4$\frac{5}{8}$ annas per yd.; 2 rupees 6$\frac{1}{2}$ annas per yd.
(24) 52,571·55 lire; 53,919·54 lire
(25) £1 5s. 5d. per pair
(26) 86·42 lire per pair
(27) £763 8s. 4d.; 3s. 2$\frac{3}{4}$d. per yd. net; total £736 16s.; 2 rupees 7$\frac{1}{4}$ annas per yd., total: 11,448 rupees
(28) £46 11s 11d.
(29) 33,702,65 escudos
(30) 7,636 rupees 2 annas
(31) 7,583 rupees 13 annas
(32) 7,465 rupees 15 annas
(33) 26·6% greater; 25·7% greater, 23·8% greater
(34) 18·55%

III. 4 (pages 297-300)—

(1) £62 1s. 7d.
(2) £198 9s. 6d.
(3) £156 11s. 7d.
(4) Aug. 21st; £145 1s. 3d.
(5) May 27th; £74 3s.
(6) $405·30
(7) £377 4s. 6d.
(8) 44·8 days; 10·5 times
(9) (i) £174 4s. 0d. plus £11 5s. 4d. in value of stock; (ii) 939 lb; (iii) £96 1s. 10d; (iv) 14·55 days, (v) 12·54 times
(10) £2 8s. 1d.
(11) £566 11s 5d.
(12) £21 0s. 4d.
(13) 129·6 days; 2·8 times
(14) £1,740 5s.; 13·6 days, 9·9 days
(15) £3 2s.
(16) £282 7s. 8d.
(17) $2,340, $2,100, $60, $180
(18) A rec. $11,977, B rec $855; C pays $402·50, D pays $9,627, E pays $2,802·50
(19) £129 1s. loss, £10 3s. loss, £191 8s. gain; £214 12s gain
(20) Loss of £49 15s. 5d
(21) £157 15s.
(22) £4,925; £1,875
(23) £36 0s 11d.; Loss of £11 5s. 9d.

IV. 1 (page 310)—

(1) 127,196 sovereigns
(2) 19s. 9$\frac{1}{2}$d.
(3) £17 2s. 7d.
(4) 4$\frac{3}{4}$d.
(5) 17,967
(6) 132·17 gr., 5·57 gr., 1·39 gr.
(7) 1s. 0·3d.
(8) 2·1923; 52·2 cents; 24$\frac{3}{8}$d.
(9) 153 tons 10 cwt. 24 lb.
(10) £42 9s.
(11) 129·3 cents per oz. tr.; 59d. per oz. tr.
(12) 1·18 gm.
(13) 37 10-gulden pieces
(14) (i) $133·84; (ii) £170 7s. 3d.; (iii) 4757·88 yen; (iv) £4 6s. 9d.
(15) (i) 2152·68 fc.; (ii) $26·05
(16) 2·1%
(17) 80·91 yen

IV. 2 (pages 329-332)—

(1) £33 6s. 11d.
(2) (*a*) $17·61, (*b*) 94·78 francs
(3) £1 11s. 9d.
(4) £28 15s. 4d.
(5) £1,567 18s. 6d.
(6) 4s $5\frac{1}{2}$d.
(7) £65 16s. 3d., £65 13s. 9d.
(8) $2\frac{3}{26}$%
(9) £1 12s. 3d.
(10) £456 18s. 8d.
(11) 3%
(12) £493 11s. $7\frac{1}{2}$d.
(13) 2·52% per annum
(14) £5 1s. 8d ; £5 12s. 9d.
(15) £38 9s. 2d.
(16) (i) £92 3s. 8d.; (ii) £92 2s. 8d.
(17) £1 5s 10d.
(18) 2s. 6d.
(19) 11s $4\frac{1}{2}$d.
(20) £203 17s. 8d.
(21) £570 9s. 4d.
(22) (i) $179·94 ; (ii) 449·81 fc.; (iii) 117 kroner
(23) (i) £788 9s 10d.; (ii) $352·64; (iii) 1586·68 fr.
(24) 3rd Sept., 1932
(25) (i) 3·263 fr. ; (ii) $6·31; (iii) £7 8s. 5d.
(26) (i) £105 1s. 6d.; (ii) £2,171 1s. 6d.
(27) £1,428
(28) £604 3s. 4d.
(29) £3 19s. 4d.
(30) £12,694 19s. 8d.
(31) (i) £38 12s.; (ii) £16 9s. 4d.; (iii) 5·89%
(32) £455 3s. 9d.
(33) £1,167 3s. 1d.
(34) £17,180 15s.
(35) $606·84 ; $822·41 ; $1787·36
(36) 30,792·45 pesetas
(37) (i) £93 5s. 9d.; (ii) £27 14s.
(38) £1 17s. 6d.
(39) (i) 752,874·50 fc.; (ii) 335,087 pesetas; (iii) 593,558·20 lire; (iv) 91,413·33 taels; (v) 83,951·02 yen
(40) (i) £1 13s. 11d.; (ii) £1 5s. 3d.
(41) 1·2%
(42) $\frac{1}{3}$%
(43) 6·21%
(44) (i) £168 10s. 9d.; (ii) £166 7s 5d.
(45) 18·03 kr. per £
(46) £323
(47) 42·83 pesetas per £

IV. 3 (pages 342-345)—

(1) £1,947, £4,218 2s , £5,765 18s. 2d.
(2) £1,886 10s., £4,092 12s., £5,568 16s 11d.
(3) £2,863, £2,398 8s. 6d., £513 2s.
(4) £2,555, £2,264 12s. 6d., £486
(5) (i) £570 14s ; (ii) £1,526 14s. 4d.
(6) (i) £25 13s. 8d.; (ii) £45 16s.
(7) (i) £4 11s. 9d.; (ii) £4 11s. 7d.
(8) (i) 778; (ii) 924
(9) (i) £38 18s.; (ii) £55 8s. 10d.
(10) (i) £5 6s.,; (ii) £5 11s.
(11) 625 shares; £375, £5
(12) £3,425, £3,610 10s.
(13) 687
(14) £33 17s. 5d.
(15) $79\frac{5}{8}$
(16) $82\frac{1}{4}$
(17) £1,244 16s. 3d.
(18) $91\frac{1}{2}$
(19) ·23% 2·30%; ·61%; 2·44%
(20) £30 15s.
(21) Salt Union 7%
(22) £48 8s. 6d.
(23) £14,533 6s. 8d.
(24) 18%, £12 shares at $41\frac{1}{2}$; 7%, £5 shares at $7\frac{3}{16}$; 5%, £1 shares at 21s. 9d.; $4\frac{1}{2}$%, £10 shares, $£4\frac{1}{2}$ paid at $4\frac{5}{8}$
(25) £1,069 1s. 3d.
(26) $17\frac{1}{16}$% (to nearest $\frac{1}{16}$th)
(27) 667 shares
(28) 3,015; £116 5s. gain
(29) 18s. $10\frac{3}{8}$d.
(30) £884 10s. 5d.; £814 6s. 10d.
(31) £1,717; £843
(32) £4 16s. 6d.; £262 5s. 2d.
(33) $95\frac{15}{16}$
(34) £4,572 14s. 9d.
(35) £1,398 10s. 10d. ; £56 5s. , £4 0s. 5d.
(36) £7,452 15s.
(37) £312 10s.
(38) £4,156 5s.
(39) £5,680 4s. 6d.
(40) $55\frac{7}{11}$
(41) £24,387
(42) $12\frac{1}{2}$%; £4 18s. $0\frac{1}{2}$d.
(43) £7,506, £3,015; £7,200, £3,600

IV. 4 (pages 357-360)—

(1) £2,398 17s.; £2,506 5s. 3d.
(2) 5·0625%
(3) 7·7706%
(4) 5·41%
(5) 5·43%
(6) 10·26%
(7) 25th Dec., 1935
(8) 4·93%
(9) (a) £3,042 12s. 5d.; (b) £2,792 3s. 2d.; (c) £3,103 9s. 5d.; (d) £2,799 1s. 11d.
(10) (a) £1,136 19s. 9d.; (b) £1,176 16s.; (c) £4,582 10s.; (d) £888 5s. 10d.; (e) £598 10s. 8d.
(11) (a) 11·94 years' purchase; (b) 12·30 years' purchase; (c) $33\frac{1}{3}$ years' purchase
(12) 29 years
(13) £406 1s.
(14) £26 8s. 3d.
(15) £835 14s. 3d.
(16) (i) £851 7s. 8d.; (ii) £841 6s. 7d.
(17) £13,297 14s. 3d.
(18) £1,287 14s. 10d.
(19) £1,085 12s. 7d.
(20) 24·01 years; 27·87 years
(21) £16 17s. 1d.; £25 1s. 7d.
(22) (i) £35 11s. 2d.; (ii) £56 7s. 8d.
(23) (i) £20 19s. 11d.; (ii) £75
(24) £2,022 3s. 5d.
(25) £2,290 9s. 7d.
(26) (i) 1,594 dollars; (ii) 18,760 fr.
(27) £1,610
(28) £47 14s. 11d.
(29) £4,013 11s.
(30) At end of 14 yr., £25,629 0s. 10d. is available
(31) 13 yr. 298 dy.
(32) £30 4s. 8d.
(33) £703 9s.

V. (pages 368-370)—

(1) £52 11s. 8d.
(2) 3s. $3\frac{1}{2}$d. (to nearest $\frac{1}{4}$d.)
(3) £395; £1 9s. 2d.
(4) £4,770; £34 11s. 8d.
(5) £2,162
(6) £1,712 6s. 7d.
(7) 19·6%
(8) £338 14s 2d.
(9) £435 1s. 9d.
(10) £69 12s. 3d.; £365 9s. 6d.
(11) £435 1s. 9d.; £733 6s. 8d.
(12) £5,910; £3,150, £1,342 10s.; £1,556 5s.; £825; £716 5s.; £3,240; £6,000; £1,425; £2,075; £760; nil. A's underwriter receives proceeds of the damaged goods
(13) £1,342 10s.; £1,556 5s.; £825
(14) £2,677 10s.
(15) £256 10s 9d.
(16) (i) Nil; (ii) £45 15s. 8d.; £63 7s. 10d.
(17) £33 18s. 3d.
(18) £38 15s. 5d.
(19) 20·456%, 13·136%, 7·961%
(20) £29 18s. 6d.

Miscellaneous Questions (pages 371-390)—

(1) £2 2s. $3\frac{1}{4}$d.
(2) £171, £516; 172, 486; £158, £484; 45, 165
(3) 8s. 5d.
(4) 11·4
(5) 2·75%, 5·22%
(6) $7\frac{1}{2}$%
(7) 3s. 4d.
(8) £160
(9) £9 1s. 8d.
(10) £53 0s. 11d.
(11) £94 6s. 2d., £90 14s. 9d.
(12) 24′, 36′
(13) £316 17s. $3\frac{3}{4}$d.
(14) 12 hours
(15) £236 0s. 3d.
(16) 1,144 lb.
(17) 51 herrings
(18) 2%
(19) 370 rupees 6·4 annas
(20) 46·4 ac.
(21) 8,000,000 tons
(22) 5 ac. 1 rd. 15 p.
(23) £287 1s. $7\frac{1}{2}$d.
(24) £352 10s. 2d.
(25) 1s. 2d., 30%, $42\frac{6}{7}$%
(26) 186 %, 156 %, 100 %, 96% ; 973,000 sq. ft.
(27) ·72 acre
(28) $\frac{5x}{28}$ sh. per lb., $\frac{15x}{112}$ pence per oz.
(29) $227\frac{3}{11}$ ft.

Miscellaneous Questions (pages 371–390)—*contd.*

(30) £3 17s. 10d.; £2 19s. 8d.
(31) (*a*) 1·730236, (*b*) ·03475
(32) { 5 ac. 1 rd. 29 sq. pl.; £51 11s. 6d.
(33) 8s. $10\frac{1}{4}$d.
(34) 14·7 years
(35) £1 6s. $9\frac{3}{4}$d.
(36) (*a*) £474 4s. 2d.; (*b*) 110 francs 52 centimes; (*c*) 408 dollars $97\frac{1}{2}$ cents
(37) Cheaper
(38) 2s. $2\frac{1}{4}$d. in the £
(39) £292
(40) £487 3s. 10d.
(41) 9·84
(42) £966,923 1s. 1d.
(43) £1,315 6s. 8d.; £2,156 8s. 11d.; £295 17s. 4d.
(44) 40,873·60 fc.; 601,293·87 fc.
(45) £12 2s.
(46) 24%
(47) £305 5s.
(48) 10 oz. tr. 2 dwt. 15 gr.
(49) 2s. $2\frac{1}{2}$d. per lb.
(50) 2s. $0\frac{3}{4}$d.
(51) 4%
(52) £39,445 8s. 5d.
(53) 3·79 litres
(54) £4 0s. 6d.; 1·27%
(55) 2·7 acres
(56) $16\frac{7}{8}$%
(58) (i) £2,777 15s. 7d.; (ii) £2,776 10s. 6d.
(59) £3,938; £2,506, £1,074
(60) £11 12s. 1d.
(61) (i) Jones rec. £11,266 13s. 4d., Smith rec. £266 13s. 4d., Brown pays £2,733 6s. 8d.; (ii) Jones rec. £8,840, Smith rec. £1,200, Brown pays £1,240
(62) 13s. 10·05d.; £7,278 10s. 6d.
(63) Loss of 13s. 6d.
(64) 136′ 9″; 97′ 1″ (to nearest inch)
(65) 62·3
(66) £4 1s. $2\frac{3}{4}$d.
(67) £7 19s. 9d.
(68) 65%
(69) 2s. 6d.
(70) 150% per annum
(71) £1,585 12s. 4d.
(72) £2,162 14s. 5d.; £1,042 2s. 9d.
(73) £4,500
(74) $17\frac{1}{3}$%
(75) £13 15s.; £18; £15 15s.
(76) (*a*) £7; (*b*) £12 10s.
(77) 25 tons 7 cwt. $17\frac{1}{4}$ lb.; £228 17s. 8d.
(78) Only water added; 2s. $11\frac{3}{4}$d.
(79) 2′ 10·715″
(80) £3 7s.
(81) £2 4s. 7d.
(82) 23·24 lb.
(83) £425
(84) 1′ 8″
(85) £1,837 15s 7d.
(86) 6 months
(87) £2 7s. 4d.
(88) £900
(89) £6 18s.
(90) £207 12s. 6d.; £51 18s. 2d.; £4 3s 1d.
(91) $5\frac{5}{12}$%
(92) $5\frac{5}{16}$%; 6s. 4d.
(93) ·015″
(94) £46 6s. 11d.
(95) 120%
(96) £462 8s. 8d.
(97) 2·19″
(98) $4\frac{1}{4}$%
(99) 5s. 8d per lb.
(100) $2\frac{1}{2}$ times as much
(101) £4,888 2s. 6d.; 5·73%
(102) 3 min. 8 sec.
(103) £58 4s 6d.
(104) 1s. $1\frac{1}{2}$d per lb.; £3 4s. per cwt.
(105) £15,680; 2,150,400 of each; £1,444
(106) $8\frac{1}{3}$%, nil; 10%, nil; $12\frac{1}{2}$%, nil; $5\frac{1}{8}$%, 9·86%
(107) A – £1,094 8s, B – £729 12s., C – nil
(108) £80
(109) £27,000, £30,000, £109,375; 17·5%
(110) $13\frac{8}{9}$%
(111) £400 19s. 2d.
(112) £1,494 1s.
(113) £112,500; $4\frac{1}{2}$%
(114) £344 15s. 6d.
(115) 5s. $0\frac{9}{16}$d. per yd.; £396 3s. 7d.; 3 rupees $14\frac{1}{8}$ annas per yd.; 6,096 rupees
(116) (i) 750 lb.; (ii) $487\frac{1}{2}$ lb.
(117) (*a*) Gain of £21 14s. 10d.; (*b*) Loss of £23 16s. 2d.; (*c*) Gain of £25 18s. 5d.
(118) £100 9s. 3d.
(119) 3,040 articles, 17s. 5d., 22s. 3d.
(120) 18s. 6d.
(121) $23\frac{1}{4}$%
(122) 43·39 ft.
(123) £63 16s. 10d.; £65 12s. 6d.
(124) 85·50 fc.
(125) 60,553 fc.

Miscellaneous Questions (pages 371–390)

(126) 304,326·48 fc
(127) £533 19s. 11d.
(128) £2 8s. 5d.
(129) (a) 53·703; (b) 64·665
(130) 23rd Feb.; £119 18s. 4d.
(131) 280 lb.
(132) £2 2s. less; £12,600
(133) £498 18s. 10d.
(134) 7·3%
(135) 17·5%; £30 6s. 4½d.
(136) £4,085, £3,240
(137) 82·29 fc.
(138) £25 5s. 6d.
(139) 30th June; £975 6s. 6d.
(140) 16·07%; 19·15%
(141) 7th Sept.
(142) 12,254·86 fc.; 61 days; 4½%
(143) 43 c. in.
(144) £1,434 15s. 3d., £1,030 0s. 4d.; £1,690 15s. 3d., £1,630 0s. 4d.
(145) 24·25%
(146) 15s. 4d.
(147) £560
(148) $644·32
(149) 35·85%; 45 strokes
(150) 20%; 50%
(151) 136 tons 8 cwt. 4 lb.
(152) 8 dwt. 7·2 gr.; 1 dwt. 20·5 gr.; £4 4s. 8¾d.
(153) £819
(154) £14,400
(155) £73
(156) £203 2s. 6d.
(157) 20·436 gm; £1 9s. 7d.
(158) 28⅓%; £53 10s.
(159) £3,966 15s.
(160) £151,300, 42½%; 13·2%
(161) 6⅛%, £12,000
(162) £38,200; 9%
(163) 162⅝
(164) 3s. 11d.
(165) £95 16s. 1d.
(166) £24,303 13s. 5d.
(167) £154 8s. 9d.
(168) £64,273 7s. 9d.; 5% 1st half-yr. 7½% 2nd half-yr.
(169) £2,490 17s. 1d.
(170) £69 13s. 5d.
(171) 28 years 4 days
(172) £50; 18·52%
(173) 1s. 10·8d.; 2s. 1·44d.; 2s. 8·7d.; 56⅔%
(174) £55 0s. 7d.
(175) 3$\frac{11}{13}$%; 4·6%
(176) £132 8s. 7d.
(177) £1,558 10s. 8d.
(178) (i) 14·90; (ii) 18·6
(179) £100,000; £2,340 4s. 1d.; £2,500
(180) £366
(181) £417 18s. 10d.
(182) £3,380, £3,125
(183) £103 12s. 6d.
(184) £16 13s. 4d.
(185) 808 payments
(186) £2,198 6s. 4d.
(187) 61 years (last payment approx. £902)
(188) £1,839,547
(190) £16 0s. 7d.
(191) £338 17s. 5d.
(192) £2,438 9s. 2d.; £623 9s. 1d.; £78 13s. 2d.; £209 15s. 3d.; £1,019 13s. 4d.; £279, £308
(193) £7 19s. 11d.; 82·04¼ fc = £1
(194) (a) 5·0945%; (b) 4·9044%
(195) 10 cwt. 10 lb.; £10 3s. 9d.; 1·365″
(196) (i) £35 17s. 6d.; (ii) £16 9s. 8d.; 8·69%, 3·99%
(197) £38 6s. 1d.
(198) Cheaper machinery; 5·26, 4·93
(199) 190 days
(200) £7,022 6s. (to nearest shilling)
(201) £20,731
(202) £373 13s. 3d.
(203) 16 years 283 days, by setting aside £2,000 at end of first 16 years
(204) £60,581 4s. 5d.
(205) £644 1s. 6d.